ADVANCED MATHEMATICS
FOR
ELECTRONICS TECHNICIANS

Prepared by

U.S., **BUREAU OF NAVAL PERSONNEL**

NAVY TRAINING COURSES

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PREFACE

This book has been prepared for the use of enlisted men of the Naval Reserve who seek training and advancement in ratings involving electronics. It is a companion volume to *Essentials of Mathematics for Naval Reserve Electronics*, NavPers 10093. Carrying forward the introduction to mathematics provided in the earlier book, this volume includes practical applications of geometry, mechanical drawing, graphic representation, trigonometry, and vectors in the fields of electricity and electronics. It concludes with an elementary treatment of the processes of integration and differentiation in calculus and an explanation of how these processes may be accomplished by electronic circuits.

A pretest appears near the beginning of each chapter of the book. Performance on these pretests will show the extent to which the reader needs to study the chapter. Intensive study of the material covering items correctly answered in the pretest may be omitted. The glossary-index provides short definitions of technical terms, and valuable mathematical tables are found in appendix D.

Acknowledgment is accorded to Dr. Ralph Shorling, whose teaching methods and procedures are incorporated in the Navy Training Courses *Mathematics*, Volume 1, NavPers 10069, and *Mathematics*, Volume 2, NavPers 10070, upon which this text is patterned. Acknowledgment is likewise accorded to the World Book Company for the use of materials found in its publications; to the Federal Telephone and Radio Corporation for permission to use material from the *Reference Data for Radio Engineers*,

3d edition; to the Radio Corporation of America for permission to use material from the *RCA Receiving Tube Manual;* and to The American Radio Relay League, Inc., for permission to use material from *The Radio Amateur's Handbook,* 23d edition.

As a NAVY TRAINING COURSE especially adapted for the Naval Reserve, this book was prepared by the Navy Training Publications Center of the Bureau of Naval Personnel.

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CHAPTER 1

USING ANGLES TO TELL DIRECTION

Every day you see an angle of some kind; it may be on the gable of a house, the grill of an automobile, or the place where two streets intersect. Nature herself has many natural angles—a tree can be at right angles to the earth, and the flat surface of a stone can be at any angle to another stone. The sun, moon, and stars form angles with the earth. Many interesting problems can be solved through a clear understanding of angles. For example, a surveyor can find the distance across a river, the height of a tree, and so forth. By the use of angles a navigator can determine his position on the earth's surface.

Like its companion volume, NavPers 10093, this book is designed for self-teaching. The method of attack is to take the pretest at the beginning of each chapter.

If you get 75 percent or more of the problems of a pretest right, you probably know the subject well enough to skip the rest of the chapter and go on to the next pretest.

If you get less than 75 percent of the problems right—

1. Study the rest of the chapter.

2. Take the pretest again; if you pass it, turn to the next pretest.

3. If you still have difficulty with the pretest, work the corresponding problems in appendix A, Remedial Work.

4. Take the pretest again; by then you should be able to work all the problems.

PRETEST 1

1. (a) The size of an angle depends upon the .... of a line from a stationary line about a fixed point.

(b) A circle is divided into nine equal parts. Each part contains .... degrees.

(c) How many degrees are generated by a turn of $\frac{2}{3}$ of a circle; $\frac{5}{6}$ of a circle; $\frac{2}{5}$ of a circle?

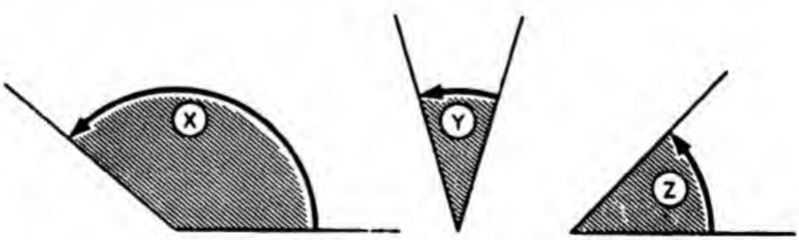
(d) What part of a circle is a turn of 270°; 60°; 30°?

(e) An angle less than 180° but greater than 90° is an .... angle.

(f) From 58° 46′ 29″ subtract 31° 50′ 48″.

(g) Change the following from degrees and minutes to degrees and their decimal parts: (15° 30′), (20° 12′), (25° 27′).

(h) Using the protractor, measure the magnitude of the following angles:



(i) A ship is on course 300°. The ship turns right 80°. What is the new heading?

(j) Your ship is on course 215° true. Radar reports a plane at 40° relative. What is the true bearing of the plane?

(k) A submarine is on a course of 240° true. The relative bearing of a tanker to the submarine is 140°. What is the true bearing of the tanker?

(l) Heading on a course 350°, a ship makes a positive turn of 45°. What is the true bearing of the ship's new course?

(m) The bearing of an object with respect to .... is a true bearing.

(n) The bearing of an object with respect to the ship's .... is a relative bearing.

(o) The bearing of a lighthouse to a ship is 300° true. If the ship is on course 060°, what is the relative bearing of the lighthouse to the ship?

STUDY GUIDE ON USING ANGLES TO TELL DIRECTION

2. When a line turns from one position to another an angle is formed. The size of the angle is determined by the amount of turning necessary to make one position of the line coincide with (exactly fit) the other position.

Figure 1.

Figure 1 illustrates a pocketknife partly opened. The blade has turned from its original position in the case, pivoting on the hinge end, which is the fixed point. The distance the blade turns determines the size of the angle which is generated. You can see from this figure that the length of the blade has nothing to do with the size of the angle.

3. Figure 2 shows an angle. Points are designated by letters, *A*, *B*, and *C*. You read one line as *AB* and the other as *AC*. These two lines are called the SIDES of the angle. Point *A*, where these sides meet, is called the VERTEX of the angle.

4. The symbol for an angle is $\angle$. The angle in figure 2 can be referred to in three ways: $\angle BAC$, read angle BAC; $\angle$A, read angle A; or by placing a small letter inside the angle at the vertex as $\angle x$, which is read angle x. When using the three letters, the letter at the vertex of the angle must be placed in the middle.

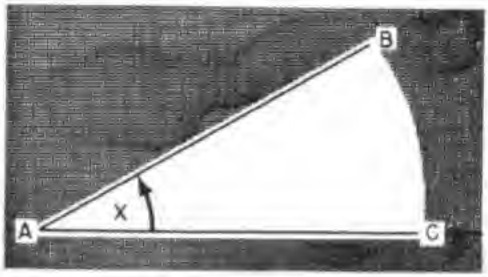


Figure 2.—An angle.

5. The major units used to measure angles are called DEGREES. The symbol for degree is a small circle written above and to the right of the numerical value. A circle, or one complete revolution of a line, contains 360 degrees, written 360°. Therefore, one degree is equal to 1/360 of a complete revolution. Figure 3 is a picture of a hinge. With the hinge closed there is no angle but as the hinge turns to position 1 it has moved one quarter of a circle or 90°. In position 2 it has moved half the distance of a circle or 180°. You might think that since a circle contains 360°, the largest angle that could be generated would be 360°. However, the rotating line doesn't have to stop at 360°. In figure 4 the line has rotated once completely and then 40° more. The size of the angle is 400°. A Mk 37 gun director may turn through 740° before it must be reversed.

6. The four kinds of angles most widely used are shown in figure 5. A RIGHT angle is an angle of 90°. A STRAIGHT angle is an angle of 180°. Any angle less than 90° is an ACUTE angle and any angle greater than 90° but less than 180° is an OBTUSE angle.

7. Angles are read in degrees, minutes, and seconds. A degree is divided into 60 minutes and a minute into 60 seconds. These also have symbols; 30 degrees 25 minutes

30 seconds is written 30° 25′ 30″. To add or subtract angles, degrees are added to degrees, minutes to minutes, and seconds to seconds. For example—

$$\begin{array}{r} 24^\circ\ 30'\ 30'' \\ 14^\circ\ 45'\ 50'' \\ \hline 38^\circ\ 75'\ 80'' \end{array}$$

However, this answer should now be simplified. The 80 seconds equals 1 minute 20 seconds. Carrying the 1 minute

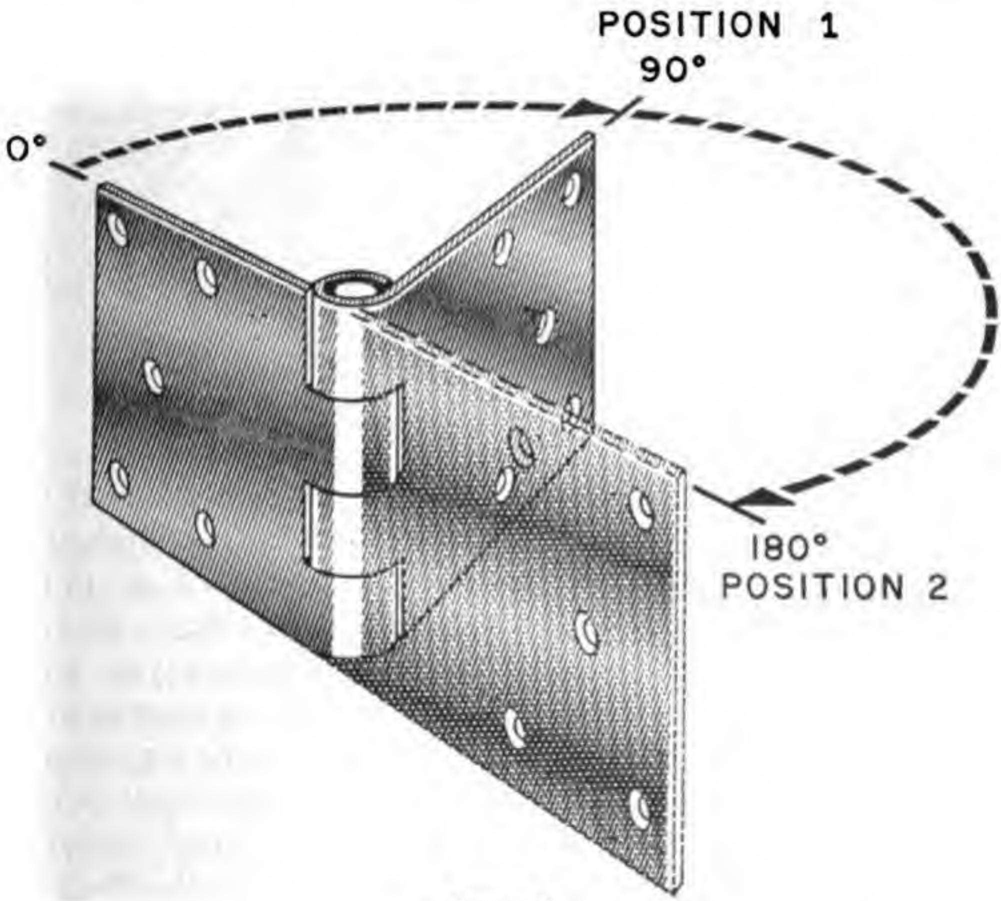


Figure 3.

over you get 76 minutes 20 seconds. Also the 76 minutes is equal to 1 degree 16 minutes. Again, carrying the degree goes over to the degree column, the completed answer is 39° 16′ 20″. Review this problem to be sure you understand the method used for obtaining the answer in its simplest form. In the preceding addition problem you

found it necessary to carry over units. In the process of subtraction, however, you may have to borrow units. Consider the problem:

$$\begin{array}{ll} \text{From} & 72^\circ\, 15'\, 32'' \\ \text{Subtract} & 60^\circ\, 18'\, 20'' \end{array}$$

The seconds can be subtracted using simple arithmetic. However, 18′ cannot be subtracted from 15′ without getting a negative answer. By borrowing 1° (which is 60′) from the 72° you may write the minuend as 71° 75′ 32″. Now subtract 18′ from 75′ and 60° from 71°. The completed answer is 11° 57′ 12″.

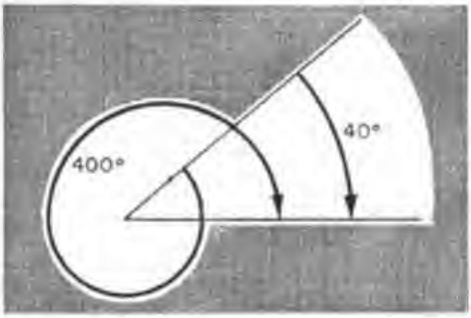


Figure 4.—Rotating line.

8. In electronic work the decimal system is used to express fractional parts of a degree instead of minutes and seconds. For example, 45° 54′ can be expressed as 45.9° because 54 minutes is $\frac{9}{10}$ of a degree. Also 20°45′ = 20.75° since 45′ is 75 percent of a degree. Suppose you had 30°40′30″. You know there are 60 minutes in a degree and 3,600 seconds in a degree. Disregard the 30° for a moment and write out the minutes and seconds as fractional parts of a degree, $\frac{40}{60}$ and $\frac{30}{3600}$, or $\frac{2}{3}$ and $\frac{1}{120}$. Using the lowest common denominator, which is 120, you write $\frac{80}{120} + \frac{1}{120} = \frac{81}{120}$. By expressing this fraction as a decimal you

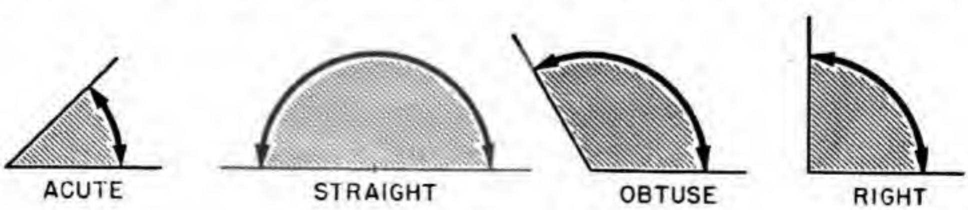


Figure 5.—Four kinds of angles.

may write $30^\circ 40' 30''$ as 30.675°. Here is another example —convert $20^\circ 16' 40''$ to decimal form.

SOLUTION: $20^\circ + (\frac{16}{60} + \frac{40}{3600}) = 20^\circ + \frac{4}{15} + \frac{1}{90} = 20^\circ + \frac{25}{90} = 20.278^\circ$. In these examples you have worked the answers out to five significant figures. If the answer to the last example is rounded off to three significant figures, the angle becomes 20.3°.

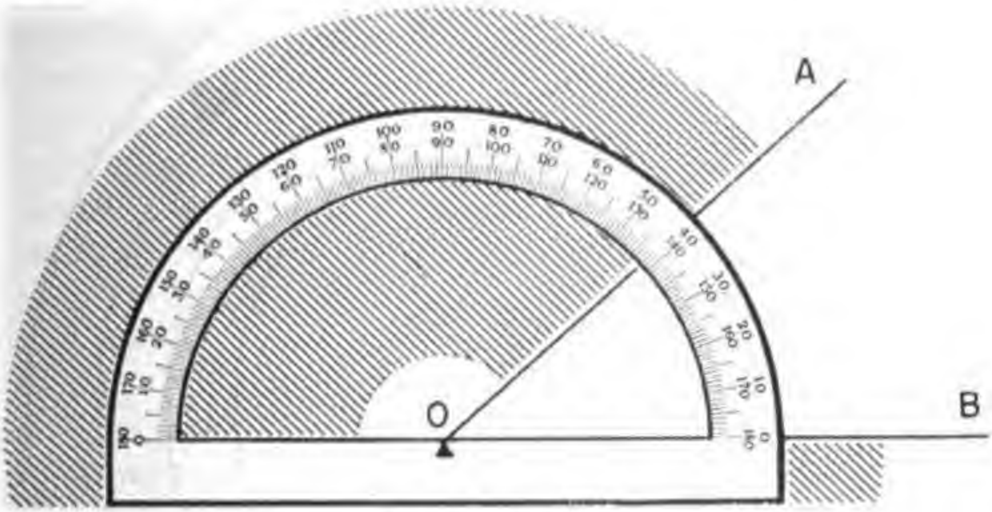


Figure 6.—Protractor.

9. A PROTRACTOR is an instrument used to measure angles. (See fig. 6.) The semicircular edge is marked off in degrees from zero to 180. Notice that it has two scales for convenience, depending upon where the stationary side of the angle rests. To measure an angle, the center of the protractor is placed exactly at the vertex of the angle and the zero of the scale is placed on one side of the angle. The protractor in figure 6 is being used to measure an angle of 42°.

10. If you take two protractors and place them together on a flat surface, the straightedge of one against the straightedge of the other you will have a circle divided into 360°. If you imagine yourself in the center, with the zero point directly North, you can accurately describe the direction of any object within sight by the degree mark over which the object is sighted. If true north is used as the starting or zero direction in such measure-

ment, the bearings which result are called TRUE BEARINGS. This is the principle of the PELORUS used in navigation. The pelorus is simply a dumb-compass (compass without magnets) card. They are located at various places aboard ship to take bearings on objects that cannot be seen from

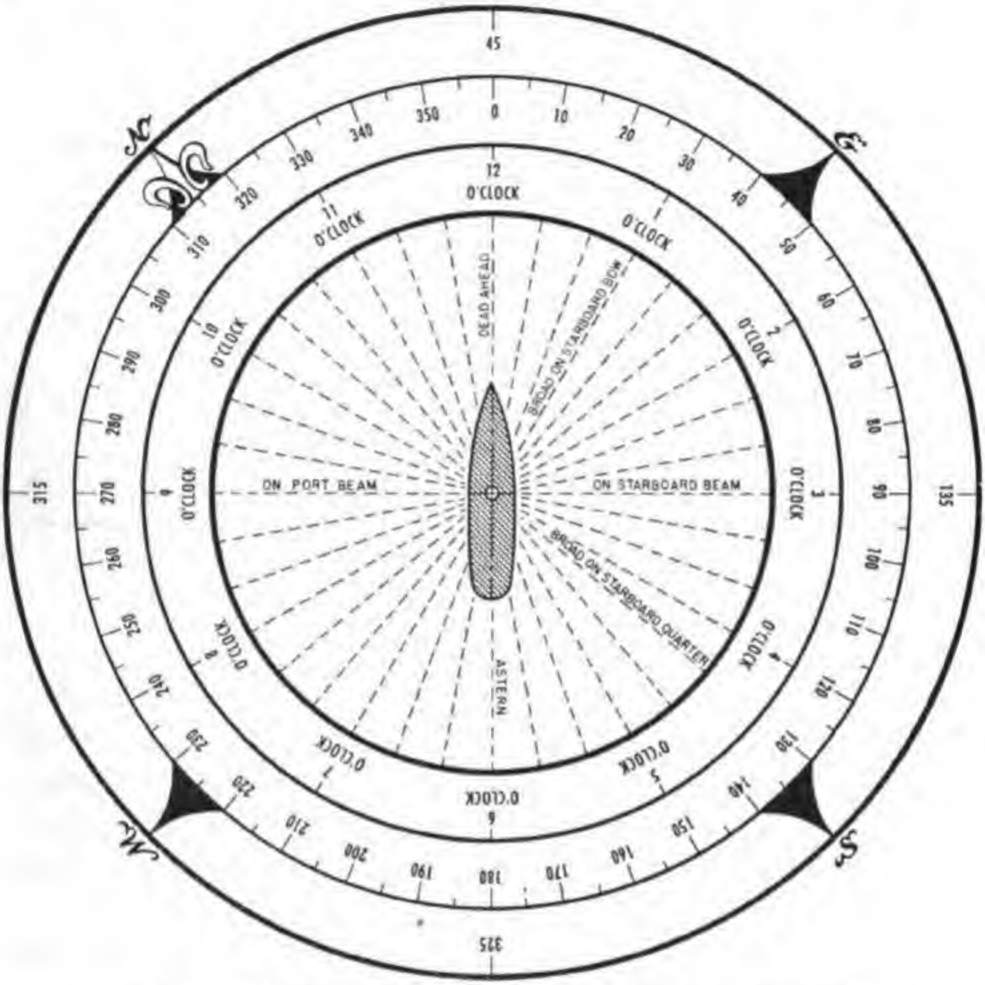


Figure 7.—Methods of designating true and relative bearing.

the compass position. Frequently it is not convenient to use true bearings, and the zero point of the pelorus card is pointed dead ahead on the center line of the ship and secured. Then the bearings taken on the pelorus are termed RELATIVE BEARINGS. The outside ring, shown in figure 7, is similar to a standard Navy compass card. It is marked off in degrees from zero to 360 in a clockwise direction beginning at North. In navigation, angles are

usually measured in a clockwise direction. This contrasts with the practice in mathematics and in alternating-current theory of measuring angles in a counterclockwise direction.

11. The compass card is divided into four main parts separated by CARDINAL POINTS. These points are North at 0°, East at 90°, South at 180°, and West at 270°. Halfway between the adjacent cardinal points are the intercardinal points NE, SE, SW, and NW. Locate these points on the card shown in figure 7 and note how many degrees each signifies. The angle formed between an imaginary line due north from a ship's position and another line in the direction the ship is moving (ship's course) is called the true bearing of its course. This angle is always measured from the true-north direction and is the number of degrees of clockwise turning necessary to reach the line of the ship's motion. Thus, the true course of a ship sailing from point *O* on course *A*, as shown in figure 8, is 030°; from *O* on course *B* is 130°; and from *O* on course *C* is 290°. These angles are referred to as true bearings. Bearings are always read with three numerals. For example, where the ship's course forms an angle of 30° with North (on course *A*) you refer to the ship's course as bearing 030° T (read ZERO THREE ZERO DEGREES TRUE). A vertical line marked on the compass bowl always indicates the direction of the ship's bow. Such a line is called a "lubber's line" of the compass. The gyrocompass usually reads directly in true bearings. That is, 000° on the compass always points directly to true north. Magnetic compass bearings were used before the invention of the gyrocompass and were always subject to errors which had to be corrected to an equivalent true-bearing value.

12. Relative bearings, mentioned above, is another term with which the electronics technician must be familiar. This uses the "ship's bow" (the ship's center line) as a reference line instead of true north. A scale of relative bearings in degrees is given in figure 7, using the second

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ring from the outside. Thus, a bearing of 090° relative is 90° right of the ship's heading, 180° relative is directly astern; and 315° is 45° left of the ship's heading. Relative bearings are used by gunnery crews for target designation since contact with the target can always be maintained relative to ship's bow regardless of how much the ship turns. Also, if the gyrocompass is damaged in battle,

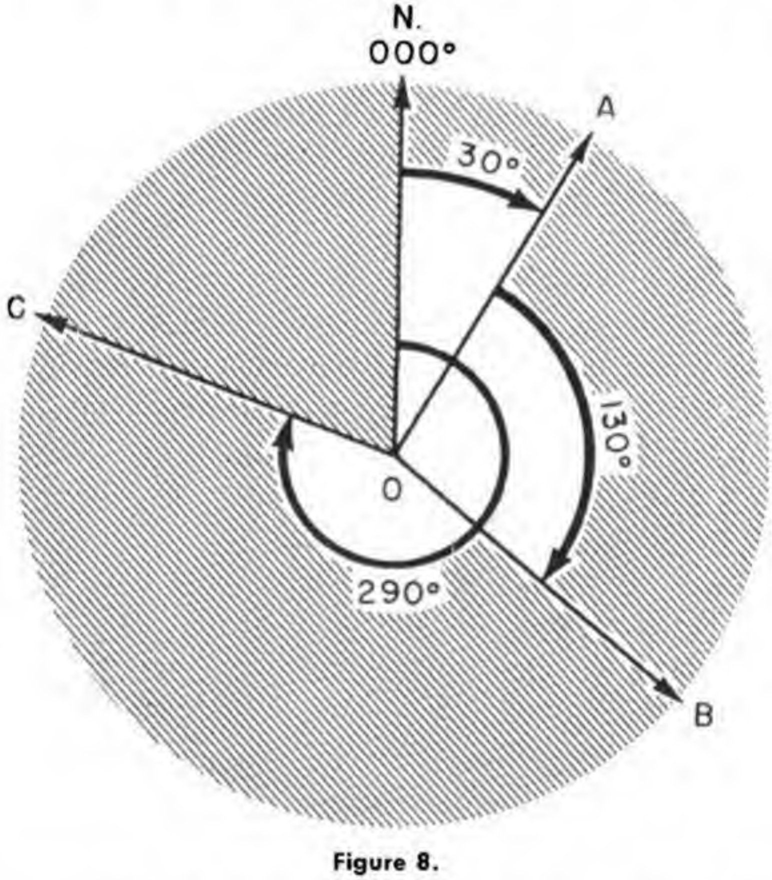


Figure 8.

targets can still be designated by relative bearings in degrees although no true bearings are available. Radar indicators usually show both relative and true bearings of targets. Two other systems of reporting relative bearings are in common naval use. These are the "point" system and the "clock" system. Lookouts usually report the location of anything they sight by relative bearings given in "points." See the inside ring shown in figure 7. The compass card is divided into 32 points each $11\frac{1}{4}$° apart with

000° relative always dead ahead. Therefore, an object sighted with a relative bearing of 135° is reported as being "broad on the starboard quarter." Aircrewmen use the "clock" system of reporting an object's relative position to their plane. It is based on the position of figures on a clock face. See the second ring from the inside shown in figure 7. If another plane is approaching at a relative bearing of 120°, the report will be "plane coming in at 4 o'clock." This method is easily learned and remembered and is sufficiently accurate for aircrew use.

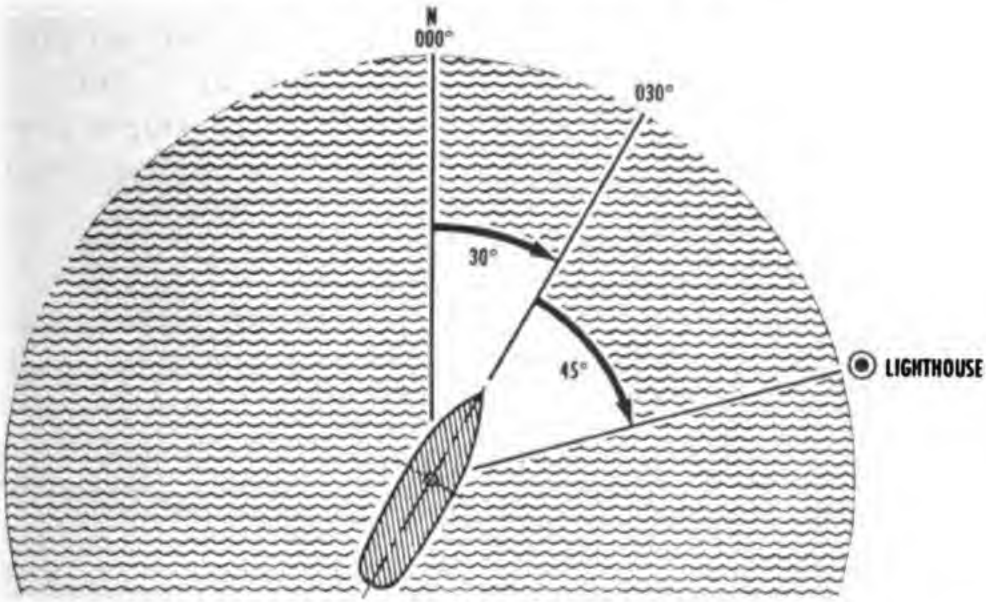


Figure 9.

13. In figure 9 we see a ship whose heading is 030°T. Across the center of the compass card, the line-of-sight to a lighthouse off the starboard bow of the ship forms an angle of 45° with the center line (or lubber's line) of the compass. The lighthouse is then bearing 045° relative. What is the true bearing of the lighthouse from the ship? When you measure true bearings the reference line or stationary side of the angle is a line pointing toward true north. You can see in figure 9 that to a person on the ship, the lighthouse bears 75° from true north, which also could have been computed by adding the heading of the ship and the relative bearing of the lighthouse from

the ship. Because a clockwise turn in navigation numerically increases a ship's course, it may be mathematically interpreted as a positive turn. A counterclockwise turn numerically decreases a ship's course and may be interpreted as a negative turn. For example, if a ship is heading 030° and then turns clockwise 30°, a positive turn results and the new course is 060°. Now, if the course is changed counterclockwise 50° from the original course of 030°, a negative turn results (fig. 10). To find the new course, you must subtract 50° from 30°. Since 50° is a larger angle than 30°, the rule is WHEN SUBTRACTING A LARGER ANGLE FROM A SMALLER ANGLE, ADD 360° TO THE SMALLER ANGLE AND THEN SUBTRACT. Thus, $30° + 360° = 390°$. Now, $390° - 50° = 340°$, which is the ship's new course. Positive and negative turns are NOT navigational terms.

DRAWING ANGLES

14. Neatness and accuracy are important when drawing lines and angles. You should know the basic rules of

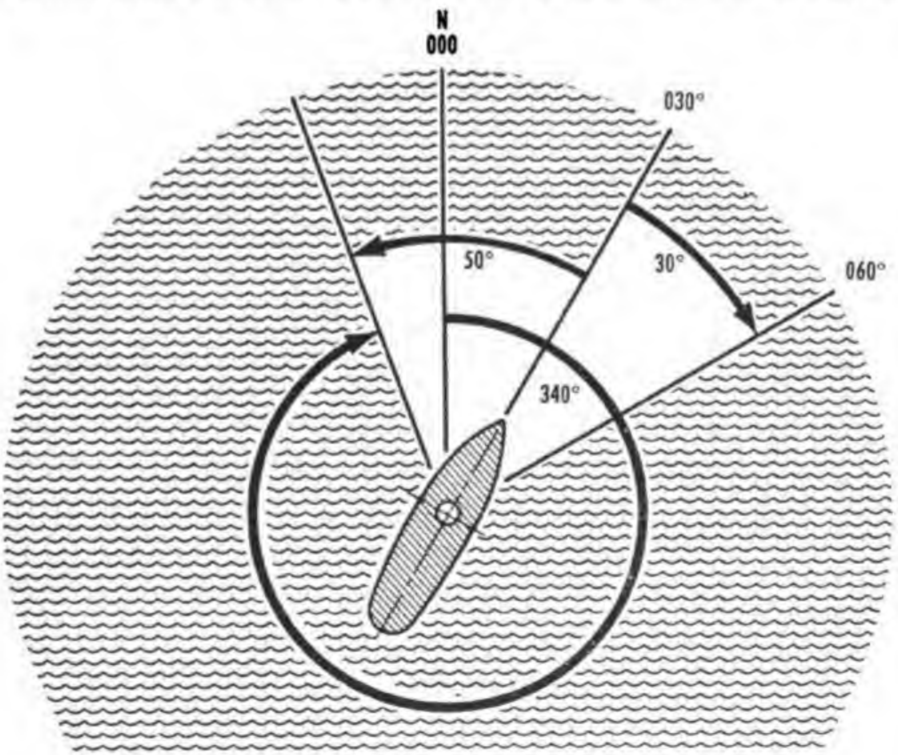


Figure 10.

drawing and be familiar with certain drawing equipment. As a guide in drawing horizontal lines a draftsman uses a T-square, shown in figure 11, a. The head of the T-square is constructed so it can be lined up accurately with the left side of the drawing board. The left side of the drawing

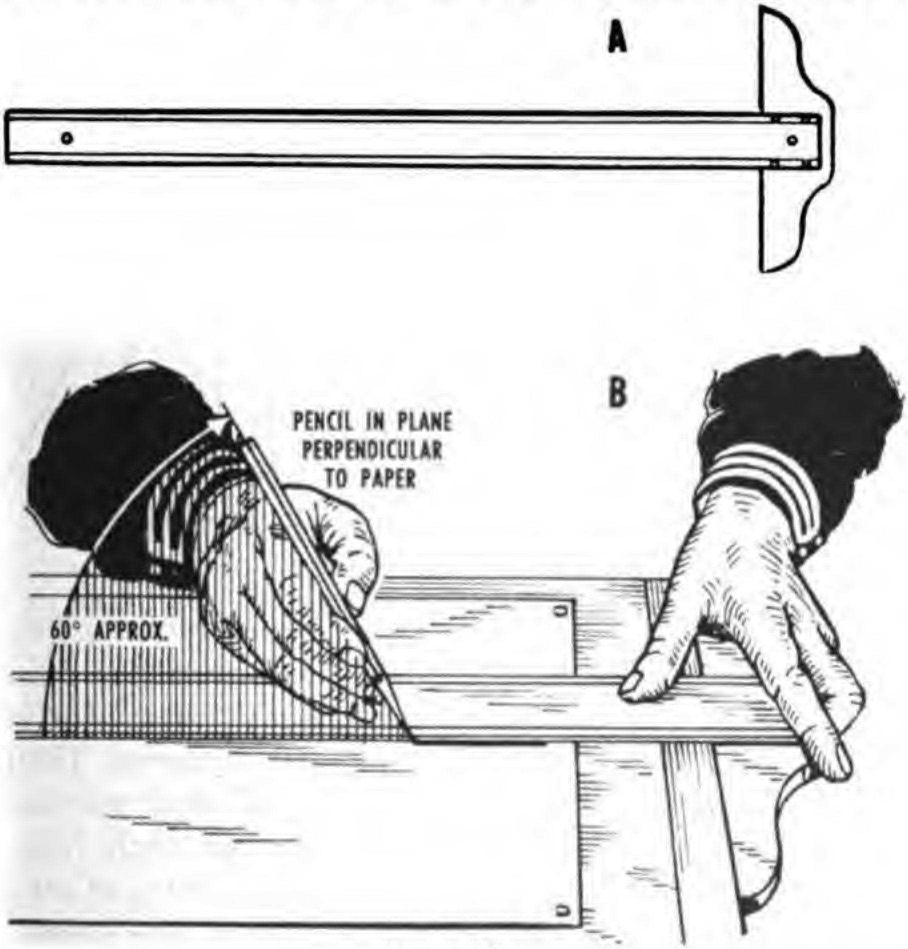


Figure 11.

board must be perfectly straight to make a true horizontal line. Avoid using your ruler as a guide unless it is equipped with a special edge for that purpose. Use a well-sharpened pencil and hold it close to the point. Incline the pencil slightly to your right (fig. 11, b) and move it from left to right. Press just hard enough to produce a good line because it must never be retraced. Vertical lines are drawn with triangles (shown in fig. 12) used in conjunction with the T-square. The 45° triangle shown on the left has an angle of 90° and two angles of 45°. The triangle on the

right is a 30°–60° triangle and has angles of 30°, 60° and 90°. Many other angles can be drawn by combining the two triangles. However, with practice, work that is just as accurate can be done with a protractor.

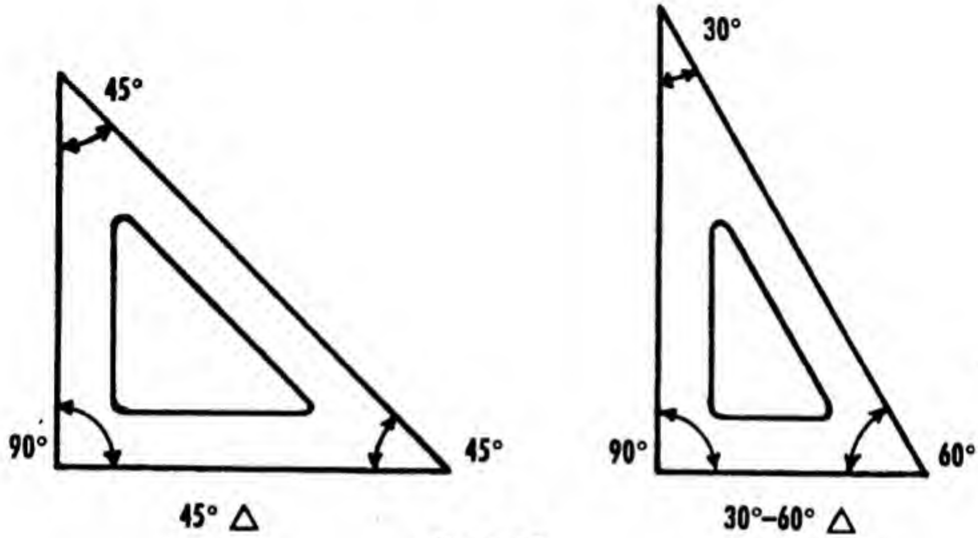


Figure 12.

15. The numbers on the scales of protractors and rulers are small but readable. However, if you want to read accurately between the lines, you must understand the effect of PARALLAX. Parallax is defined as the apparent displacement of an object as seen from two different points. For instance, if a ruler has lines dividing an inch into sixteenths and you wish to mark a point that is $\frac{1}{32}$ of an inch, this point must be exactly between two of the lines. You must look straight down at the scale, for if it is looked at from either side, an error of $\frac{1}{64}$ of an inch or more will be made. Another example of parallax can be noticed if you look at a clock from the side. An error of more than 2 minutes can be made. Later, when working with electronic equipment, you will have to read many electrical meters with great accuracy. Be sure to avoid the inaccuracies caused by PARALLAX.

16. The use of the protractor also requires careful work. To construct an angle of 65°, first draw a straight line of about 1 inch and call it AB. Position the protractor

with its center at point A and its zero mark on the line AB. With a well-sharpened pencil, place a dot on your paper right at the point indicated by 65° on the outer edge of the protractor. Now remove the protractor from the paper.

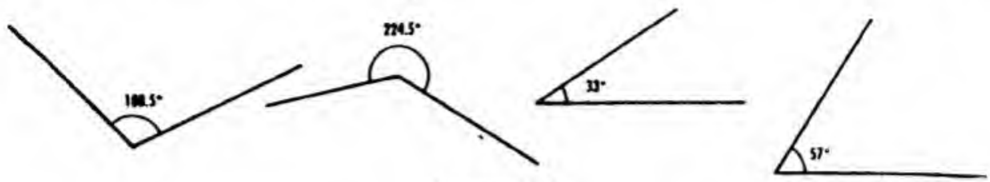


Figure 13.

Draw a straight line connecting point A and the dot you made on the paper. On a piece of transparent paper, if available, draw the following angles: $\angle 108.5^\circ$; $\angle 224.5^\circ$; $\angle 33^\circ$; $\angle 57^\circ$. When you have constructed these angles, place the transparent paper over the angles shown in figure 13 and compare your work. After you have studied this chapter in detail, work all the problems in pretest 1 that you have not previously completed and also the problems in section I, Remedial Work, at the back of the book.

CHAPTER 2

LINES AND ANGLES

Did you ever wonder how our ships with their intricate machinery are built? You know that shipbuilders and equipment manufacturers must work from plans and blueprints. These plans must first be designed by engineers, then drawn so accurately that steel beams and other parts may be manufactured at steel mills far from the shipyard, so that when delivered they fit into place without binding or stress. When requesting construction alterations at a shipyard, electronics technicians must be able to read and draw proposed changes to plans forwarded to BuShips.

As an electronics technician, you will often be required to read and understand electrical diagrams and construction layouts of electronic equipment. You must also learn to use simple drawing instruments in solving electronic problems by the graphic method. You must practice with drawing instruments to make neat calibration charts on the electronic equipment records of your ship. The ability to make neat mechanical and electronic drawings is a simple skill to learn but is one that will help you constantly in your electronic and naval career. In this chapter you will use the compass, dividers, and straightedge in making simple geometric figures which are the basis of all mechanical drawing. You will need a compass to do the construction required in this chapter. You can purchase a cheap but satisfactory compass in any five-and-ten (the cost should be less than one dollar).

PRETEST 2

17. (a) When you use a compass to draw a circle, the distance between the points of the compass legs equals the .... of the circle.

(b) Draw a line 4 inches long. Use your compass to divide the line into four equal parts. Use a ruler to check your result.

(c) Draw a line perpendicular to a straight line at its midpoint. Notice the similarity between this problem and problem (b).

(d) Construct a circle so that it passes through three previously determined points which are not on a straight line.

(e) What is the perimeter of a square whose side is 2 inches; 3 yards; S units?

(f) Use a compass and straightedge to draw a rectangle with a base 2 inches long and a perimeter of 7 inches.

(g) A closed straight-line figure having equal sides and equal angles is called a .... figure.

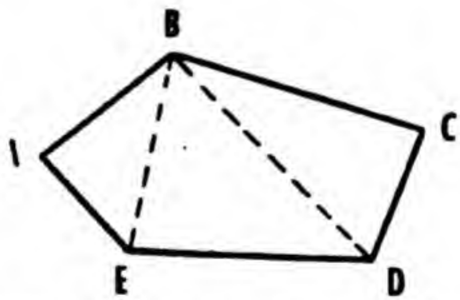
(h) Construct a 56° angle using a protractor and straightedge. Use your compass to bisect the angle.

(i) Construct an equilateral triangle.

(j) A straight line which meets the circumference of a circle at only one point is called a .... to that circle.

(k) Construct a regular hexagon (six-sided figure) within a circle so that all of its vertices are on the circumference of the circle.

(l) Use your drawing instruments to make an exact copy of the following figure—



(m) Draw a circle having a 1-inch radius. Now construct a tangent to the circle at any point on the circle.

(n) Draw a perpendicular to a straight line from some predetermined point.

(o) Use a protractor to draw an angle of 45°. Using your compass, make an exact copy of the angle.

STUDY GUIDE ON LINES AND ANGLES

18. You learned how to use the protractor and T-square in chapter 1. Two additional instruments for constructing figures, a pair of dividers and a compass, are shown in figure 14. (Do not confuse this compass with the mariner's

compass described in chapter 1.) Notice that although the pair of dividers is equipped with two needle points, the compass has only one needle point and the other leg ends in a lead point. All lines used in blueprints are either straight or curved; curved lines may be drawn as circles or parts of circles (ARCS).

A PAIR OF DIVIDERS

A COMPASS

Figure 14.

19. The correct way to use a compass when drawing a circle is illustrated in figure 15. Hold the instrument as shown and place the needle point at the center of the circle you wish to draw. The circle is completed by turning the

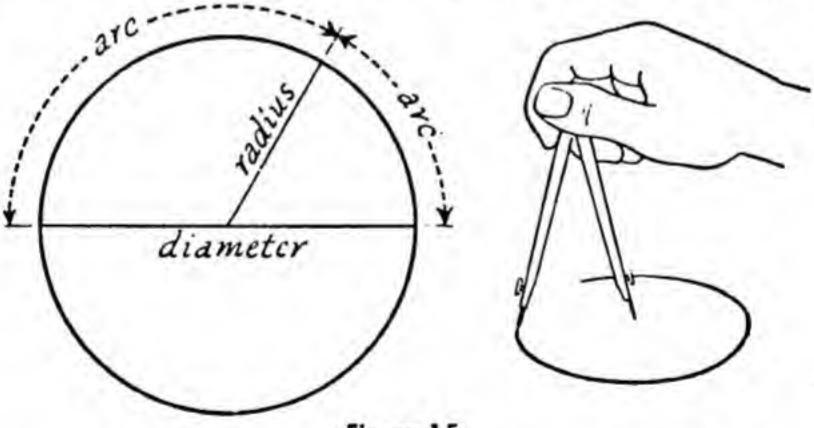


Figure 15.

compass. The distance between the pencil point and the needle point equals the radius of the circle (a straight line from the center of a circle to its circumference).

20. A pair of dividers is used for measuring distances between two points. After a given distance has been measured with the dividers, this distance can be transferred accurately by using the dividers; for example, dividing a line into equal parts. Look at figure 16. The

cross-section paper is arranged so that there are 10 divisions to the inch. Suppose you wish to know the length of

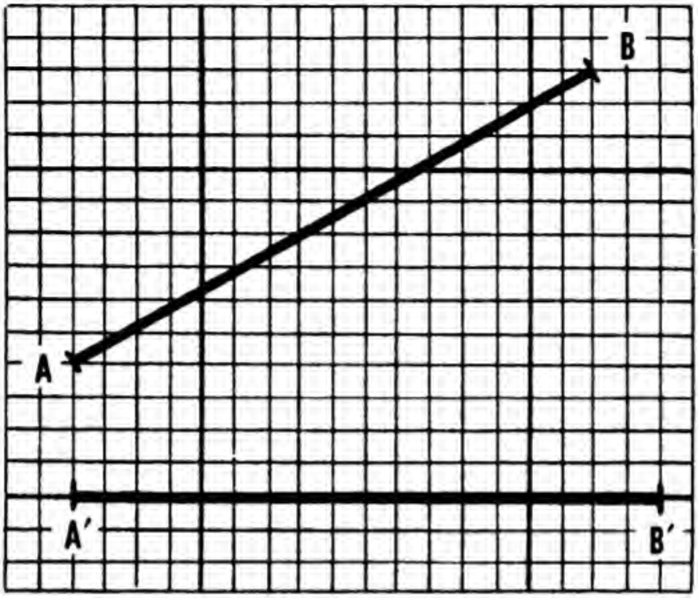


Figure 16.

line AB without using a ruler. Set the needle points of the dividers on points A and B, and transfer this distance to make the horizontal line $A'B'$. The length of the line (1.8 in.) can be determined by counting the squares between A' and B'. (A' is read A prime and B' is read B prime.) This is the way a navigator measures distances on a navigation chart. He sets the needle points at the ends of the length to be measured. He then transfers this length to a scale where each unit length represents a given distance.

21. You may use your dividers to divide a line into any number of equal parts. Figure 17, a, shows the four steps used for dividing a line into three equal parts—

(1) Open the dividers so that they span approximately one-third of the line to be divided.

(2) Place one needle point at one end of the line and the other needle point on the line. Suppose it falls at A.

(3) Turn the dividers using point A as a pivot, thus establishing point B on the line. Then turn the dividers on B as a pivot, but in the alternate direction.

(4) If the third point does not reach the end of the line, then increase the distance between the needle points by one-third the distance between the third point and the end of the line. If, however, the third point is beyond the end of the line you decrease the distance between the needle points by one-third the distance extending beyond the end

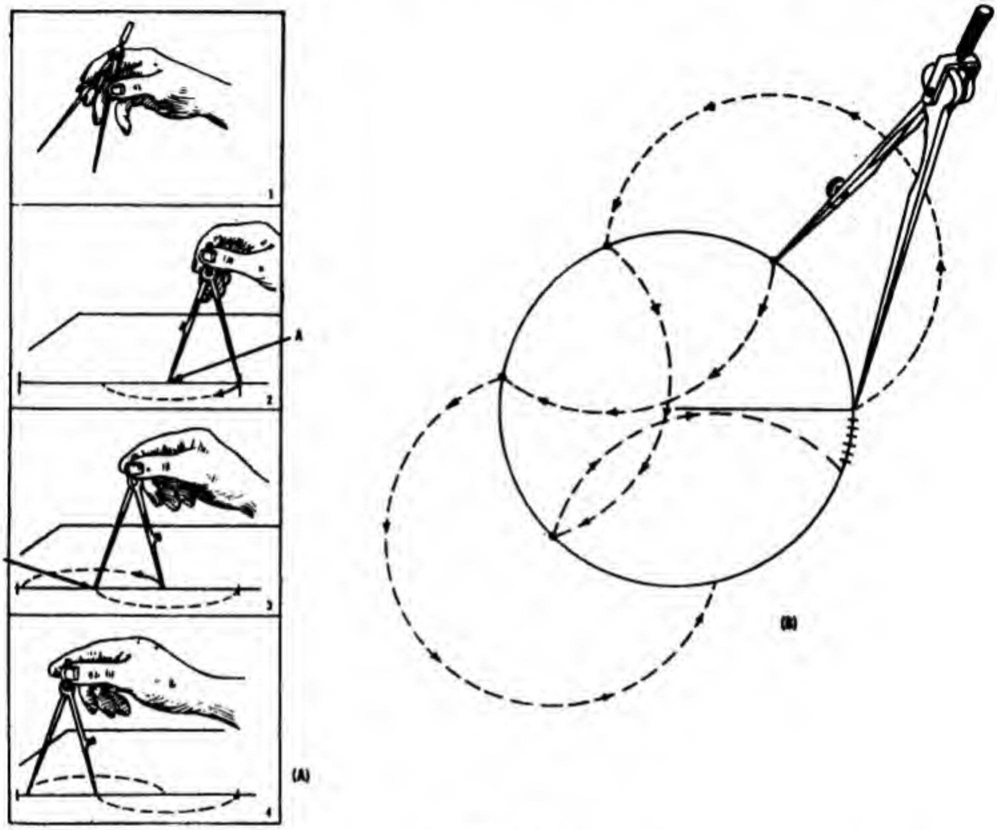


Figure 17.

of the line. This method may be used to divide a line into any given number of equal parts. You may also use this method to divide a curved line into equal parts. When working with any line, be sure to make it thin, so that the line thickness will not cause error in your measurements. Figure 17, b, shows how you may divide the circumference of a circle into any given number of equal arcs.

REPRODUCING ANGLES

22. When you construct or copy diagrams you will frequently have to draw an angle equal to a given angle. Sup-

pose you have to draw an angle equal to $\angle TAD$ shown in figure 18—

(1) Draw a working line $A'D'$.

(2) Set your compass with a radius of about 1 inch and place the needle point at A of the given angle.

(3) With A as center, strike an arc cutting the sides of the given angle at points C and B. With the same radius, transfer the compass to line $A'D'$. Now strike a similar arc using A' as the center, cutting the working line at B'.

Figure 18.

(4) Measure the opening of the given angle by setting one point of the compass at B and the other at C. With the compass fixed at this distance and with B' as center strike an arc as shown at the right in figure 18. This arc will cut the first arc at point C'.

(5) Draw a line from A' through C'. This new angle ($\angle T'A'D'$) is the same size as $\angle TAD$.

FINDING THE MIDPOINT OF A STRAIGHT LINE SEGMENT

23. You may use your compass to find the midpoint of any straight line segment such as AB in figure 19. Use a radius greater than half the length of AB. With point A as center draw arc CD. Now with point B as center and with the same radius, draw arc EF. Notice that these arcs intersect at points X and Y. Draw a straight line connecting X and Y. Point M, where XY intersects AB, is the midpoint of AB. Point M bisects AB—that is, it divides the line into two equal parts. Check your results with a ruler. Notice that XY is perpendicular to AB; this is written $XY \perp AB$. The symbol $\perp$ means "is perpendicular to." Therefore, XY is called the perpendicular bisector of AB.

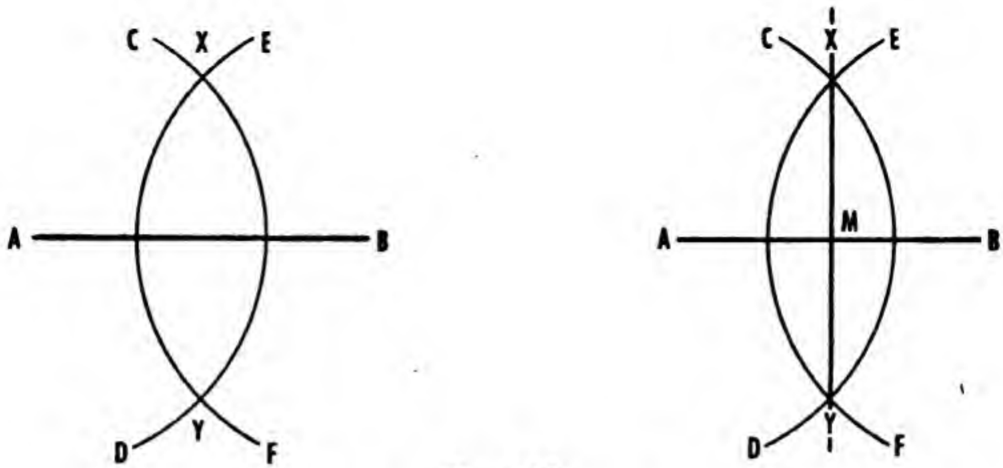


Figure 19.

FINDING THE CENTER OF A CIRCLE

24. To find the center of a circle draw any two chords such as *PR* and *PB* (fig. 20). (A chord is a straight line connecting two points on a circle.) Construct the perpendicular bisector of each chord. The point *C*, where the two perpendicular bisectors meet, is the center of the circle.

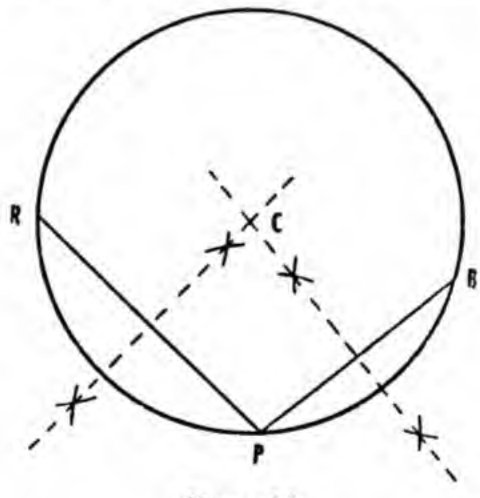


Figure 20.

CONSTRUCTING THE PERPENDICULAR TO A STRAIGHT LINE AT A GIVEN POINT ON THE LINE

25. On any straight line such as *EF* shown in figure 21, mark the point *P* at which you wish to construct the perpendicular. Then take the following steps—

(1) Set the compass for a radius less than the shorter segment of *EF*. (A segment of a line is any portion of the line; so the shorter segment of line *EF* is *EP*.)

(2) With *P* as center, draw two arcs cutting EF, one on each side of *P* (points *G* and *H* in fig. 21).

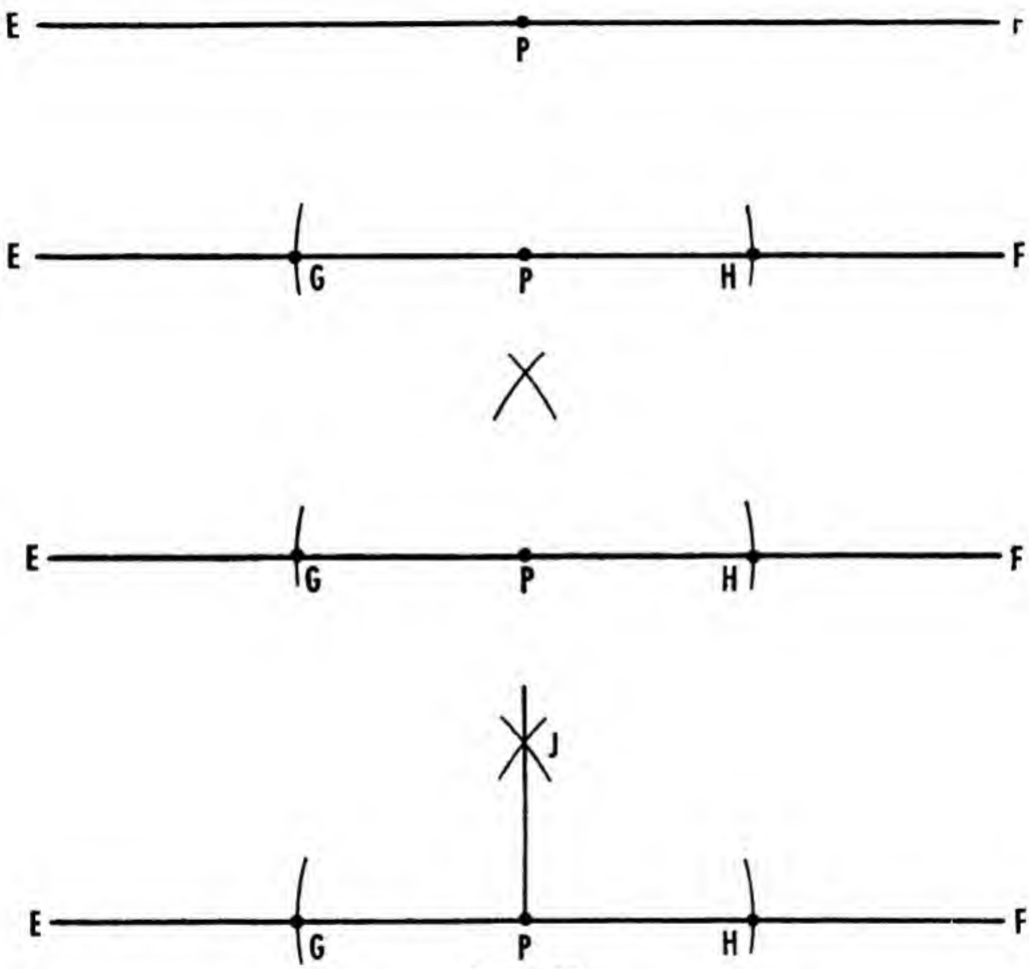


Figure 21.

(3) Now set the compass for a radius greater than *PG*.

(4) With *G* as center, draw an arc above or below point *P*. Keeping the same compass setting but with *H* as a center, draw another arc intersecting the first arc. These two arcs cross at point *J*.

(5) Draw a straight line from *J* through point *P*. The line *PJ* will be perpendicular to *EF*. Notice that $\angle JPE$ and $\angle JPF$ are right angles.

DRAWING A PERPENDICULAR TO A STRAIGHT LINE FROM A POINT NOT ON THE LINE

26. Suppose you wish to draw a perpendicular from P in figure 22 to the line AB. There are three steps—

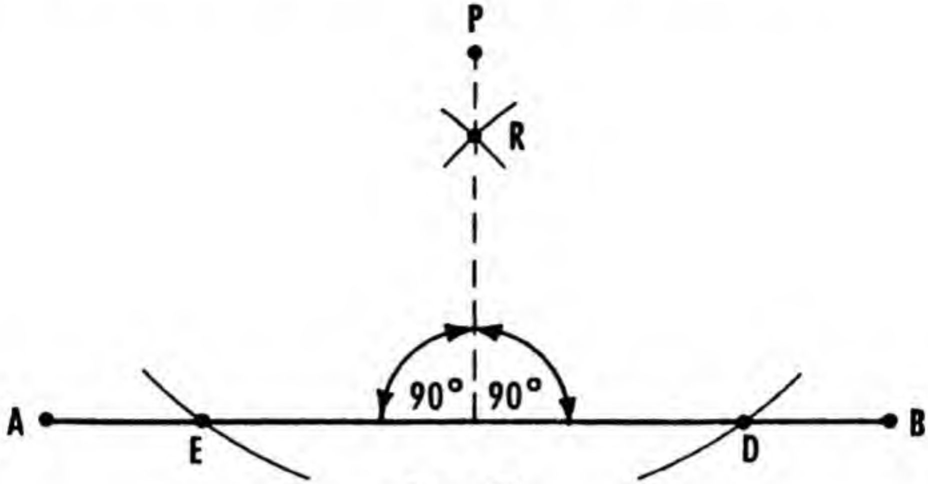


Figure 22.

(1) Set your compass to a suitable radius and, with point P as center, draw an arc cutting the line AB at points E and D.

(2) Using a radius greater than one half the distance ED and with points E and D as centers, draw arcs intersecting at R.

(3) Now draw a line from P through R to the line AB. This line will be perpendicular to AB.

BISECTING AN ANGLE

27. You can use your compass to divide any angle, such as $\angle AOB$ in figure 23, into two equal angles.

(1) Using O as a center, draw an arc cutting the sides of $\angle AOB$ at R and T.

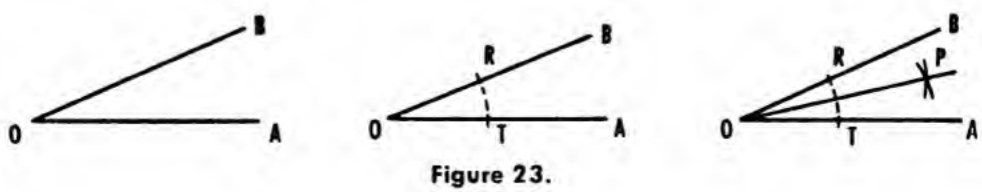


Figure 23.

(2) With R and T as centers and with a radius of convenient length, draw arcs intersecting at P.

(3) Draw a straight line from O through P. The line OP divides $\angle AOB$ into two equal angles and is called the bisector of $\angle AOB$.

DRAWING A SQUARE

28. If you know how to construct a right angle, you can easily draw squares and rectangles. The square is a closed figure consisting of four right angles and four equal sides

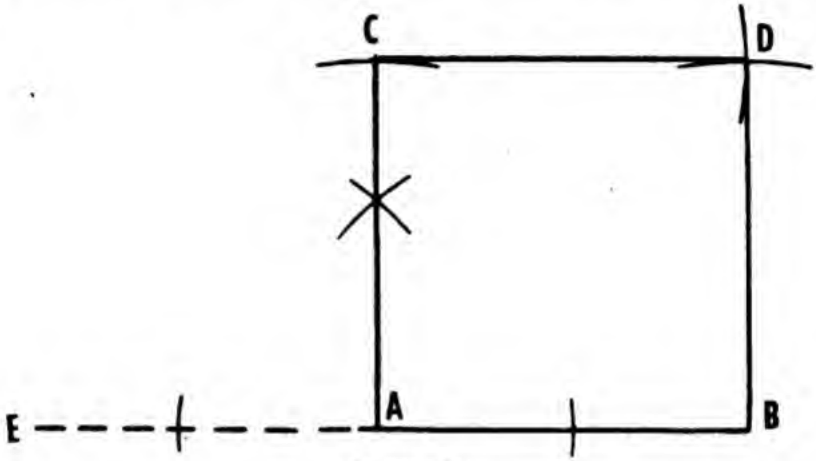


Figure 24.

(fig. 24). All you have to know to construct a square is the length of one side; for example, AB in figure 24. The steps are—

(1) Extend BA to E to provide a working line. Draw a perpendicular to line AB at A.

(2) With A as the center and distance AB as the radius, draw an arc cutting the perpendicular at point C.

(3) With points B and C as centers and again using AB as the radius, strike two arcs intersecting at D.

(4) Draw lines CD and BD to complete the square.

29. The distance around any geometric figure, or the sum of all its sides, is called the perimeter. What is the perimeter of a square whose side is 2 inches? Since a square has four equal sides, the formula for its perimeter is $p = 4s$, meaning that p, the perimeter, is equal to four

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times s, the length of one side. A square having one side equal to 2″ has a perimeter of 8″. You may use the preceding equation to find the perimeter of any square when you know the length of one side.

30. Figure 25 shows a rectangle. This figure, like the square, has four right angles. The opposite sides of a rectangle are equal. You can draw a square if you know the length of one side. How many dimensions (side lengths)

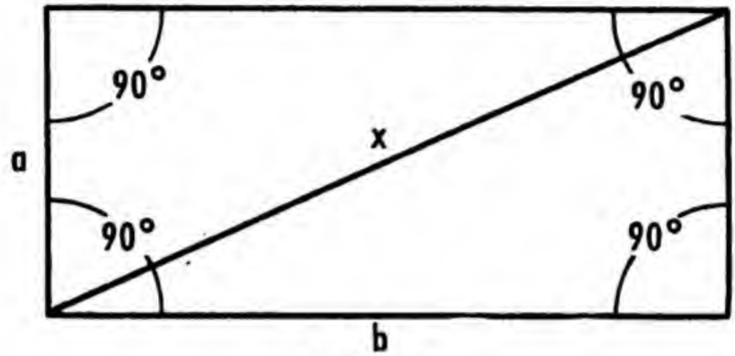

Figure 25.

must you know to draw a rectangle? In the rectangle shown in figure 25, line b is called the base of the rectangle and line a is the altitude (height). Since the base and the altitude of a rectangle are not equal, you must know the length of two sides to construct a rectangle or find its perimeter. The equation for the perimeter of any rectangle is $p = 2a + 2b$, where p is the perimeter, a is the altitude, and b is the base. After factoring, this formula becomes $p = 2\ (a + b)$. You can find the perimeter of any rectangle by substituting the numerical values of the base and altitude for the symbols in this equation.

31. Line x in figure 25 is called a diagonal. The diagonal of a quadrilateral (four-sided figure) is a straight line joining any two opposite vertexes. The diagonal of a rectangle divides it into two equal triangles.

32. A geometric figure that is constructed entirely of straight lines is called a straight-line figure. Squares, rectangles, and triangles are all straight-line figures. Any straight-line figure that has equal sides and equal angles is called a regular figure. All squares and those triangles

having three equal sides are regular figures; a rectangle is not a regular figure because not all of its sides are equal. You already know how to construct a square. In the next few paragraphs you will learn how to construct several other regular figures.

HOW TO CONSTRUCT A REGULAR OCTAGON

33. A regular octagon (an eight-sided figure) must have equal sides and equal angles. To construct a regular octagon you begin with a circle as shown in figure 26.

Figure 26.

(1) Draw a circle and its diameter AB.

(2) Construct the perpendicular bisector of this diameter and extend it to form a second diameter.

(3) Bisect the four right angles formed by the two intersecting diameters; extend these bisectors to the circumference of the circle. You will now have eight radii cutting off equal arcs of the circumference between them.

(4) Draw chords connecting adjacent intersections of the radii and the circumference. When you have drawn all eight chords you will see that they form a regular octagon. Use a pair of dividers and a protractor to check the equality of the sides and of the angles in the figure. If you know the length of one side of a regular octagon you can find its perimeter from the formula $p = 8s$.

THE REGULAR HEXAGON

34. The regular hexagon (a six-sided figure) is a very common figure. The heads of many bolts and the nuts used

with them have hexagonal shapes. You will often find hexagonal figures in machinery and electronic equipment blueprints. A regular hexagon may be constructed by using the radius of a circle to locate six equidistant points on the circumference (fig. 27).

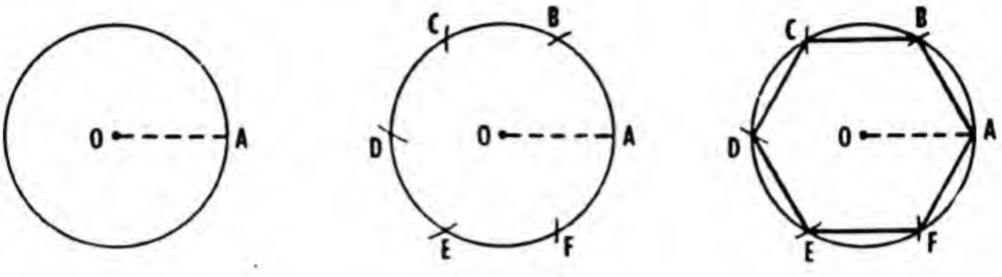

Figure 27.

(1) Draw a circle and mark any point *A* on its circumference.

(2) Using the radius of the circle and point *A* as center, draw a small arc cutting the circumference at *B*.

(3) Using the same radius and point *B* as center, draw another arc cutting the circumference at *C*.

(4) Locate points *D*, *E*, and *F* on the circumference in the same way as you located *B* and *C*. (See fig. 27.)

(5) Draw chords connecting adjacent points on the circumference. These six chords form the sides of a regular hexagon. Check your work with a pair of dividers and a protractor.

CONSTRUCTING AN EQUILATERAL TRIANGLE

35. An equilateral triangle is one in which all three sides are equal. One way to construct an equilateral triangle (similar to the method used in constructing the regular hexagon) is shown in figure 28, a; a shorter method is illustrated in figure 28, b.

(a) Constructing an equilateral triangle by using a circle–

(1) Draw a circle and mark any point *A* on its circumference.

(2) Using OA, the radius of the circle as a radius and with A as center, draw a small arc cutting the circumference at B.

(3) Using B as a center and with the same radius, draw another arc cutting the circumference at C. Locate D, E, and F in the same manner, as shown in figure 28, a.

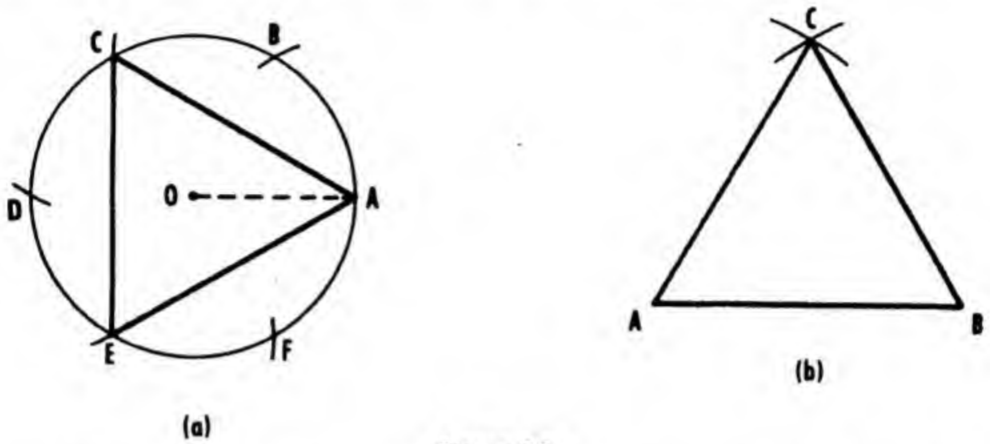

Figure 28.

(4) Draw AC, CE, and EA to form the equilateral triangle.

(b) The short method—constructing an equilateral triangle without using a circle—

(1) Draw line AB.

(2) Set the compass legs to the distance AB and, using A and B as centers, draw two arcs intersecting at C.

(3) Draw AC and BC to complete the triangle.

HOW TO CONSTRUCT A REGULAR PENTAGON

36. The regular pentagon, shown in figure 29, is not quite so easy to draw as the triangle, square, hexagon, or octagon.

(1) Draw a circle and any diameter AZ.

(2) Construct diameter YX perpendicular to AZ.

(3) Find the midpoint of one of the four radii. Suppose that H is the midpoint of radius OX.

(4) Set your compass to the distance AH and, with H as center, strike an arc cutting radius YO at G.

(5) Now use distance AG as a radius and, with A as center, strike an arc cutting the circumference at B.

(6) Using B as center and the same radius AG, strike an arc cutting the circumference at C. Locate points D and E on the circumference (as shown in fig. 29) in the same way as you located B and C.

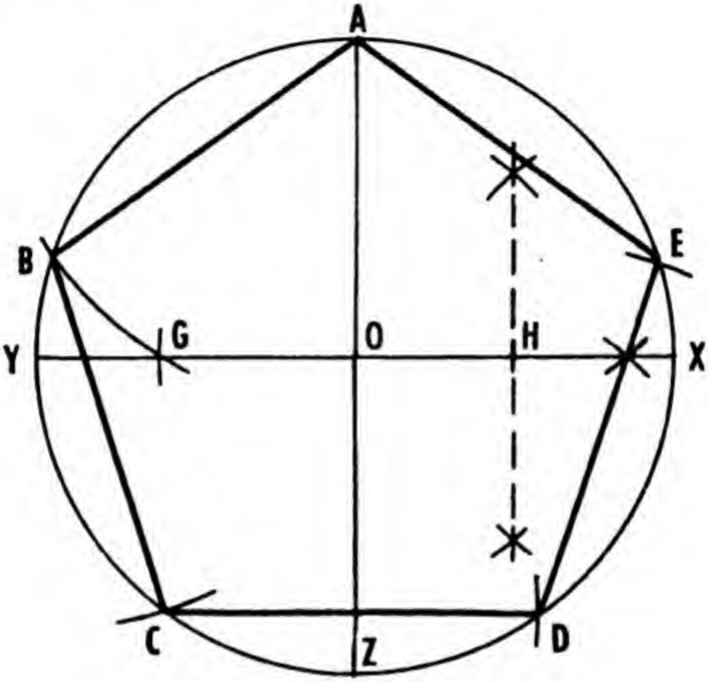

Figure 29.

(7) Draw chords connecting adjacent vertices to form the pentagon.

37. You have learned how to divide both straight and curved lines into equal parts by using a pair of dividers. Figure 17, b, shows how to divide the circumference of a circle into six equal parts. This construction locates the vertexes of a regular hexagon. Using this method for dividing a circumference into equal parts you can construct a regular figure with any given number of sides. Suppose you wished to draw a regular seven-sided figure, as shown in figure 30. Set your dividers to approximately $\frac{1}{7}$ of the circumference. Move them around the circumference until you have only the remainder, $H'A'$ in figure 30. Increase the span of your dividers by $\frac{1}{7}$ of $H'A'$ and

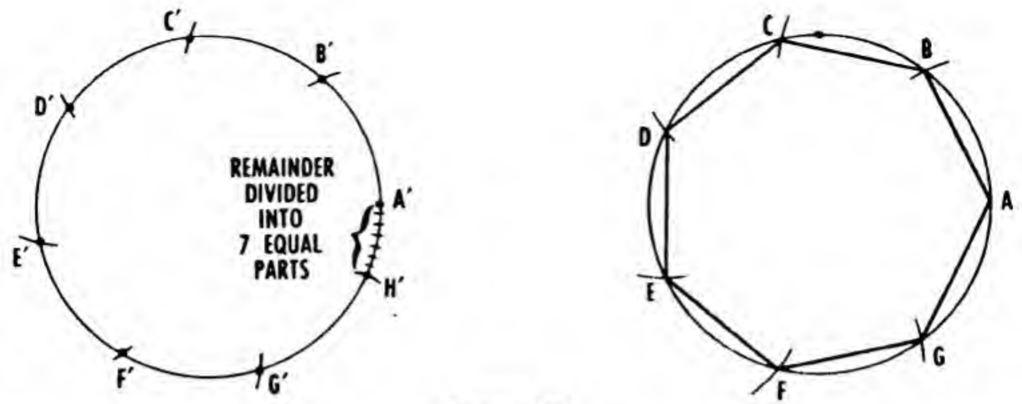

Figure 30.

try dividing the circumference again. When you have divided the circumference into seven equal arcs, draw the chords across them to form the regular seven-sided figure.

HOW TO CONSTRUCT THE TANGENT TO A CIRCLE

38. A tangent to a circle is a straight line that touches the circle at only one point, no matter how far it is extended in either direction. The construction of a tangent to a circle is illustrated in figure 31. To construct the tangent to circle *O* at point *P*—

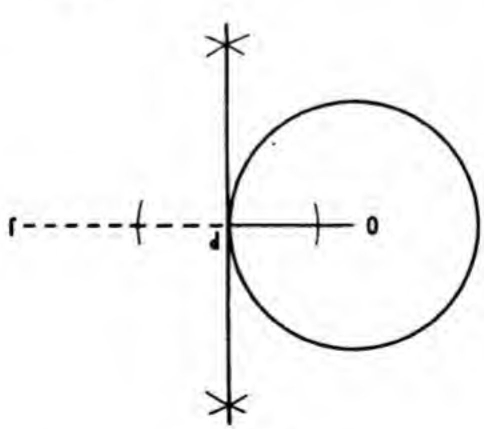

Figure 31.

(1) Draw the radius *OP* and extend it a convenient distance beyond the circumference to *J*.

(2) Construct the perpendicular to *OJ* at point *P*. This perpendicular is the tangent to circle *O* at point *P*.

HOW TO REPRODUCE A TRIANGLE

39. Since you know how to transfer lines and angles to new positions, you can make exact copies of many geometric figures. You will first learn to copy triangles, because any straight-line figure can be divided into a series

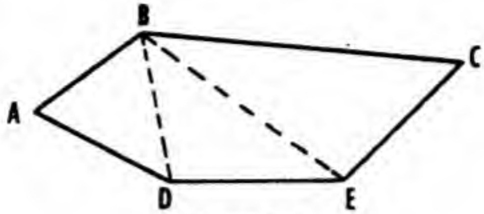

Figure 32.

of triangles. For example, figure 32 consists of three triangles. If you know how to copy triangles, you can easily construct a duplicate of this figure, using a compass and straightedge. Suppose you wish to make an exact copy of $\triangle ABC$ ($\triangle$ is the symbol for a triangle), shown in figure 33.

Figure 33.

There are three ways to do this—

(1) Using two angles and the included side. (See fig. 33.)

- (a) Draw line $A'B'$ equal to AB.
- (b) At A', construct an angle equal to $\angle A$.
- (c) At B', construct an angle equal to $\angle B$.
- (d) Extend the sides of $\angle A'$ and $\angle B'$ until they intersect at point C'. $\triangle A'B'C'$ is an exact copy of $\triangle ABC$; the two triangles are congruent, meaning they are identical. In this method you used two angles and the side included between them for the construction.

(2) Using two sides and the included angle. (See fig. 34.)

(a) Draw a line $A'B'$ equal to AB.

(b) At A' construct an angle equal to $\angle A$.

(c) Extend the line drawn to form $\angle A'$ to a point C', so that $A'C'$ is equal to AC.

Figure 34.

(d) Connect B' and C' with a straight line. $\triangle A'B'C'$ is congruent to $\triangle ABC$. In this method two sides and the included angle were used for the construction.

(3) Using three sides. (See fig. 35.)

Figure 35.

(a) Draw a line $A'B'$ equal to AB.

(b) With AC as radius and A' as center, strike an arc.

(c) With BC as radius and B' as center, strike a second arc to intersect the first arc at point C'.

(d) Draw $A'C'$ and $B'C'$ to form $\triangle A'B'C'$ which is congruent to $\triangle ABC$. Three sides were used for this construction.

If you have studied this chapter in detail refer back to pretest 2 and work any problems you have not previously completed together with the ones in section II, Remedial Work.

CHAPTER 3

PRACTICAL CONSTRUCTIONS INVOLVING PARALLEL LINES

In your previous assignments you have used parallel lines many times, although you may not have recognized them as such. The square and the rectangle are made up of parallel lines. In this chapter you will learn to recognize parallel lines and how to construct them. Railroad and streetcar tracks are familiar examples of parallel lines. When they are laid, a gauge is used to keep them equidistant at all points. To maintain correct electronic characteristics, the metal plates of radio capacitors must be parallel. This chapter also covers the interrelationships of angles. In addition to the compass, dividers, and straightedge, you will use triangles in making the geometric figures required in this chapter.

PRETEST 3

40. (a) Parallel lines are straight lines in the same .... which will not meet no matter how far they are extended.

(b) Draw a line parallel to a given line through a given point outside that line: (1) Using a compass and straightedge; (2) using a protractor and straightedge; (3) using a triangle and straightedge.

(c) Use a compass and straightedge to construct a rhombus having 2″ sides.

(d) Use a compass to divide any given line into five equal parts.

(e) If two interior angles of a triangle are 40° and 70°, what is the third angle? If the sum of two angles in a triangle is 130°, what is the third angle?

(f) What is the greatest number of right angles possible in a triangle? The greatest number of obtuse angles? The greatest number of acute angles?

(g) If one of the acute angles of a right triangle is 20°, what is the size of the other acute angle?

(h) Can an isosceles triangle be a right triangle?

(i) The sides of a triangle measure $4\frac{1}{2}''$, $6''$, and $11''$. What is the perimeter?

(j) The sum of the interior angles of a pentagon is .... degrees.

(k) What is the complement of an angle of $30°$? of $90°$?

(l) What is the supplement of an angle of $90°$? of $40°$? of $130°$?

(m) The smaller an angle, the .... is its complement.

(n) The larger an angle, the .... is its supplement.

(o) Two angles are supplementary. The smaller is $30°$ less than one-half the larger. What are their values?

STUDY GUIDE FOR PARALLEL LINES

41. Parallel lines are straight lines IN THE SAME PLANE which will not meet no matter how far they are extended. Why is the phrase "in the same plane" emphasized here? If two straight lines are NOT IN THE SAME PLANE, it may be possible to extend them without their meeting, and yet they will not be parallel. For example, a front horizontal edge of a radio cabinet, and a rear vertical edge, are straight lines which do not meet when extended. However, they are not in the same plane and, therefore, are not parallel. For your work, think of a plane as a flat surface. The figures you have constructed so far were plane figures since you considered only length and width. A rectangular solid, for example, has thickness, as well as length and width.

42. In figure 36, lines AB and CD are parallel. They have the same direction, and they make equal angles x with reference line EF. A TRANSVERSAL is a line, such as EF in figure 36, which intersects two or more lines. Corresponding angles (angles x in fig. 36) are angles on the same side of the transversal, and on the same side of the lines cut by the transversal. The symbol for "is parallel to" is $\parallel$. In figure 36, $AB \parallel CD$.

43. The corresponding angles x can be made any size by varying the position of the transversal. As long as the angles remain equal, the lines AB and CD are parallel. You can prove this by placing a pencil over line AB and another

over CD. Now rotate the pencils in the same direction about the points where they cross EF; if you keep the corresponding angles equal, the pencils will remain parallel. The principle of keeping corresponding angles equal suggests one of the most practical ways to draw parallel lines. PARALLEL RULERS were designed on this principle to

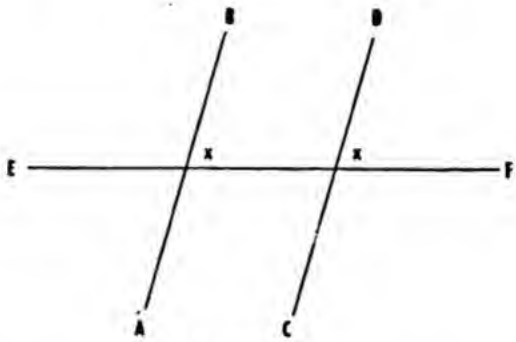

Figure 36.

help you draw parallel lines, and are widely used in navigation. For example, a navigator wants to plot course 042° on a chart (fig. 37). A compass rose is practically a reproduction of the compass card covered in chapter 1. They are placed at various places on a chart for ease in plotting.

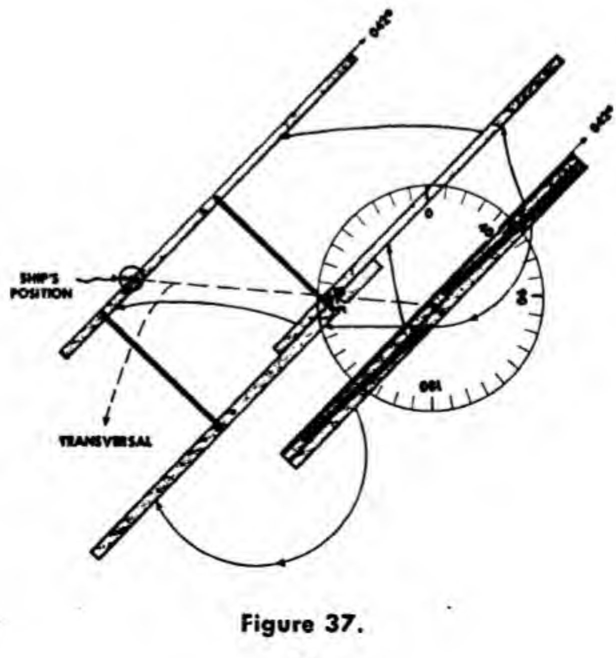

Figure 37.

The navigator places an edge of one of the rulers on the COMPASS ROSE at the desired bearing. Then he slides the other parallel ruler over so that it will fall along the center point of the ship. Now, by drawing a line from the ship along the straightedge, he can plot a course parallel to the

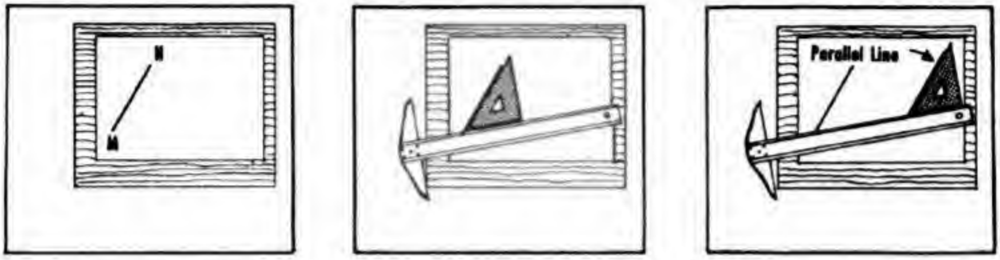

Figure 38.

bearing he selected from the compass rose. It is important to hold one of the rulers firmly against the chart before sliding the other ruler. The parallel rulers are constructed so that the corresponding angles they make with any one transversal cannot change. Therefore, only one transversal should be followed when transferring a bearing from the compass rose to the center line of the ship.

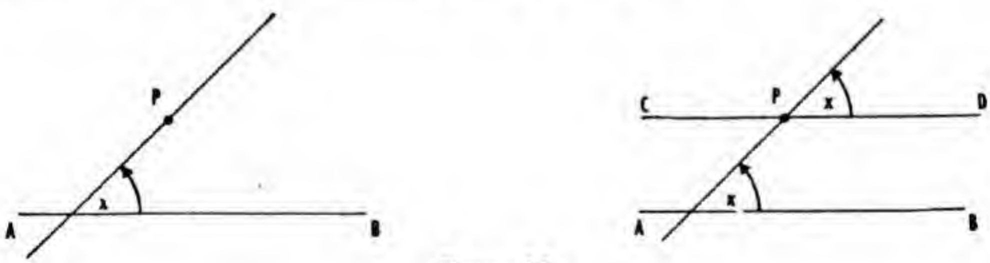

Figure 39.

44. A draftsman uses a T-square and a triangle to draw parallel lines. For example, suppose you want to draw a line parallel to *MN* in figure 38. Place the triangle against the T-square and tilt the T-square until the hypotenuse of the triangle lies along the line *MN*. Holding the T-square firmly against the drawing board, slide the triangle up to the correct position for drawing the parallel line. Keep in mind the proper way to hold your pencil for drawing lines, as was illustrated in figure 11, b, in chapter 1.

USING A COMPASS TO CONSTRUCT A LINE PARALLEL TO A GIVEN LINE

45. Suppose you want to construct a line through point P parallel to AB (fig. 39).

(1) Draw a line through P intersecting AB.

(2) Using point P as a vertex and the line through P as one side, construct a corresponding angle equal to $\angle x$.

(3) Line CD, the other side of the new angle, is parallel to AB.

46. Any four-sided figure in which the opposite sides are parallel is called a PARALLELOGRAM. Parallel lines intercepted by parallel lines are equal in length. For this reason the opposite sides of a parallelogram are also equal.

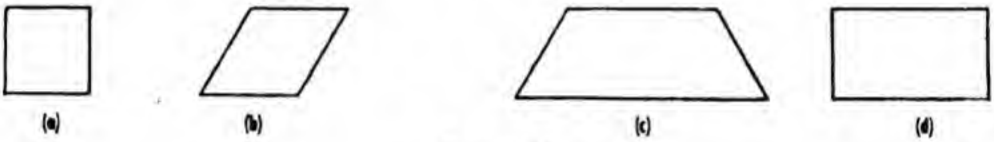

Figure 40.

Which of the drawings shown in figure 40 are parallelograms? Both the square and the rectangle are parallelograms in which all the angles are right angles. But all parallelograms do not have right angles. The parallelogram in figure 40, b, has all sides equal and is called a RHOMBUS. Many times while working with radio communication equipment you come across the familiar figure of the rhombus. Engineers have designed a very good antenna called a rhombic antenna because its shape is similar to that of a rhombus. (See fig. 41.) The sides of a rhombus are equal, and its angles are usually oblique (two of them acute and two of them obtuse). The square, however, is a special kind of rhombus with four right angles. The third figure (fig. 40, c) illustrated is the only one that is not a parallelogram. It has only one pair of opposite sides parallel. The other two sides are not parallel and would meet if extended. This figure is called a TRAPEZOID. You will study the trapezoid in chapter 6.

Figure 41.—Rhombic antenna.

HOW TO CONSTRUCT A PARALLELOGRAM

47. You can construct or copy a parallelogram by using a compass and a straightedge (fig. 42). To construct a parallelogram—

(1) Draw any line AB for the base of the parallelogram.

(2) At any angle with AB lay off AC as a second side of the parallelogram.

(3) With the compass set for a radius equal to AC and with B as center strike an arc. (See fig. 42.)

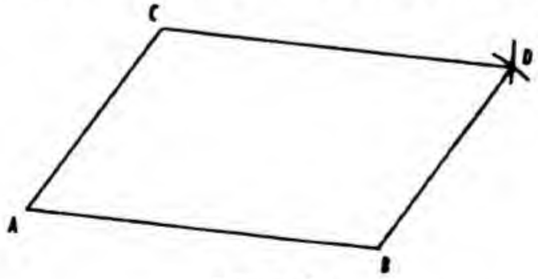

Figure 42.

(4) With C as center and a radius equal to AB strike a second arc intersecting the first arc at D.

(5) Draw lines CD and BD. Then $ABDC$ is a parallelogram since both pairs of opposite sides are equal.

HOW TO DRAW A LINE PARALLEL TO A GIVEN LINE AT A FIXED DISTANCE

48. Suppose you want to construct a line parallel to some given line at a given distance from it. Figure 43 shows the correct method for this construction. To construct a line parallel to line AB at a distance of $\frac{7}{8}''$—

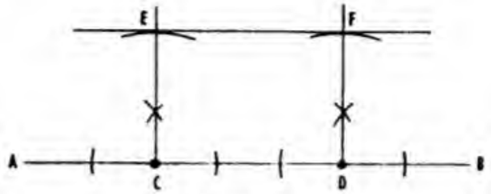

Figure 43.

(1) Erect perpendiculars at two points (C and D in fig. 43) on the given line AB. The points C and D should be separated by at least 1 or 2 inches to make your work accurate.

(2) Using your compass, mark off the fixed distance ($\frac{7}{8}''$) from AB on the two perpendiculars, locating points E and F.

(3) Draw a straight line through E and F. This line will be parallel to AB.

HOW TO DIVIDE A LINE INTO ANY NUMBER OF EQUAL PARTS

49. In chapter 2 you learned how to divide a line into any number of equal parts by using the dividers. You can divide a line into any number of equal parts by using your compass to construct a series of parallel lines. Figure 44 shows you how to divide line AB into five equal parts.

(1) Draw a line AR at any convenient angle with AB.

(2) Starting at point A, and with your compass set to a suitable radius, mark off five equal parts in succession on AR so that $AC = CD = DE = EF = FG$.

(3) Draw line BG. Through points F, E, D, and C draw lines parallel to BG. These lines will cut AB into five equal parts at points P, O, N, M, and B.

50. This method of dividing a line into equal parts may be of good use to you in various ways. In order to mount radio tubes you may want to drill holes equally spaced on a radio chassis. Likewise, in designing the front panel of a radio cabinet, you may want the various controls equally

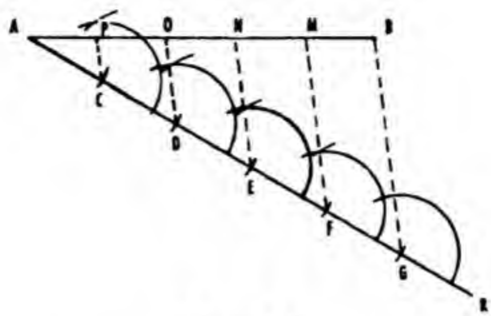

Figure 44.

distant from one another. Take a piece of cardboard or other suitable material; cut it to the proper length and use it as a template. By the given method of dividing a line into a number of equal parts with a compass, you can accurately find the centers of the holes to be drilled. (See the layout shown in fig. 45.) Place the finished template over

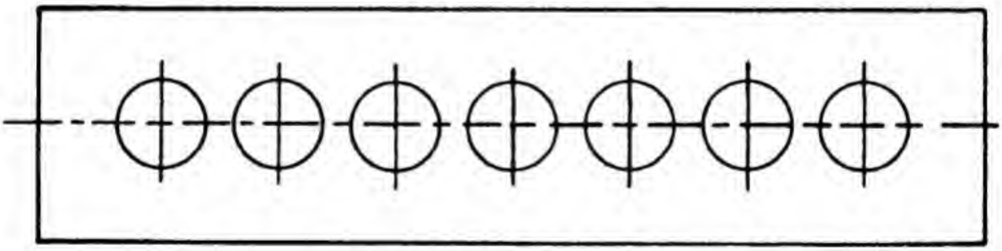

Figure 45.

the metal chassis or cabinet. With a center punch, mark in the points you have accurately laid out, thus transferring the points from the template to the chassis.

51. There is another method you may use to divide AB into a given number of equal parts. Figure 46 shows how this second method is used to divide line AB into four equal parts.

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(1) Draw lines AC and BD parallel to each other, as shown in figure 46.

(2) Using the compass, mark off three equal lengths on AC and also on BD, starting at points A and B, respectively.

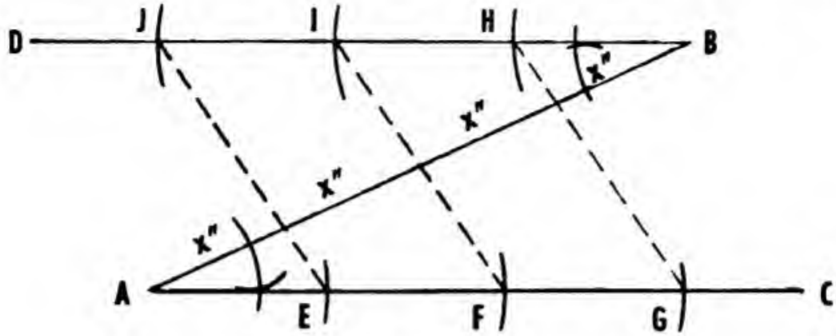


Figure 46.

(3) Draw lines GH, IF, and JE through the established points on AC and BD. The points where these three lines cut AB, divide AB into four equal parts.

STUDY GUIDE FOR THE INTERRELATIONSHIPS OF ANGLES

52. Look at the equilateral triangle in figure 47. Measure each interior angle with your protractor. In this triangle you will find that the angles, as well as the sides, are equal;

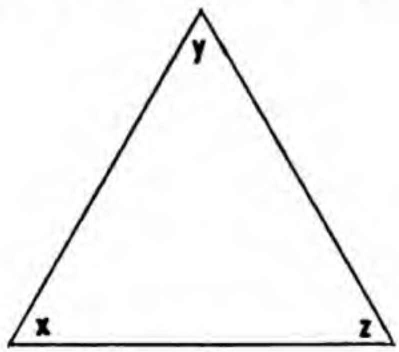


Figure 47.

each angle is 60°. Therefore, the sum of the three angles is 180°. This answer illustrates an important geometric rule, which applies to all triangles: THE SUM OF ALL THE INTERIOR ANGLES OF ANY TRIANGLE IS 180°. This relation may be expressed as an equation: $\angle x + \angle y + \angle z = 180°$. (See

fig. 48.) If you know the value of any two angles of a triangle, you may use this equation to find the third angle. Substitute the known values in the preceding equation, and you have $\angle x + 90^\circ + 45^\circ = 180^\circ$. Therefore, $\angle x = 45^\circ$.

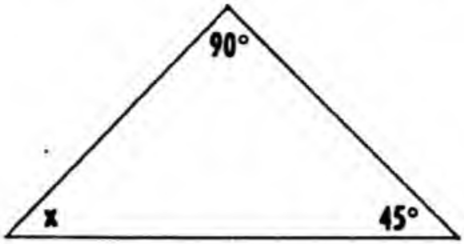


Figure 48.

53. Three types of triangles are shown in figure 49. (a) An EQUILATERAL TRIANGLE has three equal sides and three equal angles. (b) An ISOSCELES TRIANGLE has two sides of the same length, and the angles opposite the two equal sides are equal. (c) A RIGHT TRIANGLE is one in which one angle is a right angle (90°). A right triangle in which two sides are equal is known as an ISOSCELES RIGHT TRIANGLE. The two acute angles of an isosceles right triangle are each 45°.

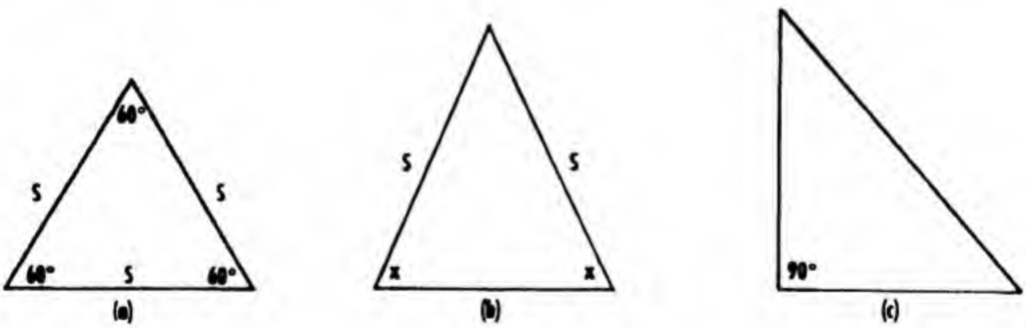


Figure 49.

54. The perimeter of a triangle, like that of any closed geometric figure, is the sum of the lengths of its sides. The formula for the perimeter of an equilateral triangle is $p = 3s$. You cannot use this formula for any triangle other than the equilateral one. The s in this equation refers to one of the equal sides of a regular figure. For triangles

that have unequal sides, the formula for the perimeter is $p = a + b + c$, where a, b, and c represent the lengths of the sides. (See fig. 50.)

55. The sum of all the interior angles of a triangle is $180°$. How many degrees are there in the interior angles of a quadrilateral? Of a pentagon. You know from your basic

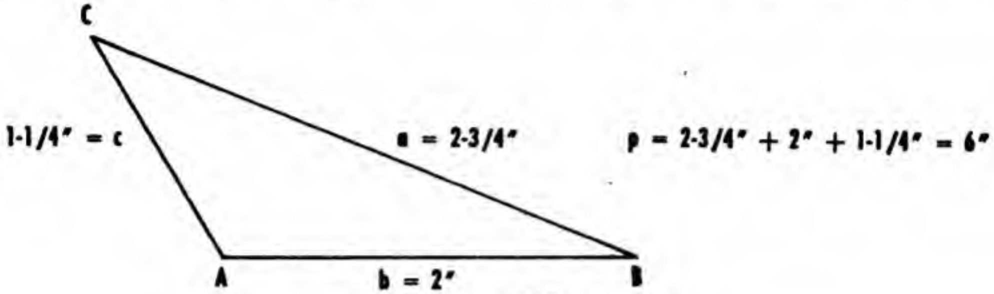


Figure 50.

constructions that any polygon (many-sided figure) can be divided into a series of triangles. Therefore, you can find the sum of the interior angles of any polygon. For example, the hexagon shown in figure 51 can be divided into four triangles. The sum of all interior angles is

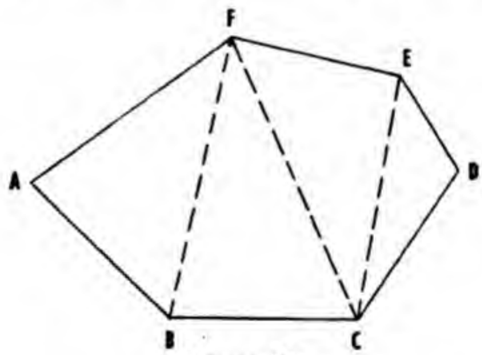


Figure 51.

$4 \times 180° = 720°$. Notice that the number of triangles formed is two less than the number of sides. The equation for the sum of the interior angles of a polygon of n sides is $(n-2)180°$ A seven sided figure contains $5 \times 180° = 900°$; an octagon contains $6 \times 180° = 1080°$.

56. When the sum of two angles is $90°$ (a right angle) they are called COMPLEMENTARY ANGLES; one angle is called the complement of the other. In figure 52, $\angle x +$

$\angle y = 90°$. Angle x is the complement of $\angle y$, and $\angle y$ is the complement of $\angle x$. If an angle is 70°, its complement is 20°. The complement of any acute angle is the difference between that angle and 90°.

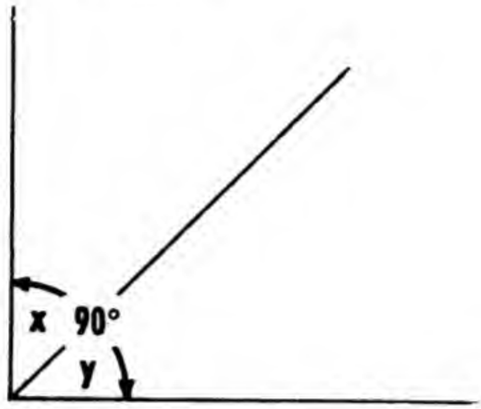


Figure 52.

57. When the sum of two angles is 180° (a straight angle) they are called SUPPLEMENTARY ANGLES. In figure 53, $\angle x + \angle y = 180°$. Therefore, $\angle x$ is the supplement of $\angle y$, and $\angle y$ is the supplement of $\angle x$. If one angle measures 45°, its supplement is 135° since the sum of the angles

Figure 53.

must be 180°. The supplement of any angle is the difference between that angle and 180°. Remember the words "complement" and "supplement." You will use them often in your study of alternating currents.

58. VERTICAL ANGLES ARE EQUAL. The two angles a and b (shown in fig. 54), formed by the two lines intersecting at E, are called vertical angles. If you will remember that an angle is an amount of turning you will easily see

that vertical angles are equal. (See fig. 54.) Place a pencil over line MN. Turn the pencil through angle a to position OP. Notice that the other end of the pencil has also turned through angle b the same number of degrees. Thus, you

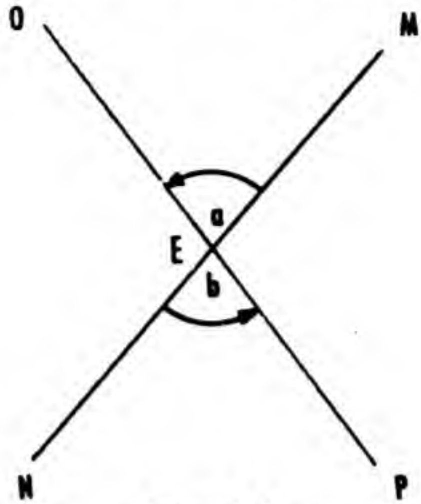


Figure 54.

have proved that $\angle a = \angle b$ (shown in fig. 54) and that vertical angles are equal.

Apply what you have learned in this chapter by working the problems in section III of the Remedial Work.

CHAPTER 4

BASIC IDEAS OF SCALE DRAWING

You have learned to recognize and reproduce the basic geometric figures used in blueprint drawing. So far, your constructions have been full-scale; the dimensions of your drawings were the same as those of the objects they represented. This method is seldom possible in practical work. Even the smallest aircraft carriers in the Navy are more than 400 feet long. A full-scale blueprint of a ship that large would probably be more of a nuisance than a help. Imagine trying to locate a particular compartment or passageway on a blueprint 400 feet long.

When an object is too large to permit a convenient full-scale blueprint, it must be represented by a scale drawing —a plan in which the correct relationship of the parts is maintained, but in which all dimensions are reduced proportionately. Likewise, drawings of very small objects, such as watch parts or small pieces of electronic equipment, must be enlarged so that you can study them easily. Such drawings—enlargements or reductions of the actual size of the object–are called SCALE DRAWINGS.

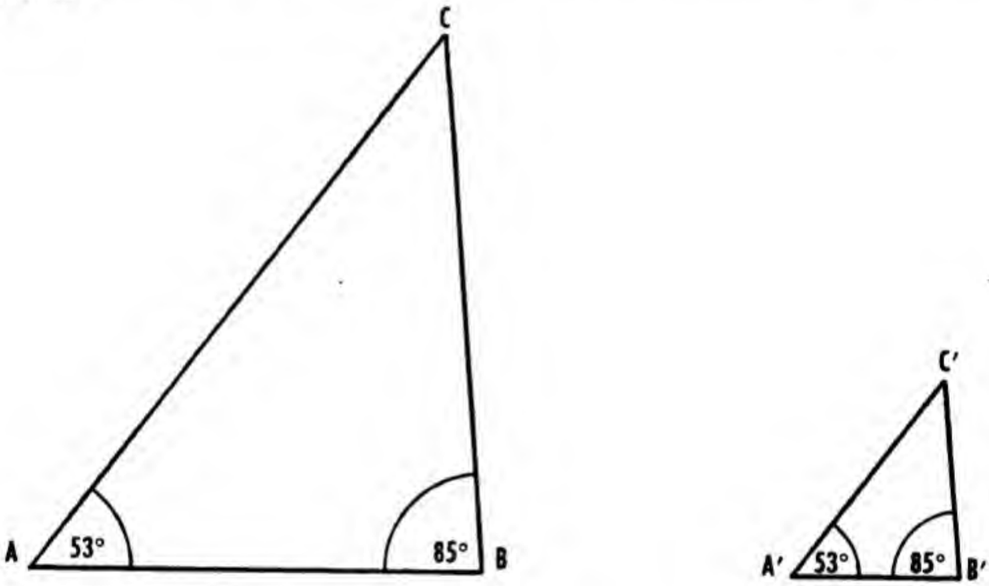
When you work with electronic equipment, you must be able to understand, work from, and prepare scale drawings. Vacuum tubes are often pictured on an enlarged scale so that you can study their parts and principles of operation. Small mechanical parts of electronic equipment are sometimes drawn larger than actual size, so that you can see their details, either for purposes of study or for making new parts in an emergency. Studying the different linear scales discussed in this chapter will improve your speed and accuracy in reading electrical meter scales. Scale drawings are useful only if you understand them.

59. (a) Two triangles are similar when the ratios of their corresponding .... are equal.

(b) Each of the following equations is a proportion. Solve for the unknown value X in each of the following equations—

$$\frac{9}{21}=\frac{X}{14},\quad \frac{8}{15}=\frac{72}{X},\quad \frac{48}{X}=\frac{120}{4},\quad \frac{35}{18}=\frac{7}{X},\quad \frac{X}{70}=\frac{24}{35}.$$

(c) The triangles shown in figure 55 are similar. Complete the ratios.



$$\frac{AB}{A'B'}=\frac{?}{A'C'}=\frac{BC}{?}$$

Figure 55.

(d) Two triangles are similar. The sides of the larger triangle measure 12″, 14″, and 20″. The shortest side of the smaller triangle is 3″. What are the lengths of the other two sides of the smaller triangle?

(e) A vertical radio transmitting antenna casts a shadow 27 feet long. A man 5 feet 10 inches tall, standing next to the antenna mast, casts a shadow 2 feet long. What is the height of the mast?

(f) Construct a triangle similar to triangle *ABC* (fig. 56) using a scale 2½ : 1. Indicate the lengths of the sides on the similar triangle.

(g) A tree casts a shadow 47 feet long and a pole standing next to it casts a shadow 11 feet long. The pole is 8 feet high. How tall is the tree?

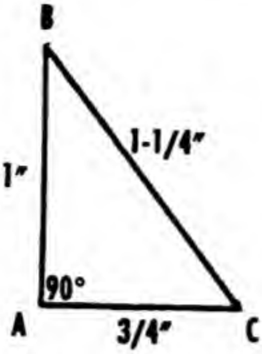


Figure 56.

(h) Express the following ratios as representative fractions. For example, 3 amperes : 5,000 milliamperes expressed as a representative fraction is $\frac{3}{5}$.

(1) 400 cycles : 20 kilocyles
(2) 10 minutes : 450 seconds
(3) 35 days : 1 year (365 days)
(4) 50 volts : 2 kilovolts
(5) 3 microfarads : 2 farads
(6) 300 tons : 2 tons

(i) The scale shown in figure 57 is exactly the same size as the one shown on a certain radio direction-finder chart. What are the actual distances that are represented on the chart by lengths of $1\frac{1}{2}$", $2\frac{3}{4}$", $3\frac{5}{16}$", and $4\frac{3}{8}$"?

Figure 57.

(j) The scale on a road map is indicated as 1 inch = 20 miles. What fraction represents the scale of the map? (1 mile = 5,280 feet = 63,360 inches.)

(k) An original radio direction-finder chart uses a scale of 1 : 2,000,000. To what distance in miles does a 1-inch line on this chart correspond?

(l) The wingspan of a model airplane is 8 inches. It was built to a scale of 1 : 72. What is the wingspan of the original plane?

(m) The following table is designed to show the scale lengths used for actual lengths on a scale of $\frac{1}{8}$" = 1'. Complete the table.

Scale Length	$\frac{1}{8}''$							
Actual Length	1′	3′	4′	6′	8′	20′	32′	40′

(n) A radio chassis is 2 feet wide and 3 feet long. Draw a plan of the chassis to a scale of $\frac{1}{4}'' = 1'$.

(o) Using a scale of $\frac{1}{2}'' = 28'$, draw lines representing the following distances: $45\frac{1}{2}$ feet, 84 feet, $150\frac{1}{2}$ feet, and 168 feet.

(p) Draw a line representing $7\frac{1}{4}''$ to a scale of $3'' = 1'$. Draw a line representing a length of 5′ 6″ to a scale of $\frac{1}{4}'' = 1'$. Check your work against the answers with the compass for (o) and (p).

STUDY GUIDE ON SCALE DRAWINGS—SIMILAR TRIANGLES

60. When an architect or engineer draws a floor plan or machinery diagram he carefully selects his scale, which is the ratio of the drawing dimensions to the actual dimensions of the object. The degree of accuracy he requires and the size of available paper guide him in selecting this scale. Thus, you see that ratio and proportion are important in scale drawing. You studied the basic principles of ratios in the chapters on fractions, and you used proportions when solving problems with a slide rule. You will use the proportional idea when you work problems involving similar triangles in this chapter. The following is a brief review of the ideas of ratio and proportion and their applications to scale drawing.

61. You know that a ratio is the quotient of one number divided by a second number. For example, the ratio of 4 to 2 may be written 4 : 2 or $\frac{4}{2}$ and has the value 2. The ratio of 6 to 9 is $\frac{2}{3}$. You also know that a proportion is a statement of the equality of two ratios. When the ratios are written in fraction form, the proportion says that the ratio of the numerator of one fraction to its denominator is the same as the ratio of the numerator of the second fraction to its denominator. The relation $\frac{12}{6} = \frac{8}{4}$ is a proportion. When you say that the sides of similar triangles are proportional, you mean that the three ratios of cor-

responding sides are equal. If the sides of one triangle are 12, 8, and 6 and the sides of a similar triangle are 6, 4, and 3, then equating the three ratios of corresponding sides gives the continued proportion—

$$12/6 = 8/4 = 6/3.$$

Instead of this continued proportion you may write three proportions as follows—

$$12/6 = 8/4,\ 8/4 = 6/3,\ 12/6 = 6/3.$$

Since the middle terms in a proportion may be interchanged, these proportions also may be written—

$$12/8 = 6/4,\ 8/6 = 4/3,\ 12/6 = 6/3.$$

62. Two angles are equal if in both cases exactly the same amount of turning is necessary to bring their sides together, regardless of the lengths of those sides. However, two straight lines are equal only if they are exactly the same length. When you make scale drawings, you copy angles exactly as they are in actual size; the object is reduced or enlarged by changing the lengths of the lines, which are called the DIMENSIONS.

63. Two figures are SIMILAR when their only difference is the scale to which they are drawn. This means that their shapes are identical, but the dimensions of one figure may all be smaller than the corresponding dimensions of the other. When two figures are similar, the ratios of all dimensions of one figure to the corresponding dimensions of the other figure are the same.

64. All straight-line figures may be divided into triangles; therefore, you will use the triangle as the basis for your study of similar figures. Often two triangles like those shown in figure 58 appear to be similar, but you will want definite proof. If two triangles are similar, their corresponding angles are equal and the sides of one triangle are proportional to the corresponding sides of the other. In figure 58 $\angle A = A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, and $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$.

If two triangles satisfy either of these conditions they are similar. For example, you may say that $\triangle\ ABC$ is similar

to $\triangle A'B'C'$ because the corresponding angles of the two triangles are equal; you may also say that the two triangles are similar because the sides of one triangle are proportional to the corresponding sides of the other. However, if you wish to prove the similarity of two triangles, you do not have to measure all the sides and angles of both figures.

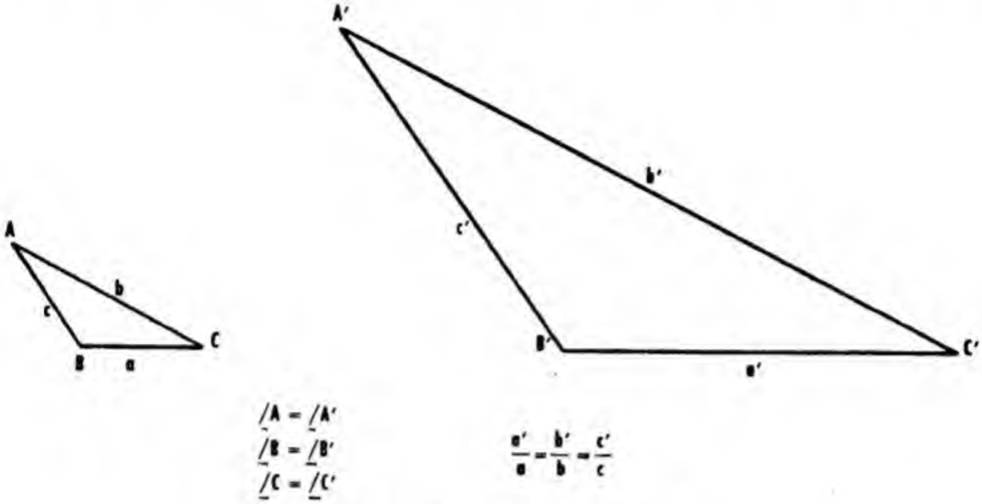
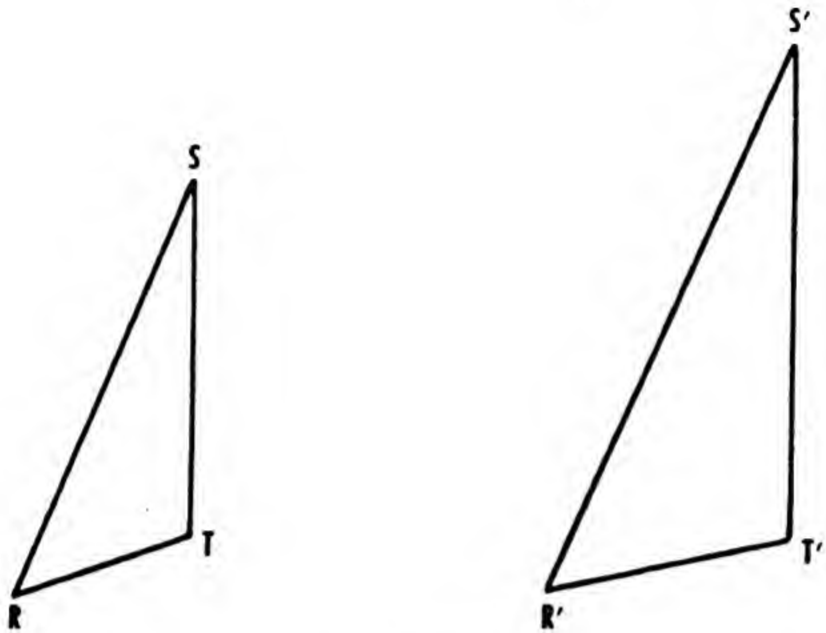


Figure 58.—Similar triangles.

65. If two angles of one triangle are equal to two angles of a second triangle you can correctly state that the two triangles are similar. Consider the two triangles shown in figure 59, in which you know that $\angle R = \angle R'$ and $\angle S = \angle S'$. Since the sum of all the interior angles of any triangle is $180°$, $\angle T = 180° - (\angle R + \angle S)$, and $\angle T' = 180° - (\angle R' + \angle S')$. Since you are subtracting equal amounts from two identical quantities, the results are equal, or $\angle T = \angle T'$. Whenever two angles of one triangle equal two angles of a second traingle, the remaining angles are also equal. Therefore, the triangles are similar.

66. Here is another way to show that two triangles are similar. It combines the "equal angle and proportional sides" ideas. Two triangles are similar if an angle of the first triangle equals an angle of the second triangle and the including sides are in proportion. This method is illustrated in figure 60.



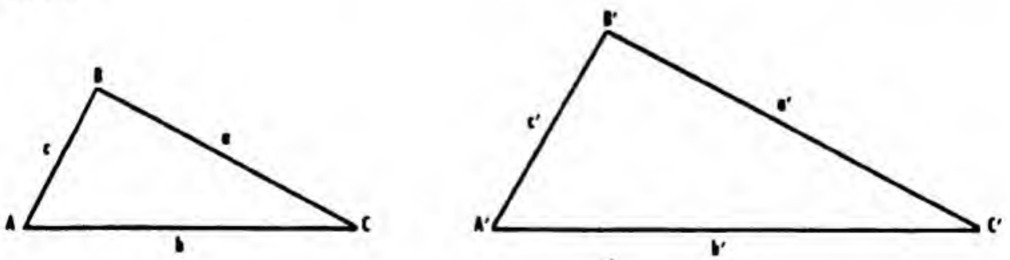
$$\angle R = \angle R'$$
$$\angle S = \angle S'$$

Figure 59.—One way to show similarity of two triangles.

67. You can now show that two triangles are similar if they satisfy any one of the following three sets of conditions:

(1) TWO ANGLES OF ONE TRIANGLE ARE EQUAL TO TWO ANGLES OF THE SECOND TRIANGLE.

(2) THE SIDES OF THE TWO TRIANGLES ARE IN PROPORTION.



$$\angle A = \angle A'$$
$$\frac{c}{c'} = \frac{b}{b'}$$

Figure 60.—These triangles are also similar.

(3) AN ANGLE OF ONE TRIANGLE EQUALS AN ANGLE OF THE SECOND TRIANGLE AND THE INCLUDING SIDES ARE IN PROPORTION.

CONSTRUCTING SIMILAR TRIANGLES

68. The three ways in which you may prove the similarity of two triangles are summarized in the preceding paragraphs. Any one of these sets of conditions may be used as the basis for constructing a triangle similar to a given triangle. In any such construction you must select the scale you wish to use, which is the ratio of the corresponding sides. Figure 61 shows how triangle $M'N'O'$

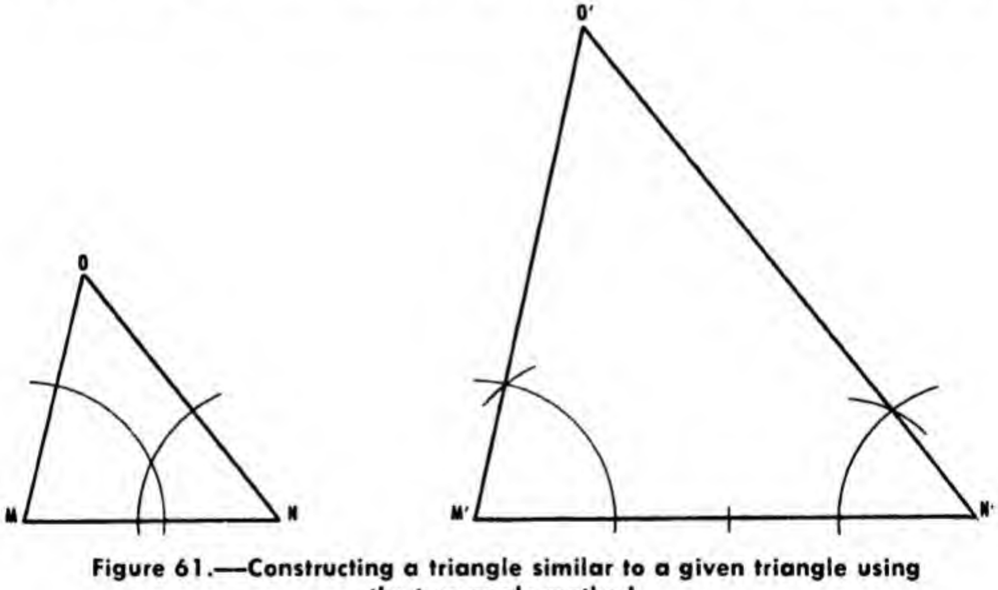


Figure 61.—Constructing a triangle similar to a given triangle using the two angle method.

is constructed similar to triangle MNO, using the two-angle method and a ratio of 2; all sides of triangle $M'N'O'$ are twice the length of the corresponding sides of triangle MNO. Side MN is measured and a line $M'N'$ equal to twice the length of MN is constructed. Then angles equal to M and N are constructed at M' and N'. By extending the sides of $\angle M'$ and $\angle N'$ to intersect each other the similar triangle is completed; the intersection of the extended sides is the vertex of the third angle O′. If the construction is accurately done, all the sides of the second triangle will

be proportional to those of the original triangle and the corresponding angles of the two figures will be equal.

69. Suppose you know the lengths of all three sides of the triangle shown in figure 62, a. You wish to construct a

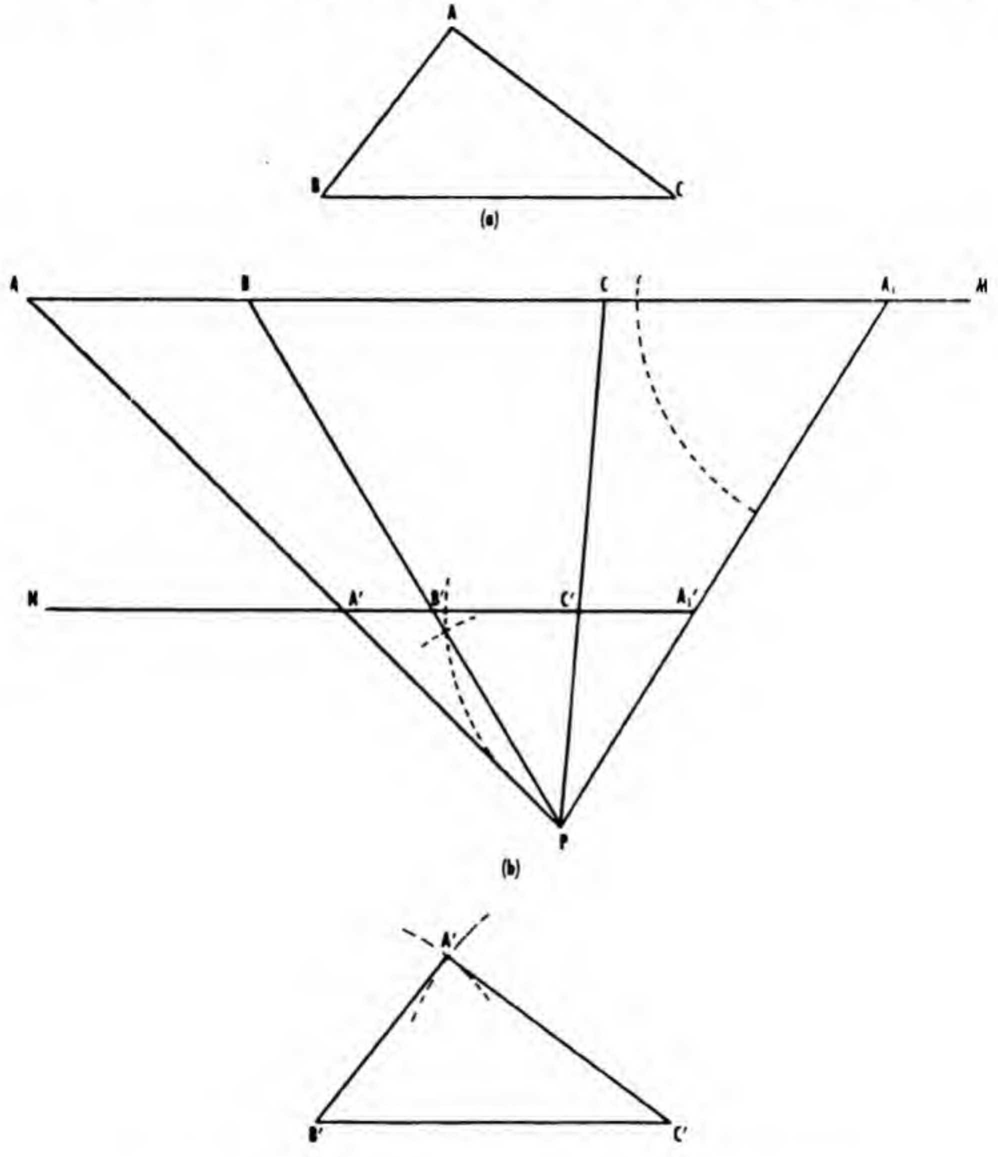


Figure 62.—Using parallel lines to construct similar triangles.

triangle similar to it with sides $\frac{2}{5}$ the length of the corresponding sides of the original triangle. You can use a simple method involving parallel lines to find the dimensions of the similar triangle. Draw a line AM slightly

longer than the perimeter of the triangle ABC in figure 62, a. Then mark off the three sides of triangle ABC on AM, just as if you had unfolded the triangle into a straight line. Look at figure 62, b. These segments are shown as AB, BC, and CA_1. Now, at A_1 draw a line A_1P at any convenient angle (about 60°) with AM, and using your dividers, divide A_1P into five equal parts. Mark the point two segments distant from P as A_1'. Through this point A_1' construct a line $A_1'N$ parallel to AM. This can be done with pencil compass or T-square and triangles. Draw PA, PB, and PC intersecting $A_1'N$ at A', B', and C' respectively, as shown on figure 62, b. The segments $A'B'$, $B'C'$, and $C'A_1'$ will then be proportional to the corresponding sides of triangle ABC in the ratio 2 : 5. Using these segments you can construct as in figure 62, c, the similar triangle $A'B'C'$ with your dividers, straightedge, and compass. By comparing with the original triangle in figure 62, a, notice that

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'A'}{CA} = \frac{2}{5}.$$

70. Another method you may use to construct a triangle similar to a given triangle is shown in figure 63. In this case you must know one angle of the given triangle and the lengths of the two sides including that

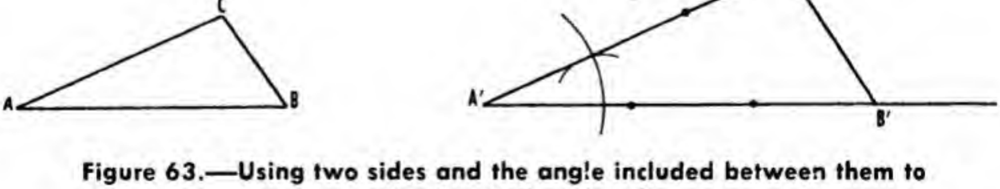


Figure 63.—Using two sides and the angle included between them to construct a similar triangle.

angle. Figure 63 shows the original triangle and the construction of the similar figure. The sides AB and AC and $\angle A$ are known. In this example the similar figure has been drawn $1\frac{1}{2}$ times the size of the original. Each side is $1\frac{1}{2}$ times the length of the side corresponding to it

in the original triangle. First construct $\angle A'$ equal to $\angle A$. Then extend the sides of $\angle A'$ to form two sides of the triangle, $A'B'$ and $A'C'$, making them $1\frac{1}{2}$ times the corresponding sides (AB and AC) in the original figure. Complete the construction by joining points B' and C'. Then $\triangle A'B'C'$ is similar to $\triangle ABC$, and each side of the newly constructed figure is $1\frac{1}{2}$ times as long as the side corresponding to it in the original triangle.

71. You may use your knowledge of similar triangles to solve many interesting and useful problems. Figure 64 illustrates one problem involving the distance between

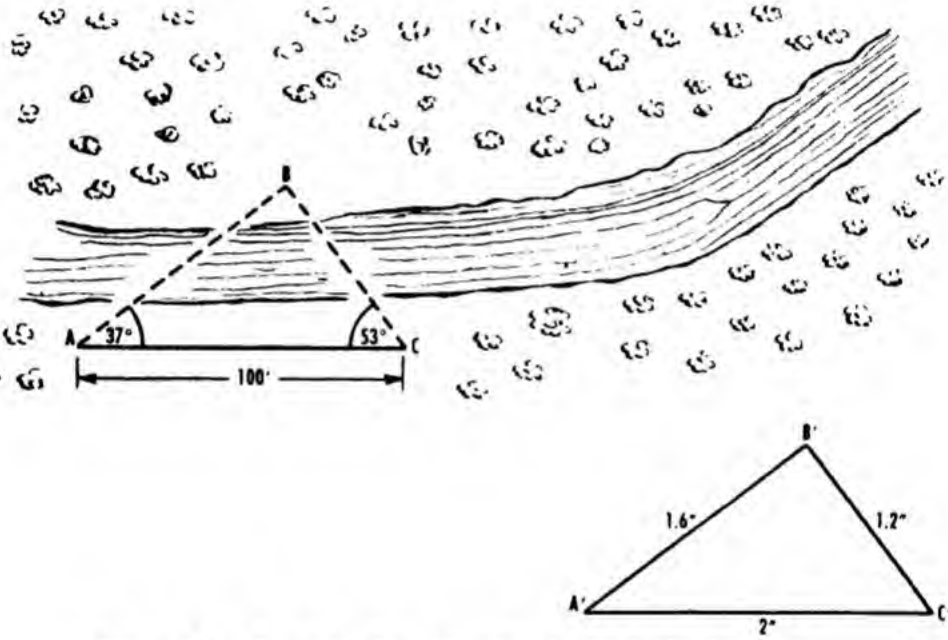


Figure 64.—A practical problem involving similar triangles.

two radio stations. A and B represent two radio stations which are located on opposite sides of a river. You would have difficulty in measuring the distance between them directly, but you could find that distance by indirect measurement. You might do this by locating any point C at a known distance from either one of the stations. Point C forms the third vertex of a triangle ABC. Suppose you know AC to be 100 feet; by taking bearings of B from both A and C you find $\angle BAC = 37°$ and $\angle BCA = 53°$. Since you know two angles, you can construct a similar

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triangle $A'B'C'$ on a smaller scale. On this similar triangle you can measure the side lengths. Since the scale drawing and the actual distances are proportional, you may now set up a proportion involving two measured sides of the similar triangle and the one known side of the actual triangle to find the unknown side AB. Suppose you construct the small triangle to the scale of $1'' = 50'$. This means that

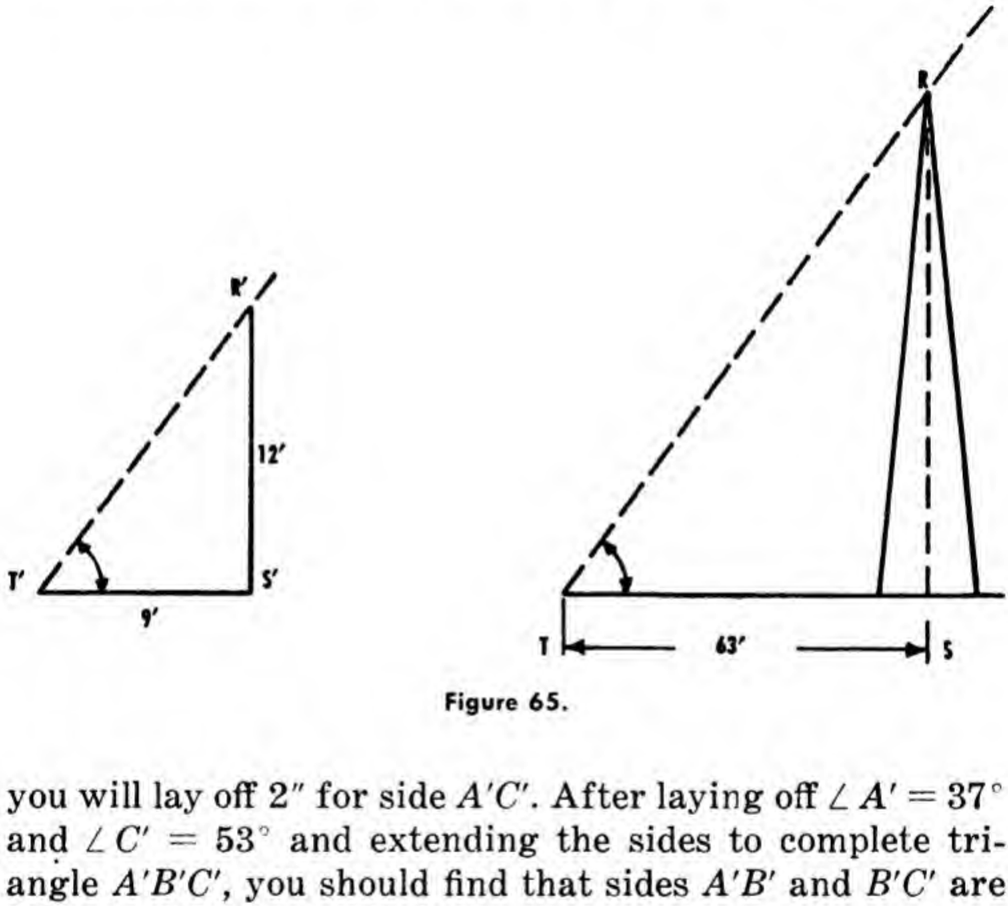


Figure 65.

you will lay off $2''$ for side $A'C'$. After laying off $\angle A' = 37^\circ$ and $\angle C' = 53^\circ$ and extending the sides to complete triangle $A'B'C'$, you should find that sides $A'B'$ and $B'C'$ are $1.6''$ and $1.2''$, respectively.

Since the triangles are similar you may write—

$$\frac{AB}{AC} = \frac{A'B'}{A'C'}$$

Replacing the letter symbols by actual values—

$$\frac{AB}{100} = \frac{1.6}{2}, \text{ or } AB = 80.$$

The distance between A and B is approximately $80'$.

72. Here is another problem that may be solved by using similar triangles. Look at figure 65. The radio beacon tower casts a shadow 63 feet long on level ground. The post, which is 9 feet high, casts a shadow 12 feet long on level ground. How high is the tower? The sun, which provides the light to cast the shadow, is so far away that you may assume $\angle T = \angle T'$. The angles between the pole and the ground, and between the tower and the ground are both right angles; therefore, they are equal. Since two angles of one triangle equal two corresponding angles of the second, the triangles are similar. You may then set up a proportion involving the sides. From this proportion you may solve for the height of the tower.

$$\frac{RS}{TS} = \frac{R'S'}{T'S'}$$

Replacing the letter symbols by the known values—

$$\frac{RS}{63} = \frac{12}{9}, \text{ or } RS = 84'.$$

The height of the tower is 84 feet.

READING AND USING SCALE DRAWINGS

73. When you draw to scale, you make dimensions proportional. This means that if you are drawing a ship, the ratio of the actual mast height to the line representing the mast on the drawing is the same as the ratio of the actual ship's length to the line representing that length.

74. In any one scale drawing, only one ratio or scale is used. Usually the chart or map indicates what that scale is. If the length of an object is 30 feet and if you draw a line 30 inches long to represent it, you say that your scale is 1 inch per foot. You then note on the drawing, Scale $1'' = 1'$, or Scale 1 : 12. The latter note indicates the scale as a ratio. Notice that the first number refers to the drawing and that the second number refers to the object. If you were making a drawing three times the size of the

object, you would write, Scale 3 : 1. This ratio may be expressed also as a fraction; in the first case it is Scale $\frac{1}{12}$ and in the second case, Scale $\frac{3}{1}$. When the scale is written as a fraction or ratio, all numbers must be expressed in the same units. For example, if the scale is 1 inch = 1 yard, you should write the scale as 1 : 36 or $\frac{1}{36}$. The fractional form of the ratio is known as the REPRESENTATIVE FRACTION (abbreviated RF).

75. There are several ways in which the scale of a drawing may be indicated. Often a fraction of an inch is used to represent 1 foot of actual length. Most mechanical and electrical drawings and a ship's compartment plans are drawn to a scale of $\frac{1}{4}'' = 1'$; usually this information is written in one corner of the drawing. For example, a note on the plan of a ship's radio repair compartment indicates the scale to be $\frac{1}{4}'' = 1'$. What is the representative fraction of the drawing? The scale may be written $\frac{1}{4}'' = 12''$, or 1 : 48. The representative fraction is $\frac{1}{48}$. If the width of the compartment on the drawing is $2\frac{3}{8}''$, what is the actual width of the radio repair compartment? You may find this dimension by setting up a proportion involving the representative fraction, the width on the drawing ($2\frac{3}{8}''$), and the actual width that you are seeking, thus:

$$\frac{1}{48} = \frac{2\frac{3}{8}}{W}, \text{ or } W = \frac{19}{8} \times 48 = 114'' \text{ or } 9\tfrac{1}{2}'.$$

Therefore, the actual width of the radio repair compartment is $9\frac{1}{2}'$.

76. Here is a similar problem which involves the drawing of a radar system layout. The diagram indicating the placement of the radar consoles and repeaters is drawn to a scale of $\frac{3}{8}'' = 1'$. On the drawing the two radar consoles are spaced $1\frac{1}{4}''$ apart. What is the RF of the drawing, and what is the actual distance between the two radar consoles? The scale indicates that $\frac{3}{8}'' = 12''$. From this you will find the representative fraction to be $\frac{1}{32}$. The

true distance may be found from the proportion set up here:

$$\frac{1}{32} = \frac{1\frac{1}{4}}{D}, \text{ or } D = \frac{5}{4} \times 32 = 40'' = 3'4''.$$

The actual distance between the radar consoles is 3′ 4″.

77. Sometimes the drawing scale is expressed directly as the representative fraction; for example, scale $\frac{1}{600}$. This may be arranged to read in inches or any other unit of length you choose; for example 1″ = 600″, or in terms of inches and feet, 1″ = 50′. If a drawing of a ship's rigging and radio antenna is made to scale $\frac{1}{600}$ and if the transmitter antenna is represented by a line $2\frac{3}{4}$″ long on the drawing, what is the length of the transmitter antenna? Set up the following proportion—

$$\frac{1}{600} = \frac{2\frac{3}{4}}{L}, \text{ or } L = \frac{11}{4} \times 600 = 1650'' = 137'6''.$$

The antenna is 137′ 6″ long.

CHARTS AND MAPS

78. The scales used on charts and maps are usually drawn in. Either a small section of the scale is drawn in the lower right-hand corner, called the KEY, where all information for use with the chart is located, or one or more entire sides of the chart are marked off in the scale units. Such scales are called GRAPHIC SCALES. Figure 66 illustrates a chart using a graphic scale. Notice that on

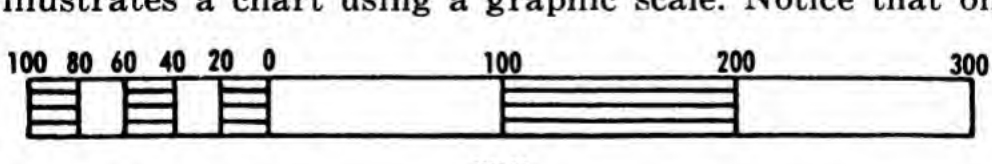


Figure 66.

this chart 1″ represents 100 miles. You may note this fact in the key of the chart and in the graphic scale, but you must be careful not to base measurements on this NOTED scale unless you are using the original drawing.

The reason for such caution is clear; if the drawing were photographed or copied in some way on a smaller or larger scale, the graphic scale would be enlarged or reduced proportionately. On any copy that had dimensions different from the original, the scale would certainly not be 1 inch = 100 miles. When using a graphic scale to find the distance between any two points, you simply span the distance with your dividers and then transfer the dividers to the graphic scale, where you can read off the distance units directly.

79. If you do not have a pair of dividers, you may use another method to measure distances on a graphic scale. Place the edge of a straight piece of paper along a line joining the two points in question. Make a small pencil mark opposite each point. Then transfer the paper so that the same edge is alongside the graphic scale and read off the distance between the two marks you have made. Notice that the graphic scale used in the key may have two parts. From the zero to the right, the scale in figure 66 is marked off in large units; from the zero to the left, it is marked off in smaller divisions. Referring to figure 66, if you had to measure a distance of 140 miles, you would place the right-hand mark on the edge of the paper opposite the 100-mile mark on the scale. The left-hand mark would then fall opposite the 40-mile mark on the left of the zero. When the graphic scale extends for the entire side of a chart, you will have to count the units on the scale between the divider points or between the marks on the paper you used for measuring. Any fraction of one division should be approximated, keeping in mind the scale of the drawing.

DRAWING TO SCALE

80. Two types of scales are used for plan drawing and blueprint reading; the engineer's scale and the architect's scale. They are usually stamped on rules of triangular cross section. In this way, six measuring edges are pro-

vided, each marked off with one or two scales. Figures 67 and 68 show the two triangular rules. Sometimes small flat rules with only one or two scales are used.

81. The engineer's scale, shown in figure 67, is divided decimally, having various scales on which 1 inch is divided into 10, 20, 30, 40, 50, or 60 parts. This scale is generally used for constructing plans of large areas, such as maps. With the engineer's scale you can lay off, measure, divide, and subdivide distances on a map or chart.

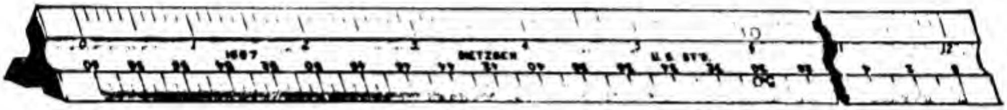


Figure 67.—The engineer's scale.

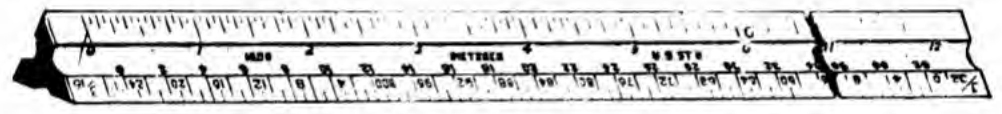


Figure 68.—The architect's scale.

The 10 scale, for example, may be used when you wish to represent 100′ by 1″. Then each division, which is $\frac{1}{10}$ inch long, represents 10 feet. The 20 scale, which has each inch divided into 20 parts, may be used where 1″ = 200′. Then each division represents 10 feet.

82. The architect's scale, shown in figure 68, is proportionately divided into feet and inches. The first scale, used when drawing an object smaller than actual size, is the half-size scale, that is, 6″ = 1′. The half-inch points are marked as inches, the quarter-inch points as half inches, and so on down to the smallest units shown on the scale. In the example shown in figure 69 a line is drawn representing an actual distance of $2\frac{3}{4}$″, using the half-size scale.

83. If the half-size scale is too large for your purpose, you may use any of the following scales. You can find all of them on the architect's triangular scale. You simply

select the scale that is most convenient for the drawing you wish to make.

$$
\begin{aligned}
3'' &= 1' \text{ (1/4 size)}\\
1\tfrac{1}{2}'' &= 1' \text{ (1/8 size)}\\
1'' &= 1' \text{ (1/12 size)}\\
\tfrac{1}{2}'' &= 1' \text{ (1/24 size)}\\
\tfrac{1}{4}'' &= 1' \text{ (1/48 size)}
\end{aligned}
$$

The next size smaller than the half-size scale is the quarter-size scale, as shown in figure 68; it is marked by a number 3 at one end, which indicates that on it $3'' = 1'$.

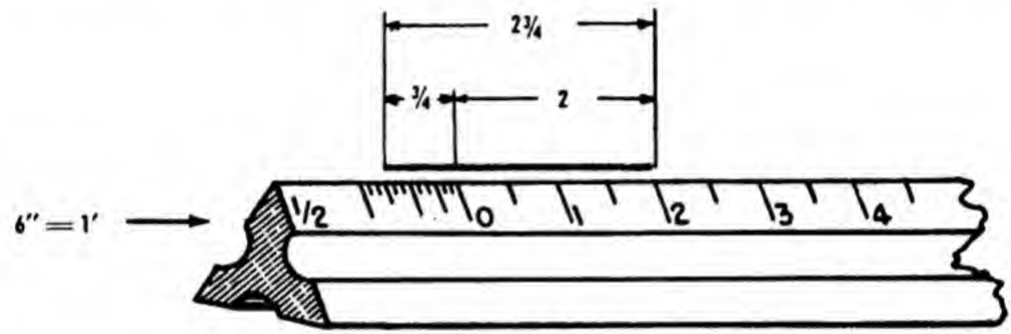


Figure 69.—Using the half-size scale.

The 3 inches are divided into 12 parts. When you use this scale for drawing, these 12 divisions represent inches. Each of the 12 divisions are subdivided into 8 parts, which represent eighths of an inch. Starting from the zero mark, you measure inches to the right and feet to the left. Three inches to the left of the zero is the number 1, meaning that on this scale a distance of 3 inches represents 1 foot.

84. Suppose you have to draw one side of an object whose length is $1'\ 5\tfrac{5}{8}''$; you wish to use the quarter-size scale. First locate the zero on the scale. Remember, you measure feet to the left and inches to the right. Your line would be from the $1'$ mark on the left of the zero to the $5\tfrac{5}{8}''$ mark on the right. This problem is illustrated in figure 70. As an electronics technician at an advanced base, you may be called upon to supervise the installation of new electronic equipment. The equipment layout plan

usually is drawn to the architect's scale. If you have an architect's scale you should practice drawing lines representing various lengths, using the different scales. In this way you will become familiar with the use of the archi-

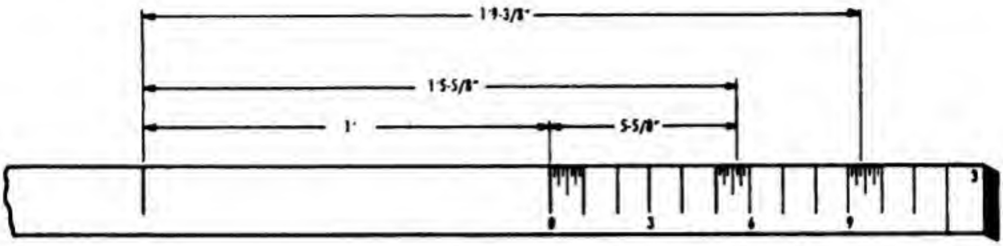


Figure 70.—Scale: 3″ = 1′.

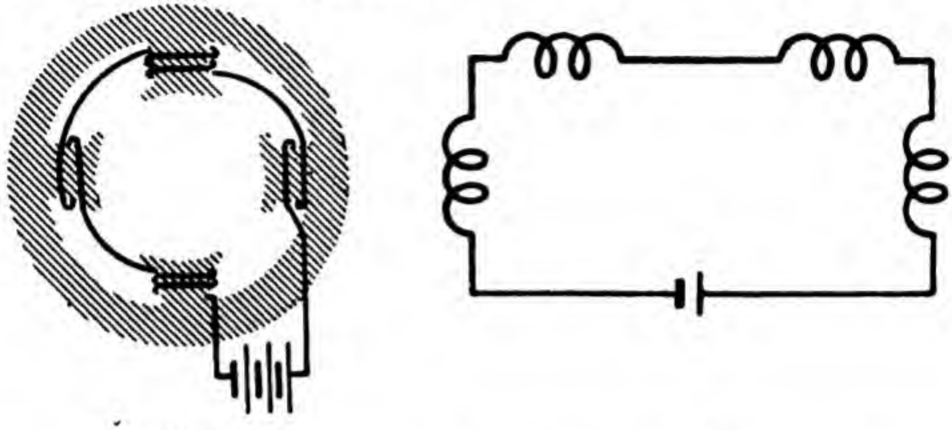
tect's scale and develop your understanding of it. You will then be able to read scale drawings or make them yourself.

ELECTRICAL SCHEMATIC AND WIRING DIAGRAMS

85. You have learned to construct and read scale drawings. Diagrams of this type provide reduced or enlarged duplicates of original objects so that you may study them conveniently. In electronics work, scale drawings are often used to show the mechanical details of parts placement. However, for wiring, trouble shooting, and circuit analysis, two other types of drawings are used.

86. If you examine the underside of your home radio, you will realize that a scale drawing of all its details would be more confusing than helpful. Such a diagram would be a difficult drawing problem and would take a great deal of time to complete. Electronics engineers and technicians use a simpler type of diagram, called a SCHEMATIC, which is easily drawn and understood. A schematic diagram is a drawing using an electrical shorthand; it shows the plan of all wiring connections and on it all electrical parts are represented by symbols. By using a schematic diagram you can show electronic equipment on a drawing of con-

venient size. Schematic diagrams of electronic devices enable you to trace out wiring circuits for trouble shooting and make it easy for you to understand and explain principles of operation. You will learn how to use these diagrams to find quickly and easily the trouble in damaged or inoperative equipment.



(a) (b)

Figure 71.—The wiring and schematic diagram of the field winding of a motor.

87. Another type of drawing used in electrical and electronics work is the WIRING DIAGRAM. The wiring diagram is somewhat similar to the scale drawings you have studied in that it shows certain structural parts of equipment; however, wiring diagrams are not necessarily drawn to an exact scale, and often symbols may be used for some parts while others are represented by pictures. Figure 71, a, is a wiring diagram of an electric motor, showing the coils, the motor frame, and the pole pieces on which the coils are wound. Figure 71, b, is the schematic diagram of the same motor. In the latter figure the coils are indicated by symbols and the battery is represented by the symbol for an electric cell.

88. When you draw schematic diagrams you use solid lines to represent wires or other metallic connections. On any schematic diagram the length of the lines representing wires or other connections does not represent their length in the actual equipment. Parts are placed on a schematic for convenience in circuit tracing, and connection lines are drawn in any convenient lengths. When

two connecting lines do not make contact with each other but happen to cross on the diagram, they are drawn as shown in figure 72, a. If the intersection of two connection lines is to represent a contact, a dot is placed at the intersection as shown in figure 72, b. Occasionally, you

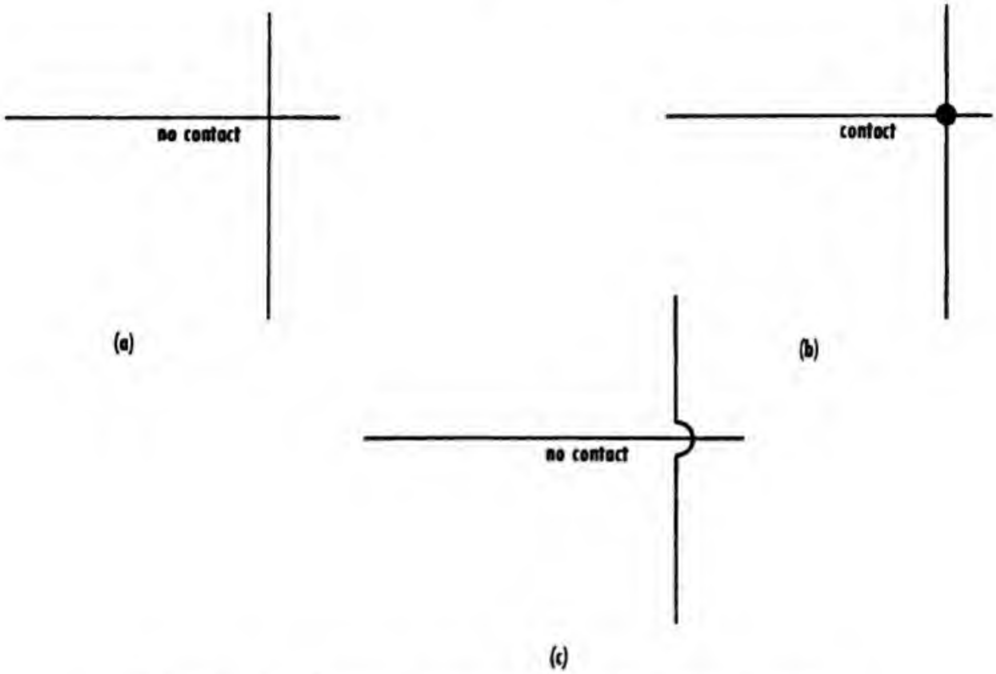


Figure 72.—Schematic representation of intersecting wires.

may see drawings in which two connection lines that do not make contact are drawn, as shown in figure 72, c, and those that do make contact are simply made to intersect. You will have to exercise care in reading and drawing schematic diagrams to avoid errors in representing connections; one of the first things you should do when you read a schematic diagram is to decide which system of connection symbols is being used for the wiring.

89. You should learn to recognize all electronic symbols at a glance so that you will be able to read schematics quickly and easily. A list of some of the symbols used in schematic drawings is shown in figure 73. Since the vacuum tube is the most vital part of any piece of electronic equipment, it is particularly important that you know the

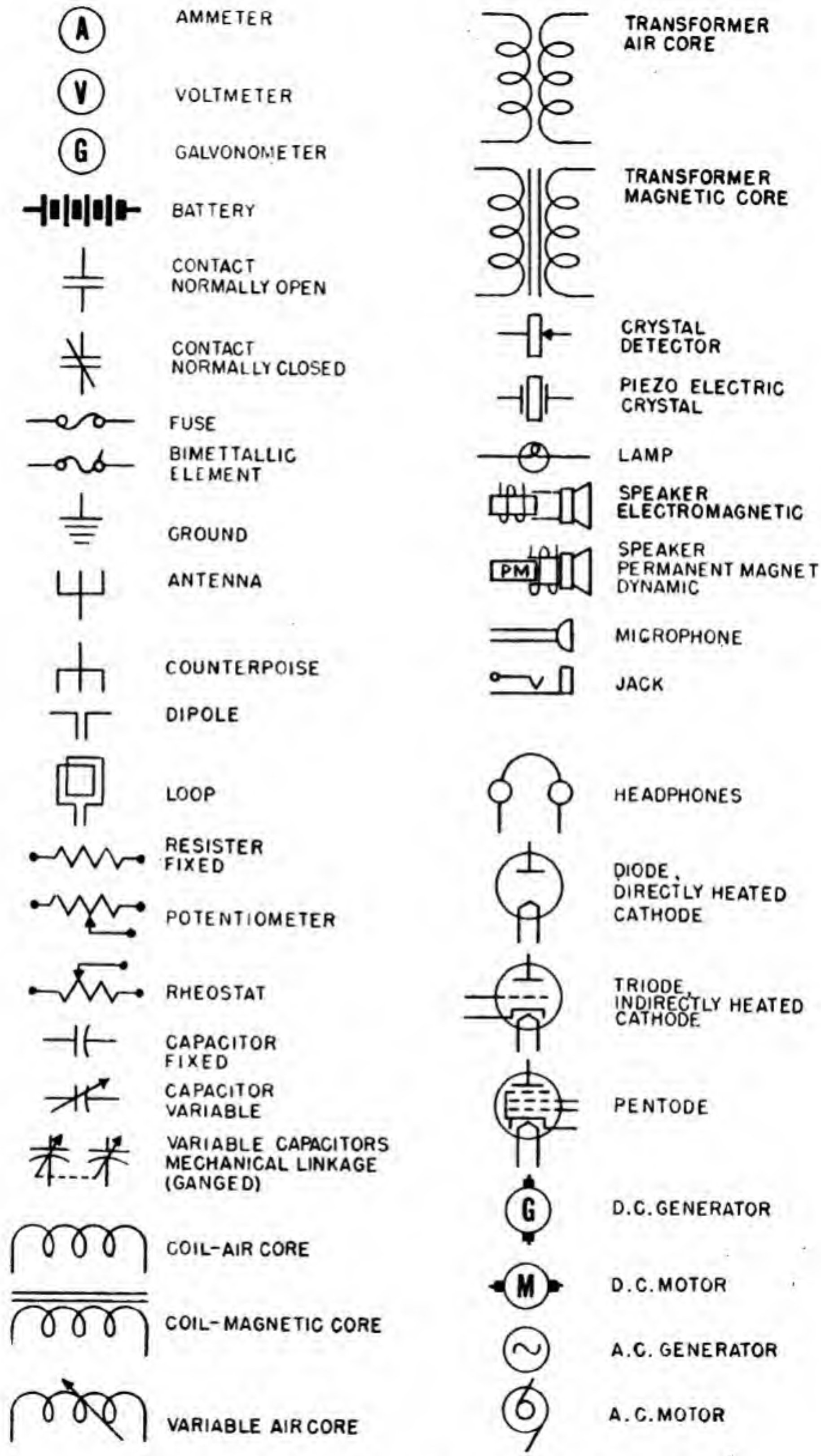


Figure 73.—Electronic symbols used in schematics.

symbols for the principal types of tubes and their parts. An experienced electronics technician often begins at one of the terminals of a tube when tracing a circuit.

90. At first when you look at schematic diagrams you may find it difficult to imagine the meaning of the symbols. Figure 74 will help you learn to associate the various symbols with the parts they represent. This illustration combines the schematic diagram of a simple four-tube receiver and a series of pictures of the component parts.

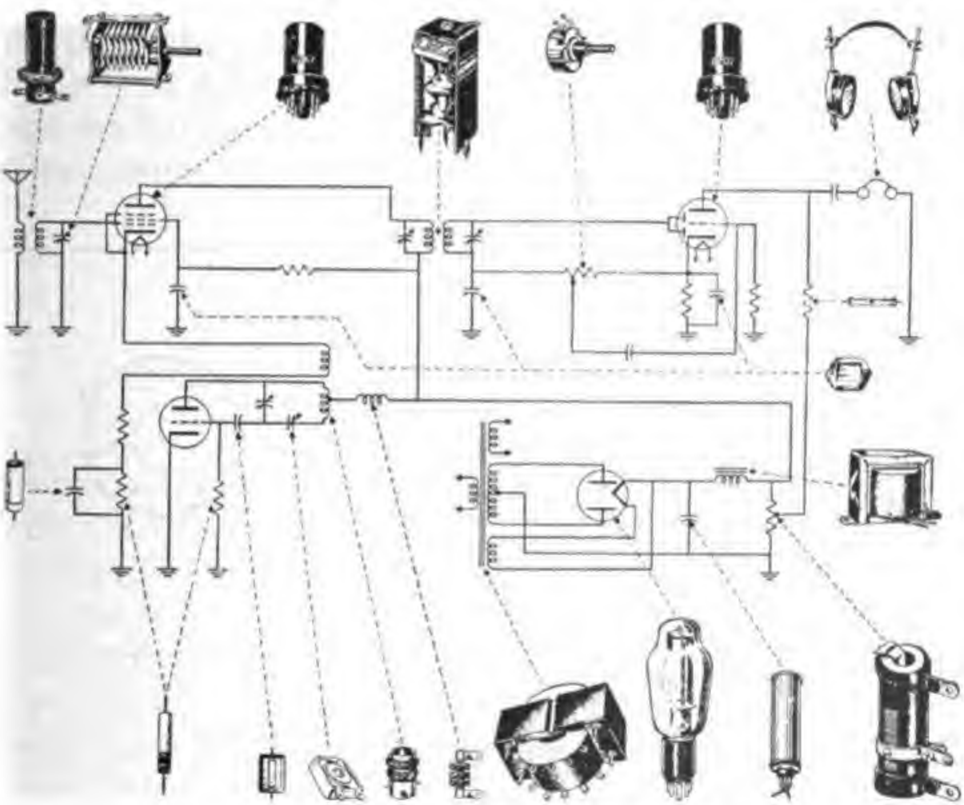


Figure 74.—What the symbols mean.

The drawings around the border of the schematic show what each part looks like. If you are able to do so, study a radio chassis at your Naval Reserve Training Center; try to identify all the parts and find the correct symbols to represent them. Another way to learn the symbols for electronic parts is to examine your home radio and try to find the symbol for each part you can identify.

THE VACUUM TUBE AND THE TUBE SOCKET DIAGRAMS

91. The elements inside a vacuum tube are tied directly to pins that extend from the base of the tube. A typical eight-pin tube, called an OCTAL BASE TUBE, is shown in figure 75, a. Figure 75, b, is a schematic representation of a 6C5G triode with an octal base, although three of the pins are not used. The diagram shows all the tube elements. The pins connected to each element are numbered. The corresponding octal socket is shown in figure 75, c. Notice that all the holes are of equal diameter. When you look at the bottom of a radio chassis you can identify the tube pins by counting from the key. Looking at the bottom and moving clockwise from the key, you will see that the first pin is number 1. The other pins are numbered in

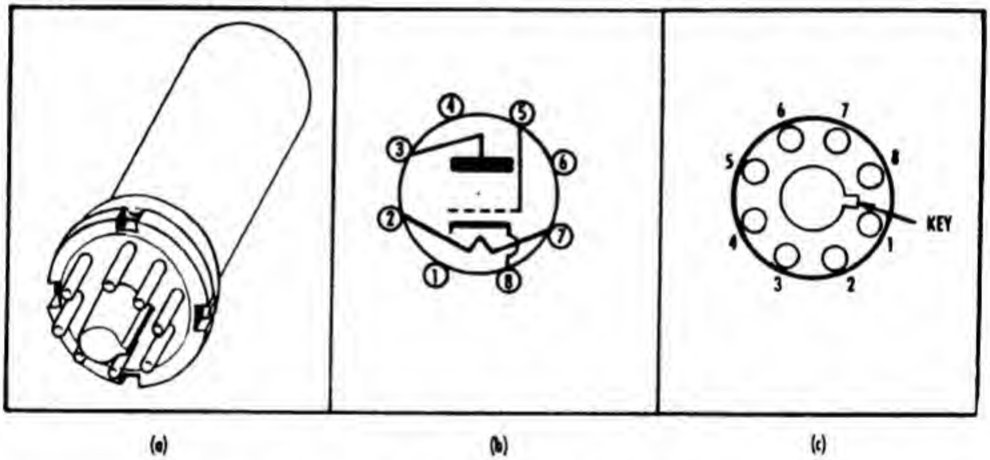


Figure 75.—Octal tube and tube socket diagrams.

clockwise order, as shown in figure 75. A six-hole tube socket is shown in figure 76. In this type of socket two of the holes are larger than the others, and the key is usually a small lug of bakelite protruding from the base between the two larger holes. The pins are numbered in a clockwise direction, as shown in figure 76. If you look at a tube socket from the TOP of a chassis the numbers corresponding to the tube pins will read COUNTERCLOCKWISE from the key.

92. You will often have to identify a circuit component on the schematic diagram after you have found that part on the radio chassis. To do this, you will have to trace wires on both the wiring and the schematic diagram, as well as the wiring inside the equipment itself. Suppose you see a burned resistor inside a radio set and want to replace it. Since the color coding (the painted marks that

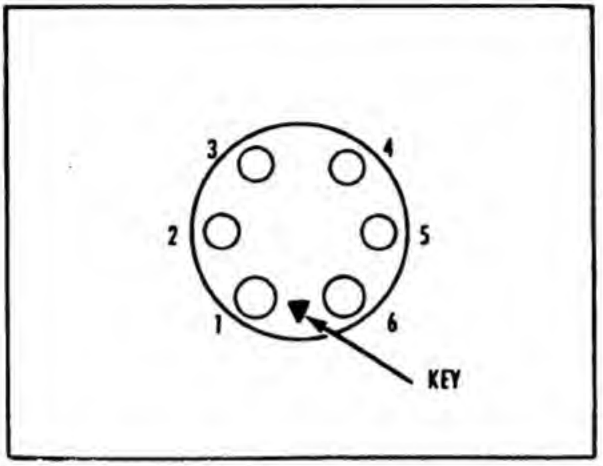


Figure 76.—A six-hole tube socket.

indicate the value of resistance) is burnt off, you do not know what size of resistor to use as a replacement. First, determine the tube element or other component to which the resistor is connected; then turn to the schematic diagram and locate the resistor on it. The diagram will tell you the correct value. You can also use the diagram to determine the purpose of that resistor in the circuit. You may then figure out what substitute resistor to use if you do not have an exact replacement on hand.

THE OHMMETER

93. Sometimes a piece of electronic equipment will not operate, even though all the parts appear to be in good condition. When you study electronic circuits you will learn to trace the trouble with various instruments. You will not be able to check electronic gear that is connected to a power line until you have studied some of the elements

of circuit design. However, you can often locate the trouble in radar or radio sets by checking the circuit when the power is shut off, using an instrument called an OHMMETER. You can use an ohmmeter to find broken connections (open circuits) or to locate parts that are connected when they should be separated (short circuits). You

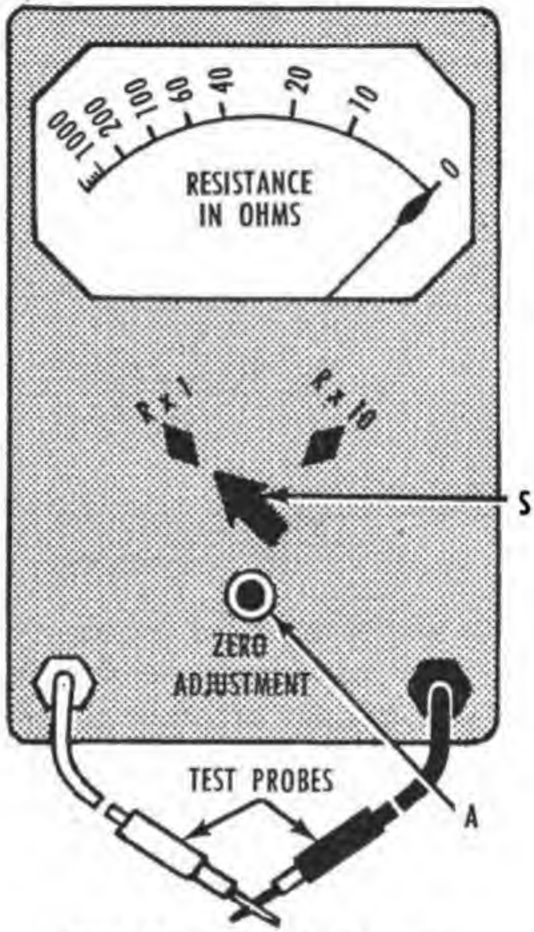


Figure 77.—A simple ohmmeter.

may also use an ohmmeter to measure resistance values or to find out whether or not a capacitor is in good condition. Figure 77 shows a simple ohmmeter. This instrument measures the electrical resistance of a part or circuit in ohms, the standard unit of electrical resistance. Note that the ohmmeter reads zero ohms at the right-hand end of its scale. Two test contacts called PROBES are attached to leads from the meter. The resistance to be measured

is placed across these probes, and the meter scale registers the resistance in ohms. A battery inside the ohmmeter provides the current for these measurements. A variable resistance in the meter circuit may be adjusted since battery pressure falls off with age. This compensating resistance is called the ZERO ADJUSTER (*A* in fig. 77). It is correctly adjusted if the meter reads zero when the two probes are touched together. Inside the meter there are several different resistances which may be included in the circuit by turning the switch marked *S*. The meter range covers low or high values, depending on which of these resistances are included. You should always use the meter on the range on which you will get the most accurate reading.

THE OHMMETER AND SCHEMATIC DRAWING

94. When an electronics technician has to work on unfamiliar equipment for which he has no circuit diagram, his first step is to sketch a schematic diagram of the circuit. Before drawing that diagram he must identify the components and the manner in which they are connected. However, there are many different electronic parts that look alike and cannot be identified accurately without instruments. Component parts that are totally enclosed (such as transformers) cannot be drawn until their internal wiring has been determined. Although the ohmmeter can be used only to measure resistance, the resistance of a part is very useful in identifying it and its internal wiring circuits.

95. The resistance of a simple cylindrical resistor, like the one shown in figure 78, a, can be measured easily with an ohmmeter. Place one probe of the ohmmeter on each end of the resistor and read the resistance on the meter scale. You can then draw the resistor and indicate its value on the drawing. The resistor shown in figure 78, a, has a resistance of 2,500 ohms, and the schematic drawing of it in figure 78, b, gives this information.

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96. Suppose you have two cylindrical parts like those shown in figure 79, a, and you do not know whether they are resistors or capacitors. You may use the ohmmeter to identify the parts, and, if they are resistors, to measure their values. Set the selector switch on the meter to the high range and place the probes on the leads to the first

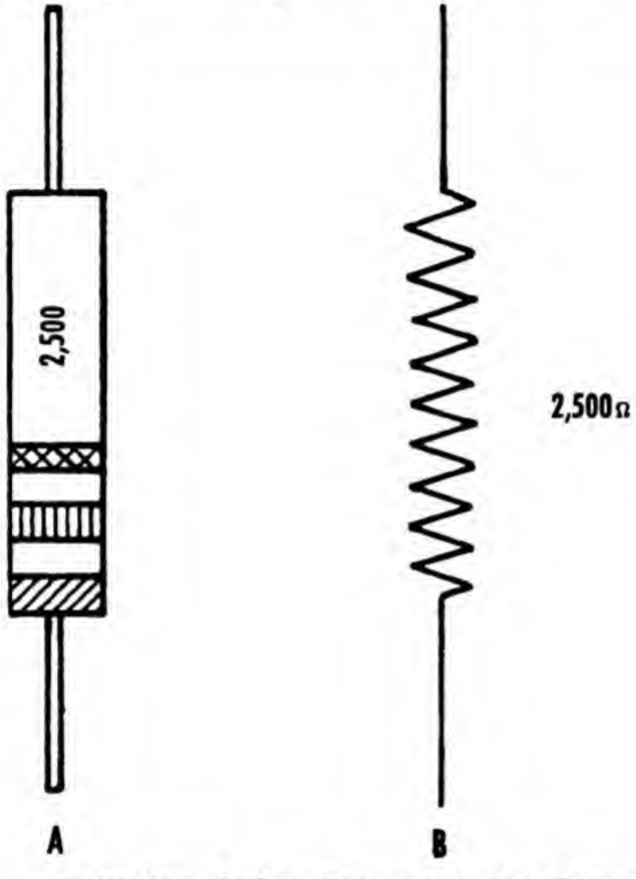
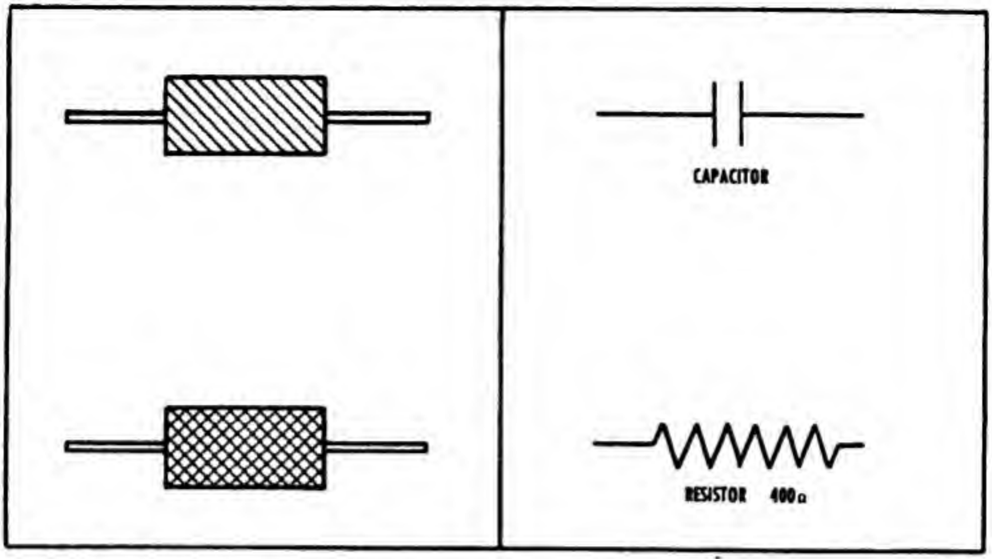


Figure 78.—A resistor and schematic representation, showing value.

part. A reading of infinite resistance on the meter scale indicates that there is no d. c. path through this part; therefore, it is probably a capacitor. Still using the high range you measure the second part and get a very low reading. Reset the switch to the low range and measure that part again. You now get a reading of 400 ohms, which indicates that the part is a 400-ohm resistor. The two parts may then be drawn as shown in figure 79, b. This assumes the parts are in good condition, since a defective capacitor might have a very low or even zero resistance.



(a) (b)

Figure 79.—Schematics of a resistor and capacitor.

97. The ohmmeter may be used also to identify the leads of transformers or other devices whose exact internal wiring is unknown. Figure 80 shows a power-supply transformer with seven leads coming from it. You wish to identify the leads so you can draw a schematic diagram of the transformer. You begin by holding one meter probe on one of the seven leads, number 5 for example, and moving the other probe from lead to lead, noting the meter readings.

98. The combination 5–1 and 5–2 both give infinite resistance readings, indicating there is no connection between either 5 and 1 or 5 and 2. However, combination 5–3 reads 50 ohms, and 5–4 reads 25 ohms. Somehow, leads 3, 4, and 5 are connected. You check combination 3–4 and again get a reading of 25 ohms. Since lead 4 appears to be halfway between 3 and 5 you conclude that all three leads connect to one coil; 3 and 5 seem to be the ends of the coil, and 4 appears to be connected to the center of the coil. As a final check on this coil, which you can refer to as coil 3–4–5, you check each of those three leads with all the other leads going into the transformer. There does not seem to be any connection between any of those three

and the other four leads. Coil 3–4–5, therefore, is a separate coil within the transformer.

99. You now place one probe on lead 1, and, placing the other probe on lead 2, you get a reading of 1 ohm. You check both 1 and 2 with all the other leads and find no further connections to either 1 or 2. Leads 1 and 2 must

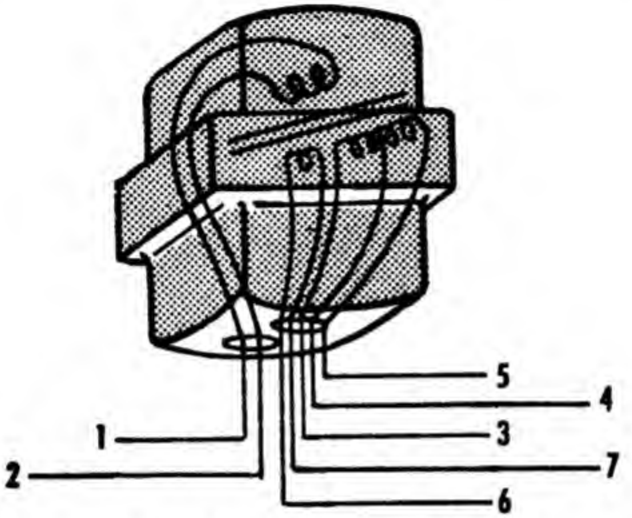


Figure 80.—A power transformer.

be the ends of a second separate coil, which appears to have less resistance and therefore must be shorter, having fewer turns than coil 3–4–5.

100. Finally, you place one probe on lead 6 and the other on lead 7, and you find that the resistance of this last combination is so small that it is barely noticeable on the meter scale. You check with other leads and find no other connection between either 6 or 7 and the other five leads. Coil 6–7 must be a very short coil of wire with a large diameter and it must offer a very low resistance to electric current from the meter battery.

101. You have now identified the seven leads to the transformer and have determined that there are three separate coils, one of which (coil 3–4–5) is center-tapped (has a lead connected to the middle of the coil). The transformer is heavy and obviously has an iron core, which means that the coils are wound around iron slugs or bars

inside the shell. This shell also appears to be metal and is therefore a shield, which means it keeps the magnetic effects of the coils inside the shell and that it keeps outside magnetic effects from getting into the shell. You can now draw the schematic diagram of the transformer, showing the three coils as you found them, the iron core, and the shield. Figure 81 shows the complete diagram of the transformer. The transformer has a metal shell or

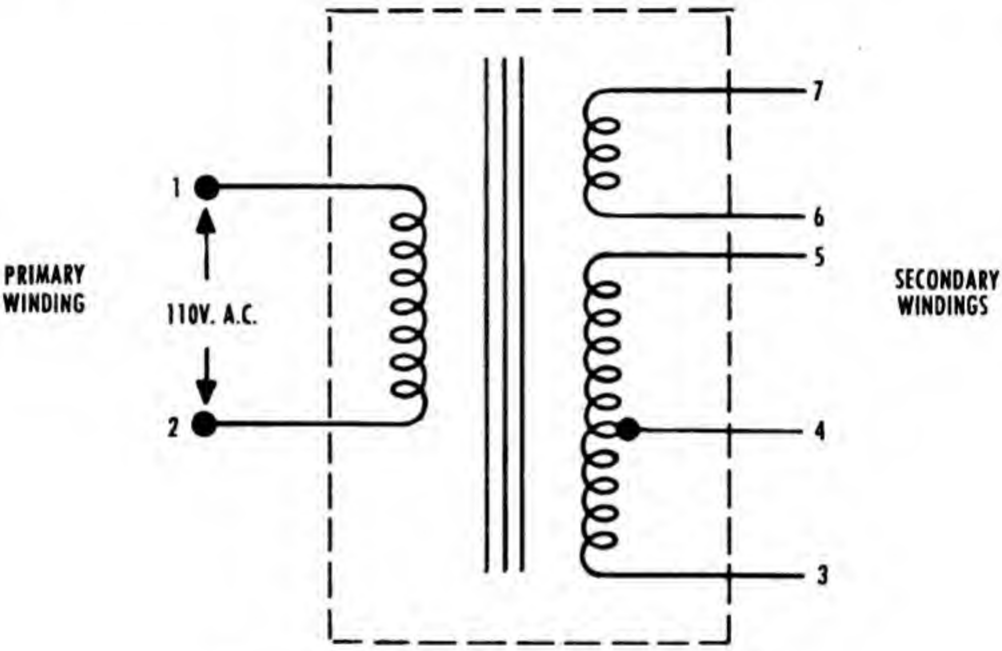


Figure 81.—Schematic of a power transformer.

shield. You represent this on the drawing by a dotted line surrounding the coils. The two heavy lines between the coils indicate that the coils are wound on an iron core, which is usually true with transformers used at low power and audio frequencies (from 25 cps to 20,000 cps). Transformers used at radio frequencies, which are hundreds of thousands of cycles per second or more, are wound on hollow paper or plastic tubes and their schematic diagrams appear like those in figure 82.

102. The resistance of a coil increases with its length or the number of turns on the coil, so you can easily judge

which coil is largest and which is smallest. You will learn later when you study electricity that the ratio of the voltages across two separate coils within a single transformer is directly proportional to the ratio of the number of turns on the two coils. Roughly, this means that a coil having 10 turns will have a voltage across it that is twice that of

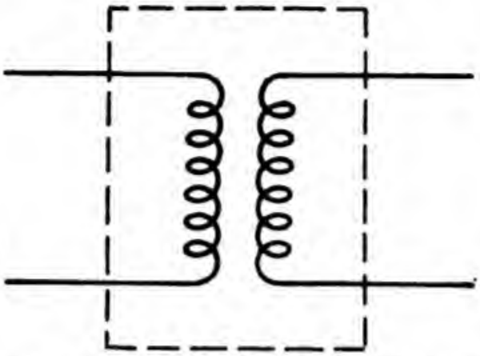


Figure 82.—A high frequency transformer schematic indicating no iron core.

a coil having 5 turns. Since the resistance of the coils is an estimate of the relative number of turns in them, you can easily judge that coil 3–4–5 in figure 81 has the highest voltage across its terminals; coil 1–2 has the next highest voltage across it; and coil 6–7 has the lowest voltage. The transformer shown in figure 81 is a power-supply transformer in which coil 1–2 is connected to the 110-volt a. c. line; coil 3–4–5 gives about 350 volts a. c. for the vacuum-tube plates; and coil 6–7 puts out about 6 volts a. c. for the filaments or heater elements of the tubes. Later, when you study electronic equipment in detail, you will learn the color code used by the Navy and commercial manufacturers to identify transformer leads. However, it is always a good idea to use testing instruments to check connections. In machines as vital as radios and radars every precaution must be taken to avoid error or damage to equipment. In this way, schematic diagrams help in the study and repair of electronic equipment.

103. Sometimes it is difficult to locate the schematic diagram of an electronic circuit that must be repaired. When this situation arises a schematic can be drawn from

the equipment itself, as in the following example. Figure 83, a and b, are top and bottom pictorial views of a radio-receiver chassis. The power-supply section, which uses a number 80 rectifier tube to change alternating current to direct current, is shown completed and mounted on the

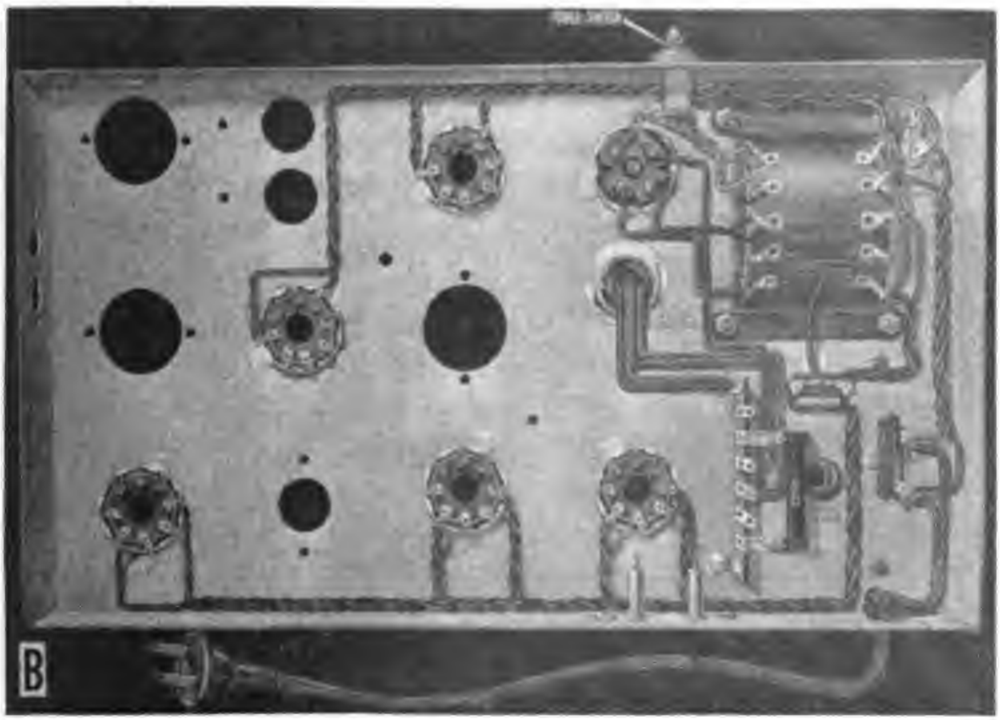


Figure 83.—Top and bottom pictorial views of a radio chassis.

chassis. By following the pictorial views, you can trace the wiring visually in much the same manner as though the tracing were done on the equipment.

104. The transformer used on this chassis may be represented by a schematic symbol similar to the one shown in figure 81. However, it has four windings instead of three, one coil being used for the rectifier tube. The 110-volt alternating current from the power line passes through one winding of the transformer known as the primary. The high-voltage winding, which is connected to the plates of the rectifier tube, forms one winding of the secondary. The filament supply of the rectifier tube comes from another winding on the secondary. The filament supply to the other tubes, on the radio chassis, is the third winding of the secondary. The high-voltage winding and the rectifier filament supply winding are both center-tapped, the center taps being connected to ground. Sketch the schematic diagram of each part of the circuit as you trace it out.

105. First trace the two wires from the power line to the chassis. On the bottom pictorial view you see a 0.001-microfarad capacitor connected from one power lead to a ground on the chassis. From its connection with that capacitor, the power lead runs to terminal 4 of the power transformer. The coil connected to terminals 2 and 4 of the power transformer forms the primary winding. Draw the schematic diagram of the power leads, the switch, the capacitor, and the primary of the transformer. The lower left corner of a blank page is a good location for these symbols.

106. Now trace out the other power-transformer leads. From terminals 1 and 3, wires run to connections 1 and 4 on the tube socket. The schematic diagram for a type-80 tube is shown in figure 84. Usually you will look for the tube diagram in a tube manual. Now copy the schematic diagram of the type-80 tube about an inch to the right of the transformer symbol. The diagram of this tube indicates that pins 1 and 4 are the filament connections. Locate

pins 1 and 4 on each side of the socket key and trace the twisted filament wires to the transformer winding which ends in lugs 1 and 3. You will also notice that the filament is the source of electrons in this tube. Now add the schematic diagram of this winding to your drawing, and draw the wires connecting the tube and filament winding.

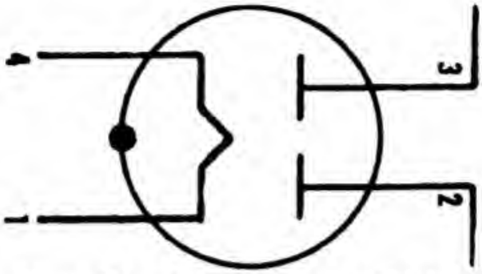


Figure 84.—Schematic type-80 tube.

Checking terminals 5 and 9 of the transformer, you find that a wire from terminal 5 goes to tube-socket connection 2 and that terminal 9 is wired to socket connection 3. The diagram of the type-80 tube shows that pins 2 and 3 are connected to the plates of the tube. Since the leads from transformer terminals 5 and 9 go to the plates, they must be the ends of the high-voltage coil. Draw a high-voltage winding on your transformer and connect the outer ends to the 80 plates. From terminal 7, located between lugs 9 and 5 on the transformer, a wire leads to a soldered ground connection on the chassis. The wire from terminal 7 to ground must then be the center tap of the high-voltage coil. If you were working with a piece of equipment instead of a pictorial diagram, you could disconnect the wires from the terminals and verify these conclusions by ohmmeter readings. Now add a center tap connected to the schematic diagram of the high-voltage coil, number it point 7 and connect it to a ground symbol showing the wiring as you have traced it.

107. Checking the last three terminals of the transformer, you find that lug 8 is connected to lug 7, which is grounded, and that lugs 6 and 10 are connected to separate lugs on a terminal strip. From these lugs the fila-

ment supply leads run to the other tubes that will be mounted on the chassis. Draw in the winding that supplies the other tube filaments with the center tap connected to the ground symbol.

108. Two more large parts remain—the choke with two wires and a capacitor can with four wires, two of which are grounded. Draw the symbol for a choke to the right of the 80-tube symbol. Since a single capacitor section has two leads, this capacitor can evidently has two sections, which you can draw below the choke symbol. You can also show the two capacitor leads connected to a ground symbol at the bottom of the drawing. On the bottom of the chassis one wire connects pin 1 of the 80 tube to lug 4 of the terminal strip. One wire from the choke also connects to this lug. Draw a wire connecting the 80 tube and the choke on your schematic diagram. Also draw the connection between the other wire from the choke and the terminal lug number 1, labeled "B+ output."

109. Now notice that two wires from the capacitor can, which are not grounded, go to lugs 3 and 4 on the terminal strip. One end of the choke coil is wired to lug 3, and the other end is wired to lug 4. You can therefore draw the wires (1) connecting the ungrounded end of one capacitor with one end of the choke coil and (2) con-

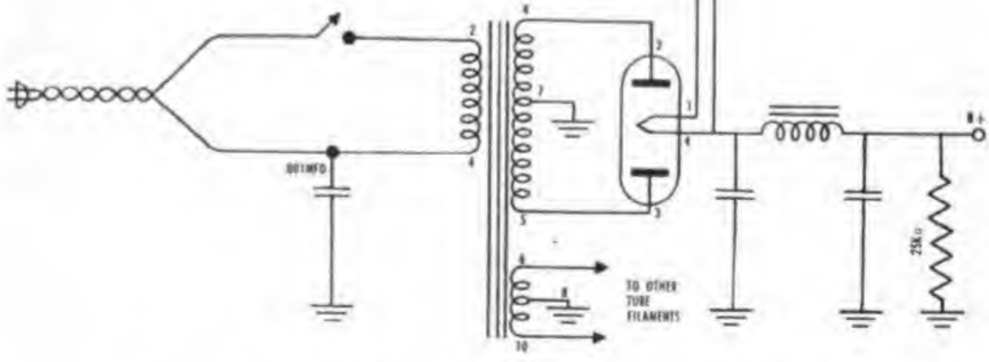


Figure 85.—The schematic of a radio receiver power supply.

necting the ungrounded end of the second capacitor with the opposite end of the choke coil. A 25,000-ohm resistor is connected between lugs 2 and 6 of the terminal strip. Lug 2 is wired to lug 1, the B+ output and lug 6 are grounded to the chassis. You can now complete your diagram by drawing the 25,000-ohm resistor connected between the B+ terminal and a ground symbol. When you have completed and checked your drawing, compare it with the correct schematic diagram of this circuit which is shown in figure 85.

LAYOUTS APPLICABLE TO ELECTRICAL INSTALLATIONS

110. In the installation of electronic gear, the location of the various parts and interconnecting cables is a very important factor. Space limitations and adequate ventilation to maintain efficient operation of tubes, motors, and generators must be considered when planning the location of electronics equipment. Usually connections and locations are planned in advance by engineers, and layout drawings are supplied with equipment or with the parts sent out for modifying existing installations. As a Navy electronics technician you will use layout drawings. You should be able to read and understand them and know how to prepare such drawings when necessary.

111. For example, one particular type of equipment may have vacuum tubes, which will break down very rapidly or may not operate at all unless they are placed in an upright position; or a plan may indicate that certain equipment must be located on a ship at a particular frame number so that it is protected from electrical or magnetic fields caused by other equipment. Radar equipment requires the use of special types of cable for interconnections, and plans will indicate not only the type of cable but often the length to be used. These are only a few examples of situations you will find when you repair and install electronic gear aboard ship. In this chapter, however, you will study only the plans used for installing or

modifying individual pieces of equipment. This will give you an opportunity to use the geometric constructions you have learned and will familiarize you with the use of plans and layout drawings.

112. A layout plan is comparatively simple, but careful measuring, accurate checking of all scribed marks against the drawing, and skillful use of tools are necessary to produce a finished layout with accurately positioned openings of the proper size. In making an installation layout, you put your mathematics to a practical test.

113. The problem illustrated in figure 86 is taken from a radar instruction book. This problem is typical of the layout drawing work an electronics technician may have

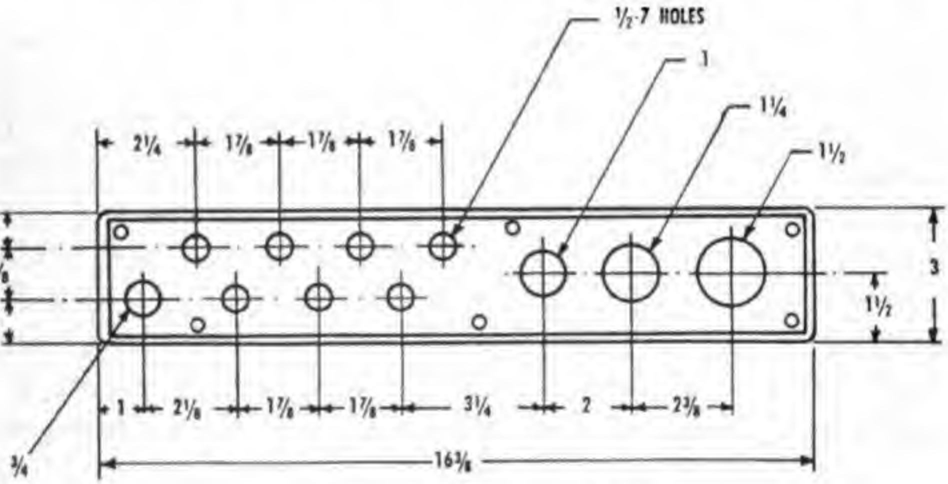


Figure 86.—Cable entrance plate.

to do. Two metal plates are supplied with a certain radar so that the holes for cable entrances may be drilled before the plates are fixed to the equipment. As an electronics technician it might be your job to drill the holes in the correct locations.

114. Two principal types of dimensions are given on the plans for this job and on most other layout work as well. SIZE dimensions are the measurements you follow when cutting a piece of metal or wood from stock to the correct shape. When you locate holes for drilling, punching, or cutting, you follow LOCATION dimensions. The

length ($16\frac{3}{8}''$) and width ($3''$) of the piece to be cut from stock are shown in figure 86; these are the size dimensions of this drawing. The many small dimensions shown in the figure which indicate the hole centers and spacings are the location dimensions. Notice that on this drawing the distances between the holes are measured from center to center. The diameters are indicated by notes with arrows pointing to the holes to which each note refers. The location dimensions of a drawing may be given in various ways, depending on where the greatest degree of accuracy is required. The correct locations of the hole centers are easily determined by the intersecting dimension lines.

115. In figure 86 CONTINUOUS LINE dimensions are used. These dimensions indicate that four holes are to be made on the lower left of the plate; the hole centers are to be on a straight line parallel to and $1''$ away from the bottom edge of the plate. The first hole, starting from the left side, will have a diameter of $\frac{3}{4}''$; the other three will have a diameter of $\frac{1}{2}''$. Using dividers and a straightedge, start by locating the centerline of the holes $1''$ from the bottom edge. Then locate the center of the hole that has a $\frac{3}{4}''$ diameter $1''$ from the left edge of the plate. Using the given dimensions, locate the centers of the other holes. Set your dividers for the distance between hole centers, and with the known hole center as the pivot point, scratch a mark cutting the centerline to locate the center of the next adjacent hole. Repeat this procedure for the next hole center, and be sure to set the dividers to the correct distance for striking each arc. Where the spacing between hole centers is the same for several holes in a row, this distance will be the same; however, be sure you have obtained the correct dimensions from the drawing before making any marks or cuts in the material.

116. By using the continuous-line method, you would carry across the plate any error in the location of the first hole. Suppose you made another type of error, in which each arc was $\frac{1}{32}''$ too long or too short. Then the errors

on each of your progressive measurements would add up and the last hole would be $\frac{1}{8}''$ off in relation to the edge of the plate. There is a procedure for using location dimensions which avoids such accumulated errors; it is called the BASE LINE METHOD. In this procedure, shown in figure 87, you use the left edge of the plate as a reference

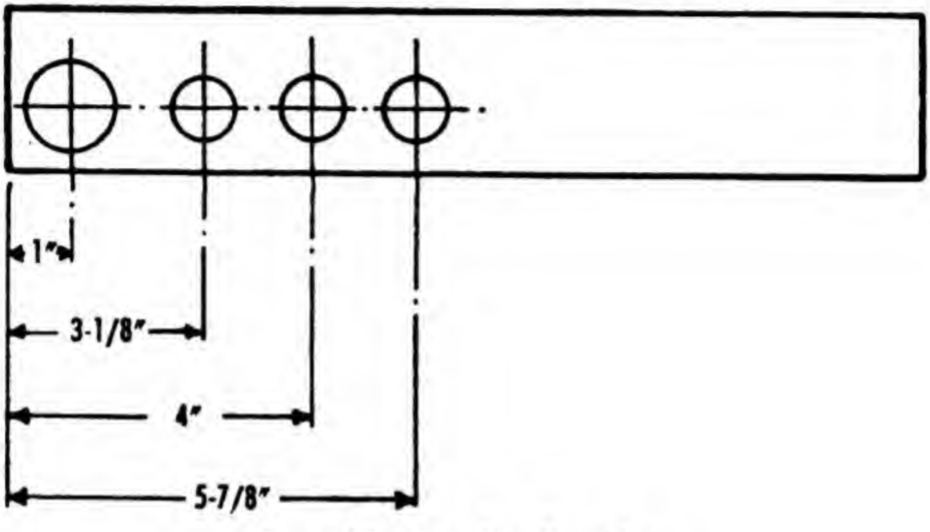


Figure 87.—Base line dimensions.

for each measurement. If you make an error of $\frac{1}{32}''$ on each hole, the last hole still is not more than $\frac{1}{32}''$ off center. Since you return to the left edge as your reference line for each dimension, the error in one measurement is not carried over to the next.

TOOLS USED IN LAYOUT WORK

117. You will need certain special tools to draw accurately the lines required when you work with metal stock. For example, you could use an instrument called a COMBINATION SQUARE when you draw the lines required for the work shown in figure 86. Figure 88 shows the combination square and a few of its uses. You cannot use such a square without a straight edge on the material as a reference. When conditions do not permit using a combination square, you will have to make your construc-

tions with a compass and a steel straightedged rule. There are several other tools you will use when working with or from layout drawings. It is important that you understand how to use these tools correctly.

118. When measuring from a steel straightedged rule, always hold it so that you avoid parallax in your readings. When you wish to make a mark opposite some point on a straightedged rule, hold the edge of the rule against the paper or piece of stock to avoid any parallax error. The correct way to avoid parallax is shown in figure 89. You may use your dividers and compass on steel or wood surfaces, just as you use them on paper for drawings. Sometimes you will wish to mark a line on a piece of metal to indicate where it is to be cut. A pencil line on metal is easily smeared. Instead, a fine scratch line on the metal is better; you will use a tool called a SCRIBER (fig. 90) for making this type of mark.

119. When making holes, use a prick punch first to make a slight dent to hold the drill centered. Drills should be ground properly, as shown in the book *Use of Tools*, NavPers 10623, if the holes you drill are to be centered true and accurately. You must clamp your work to the drill bit to prevent motion and inaccurate work. After locating a hole center by using crossed scriber lines, you should use a prick punch (fig. 90) to make a slight circular dent in the face; this simplifies correct centering and holding of a drill. Figure 91 shows the correct way to hold a drill. The important thing to remember when using a drill is (1) to hold it perpendicular to the surface you are drilling and (2) to start the drill exactly at the center point. Holes larger than 3/8″ in diameter are made with either large drills or chassis punches. Whether you use a chassis punch or a drill for these larger holes, you should proceed as follows: Place the punch exactly at the center you have located; then make a carefully centered small hole with a drill having a small diameter. Enlarge the hole to the desired size with a larger drill or chassis punch. This procedure insures accurate location

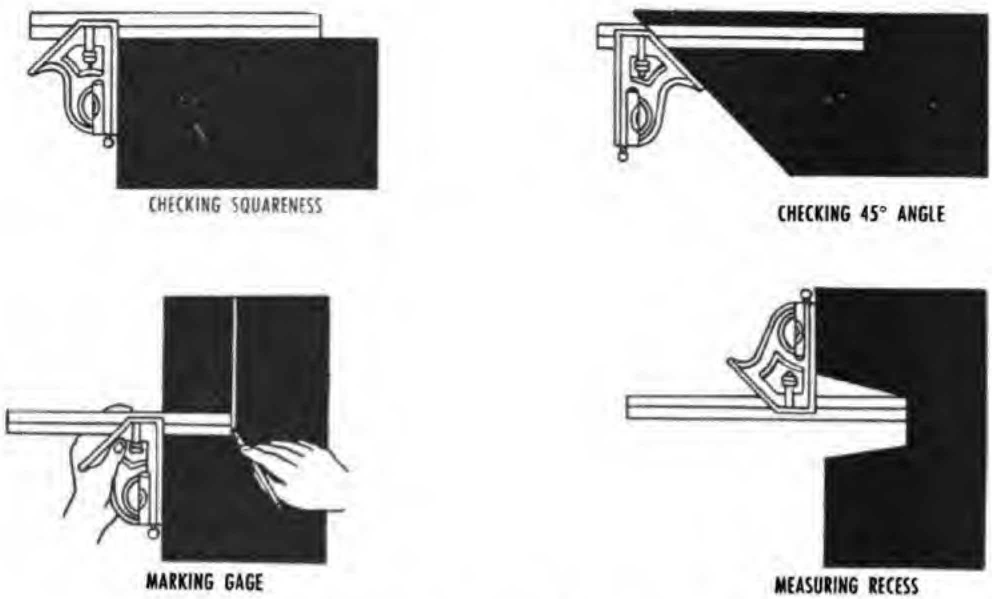


Figure 88.—Combination square and a few of its uses.

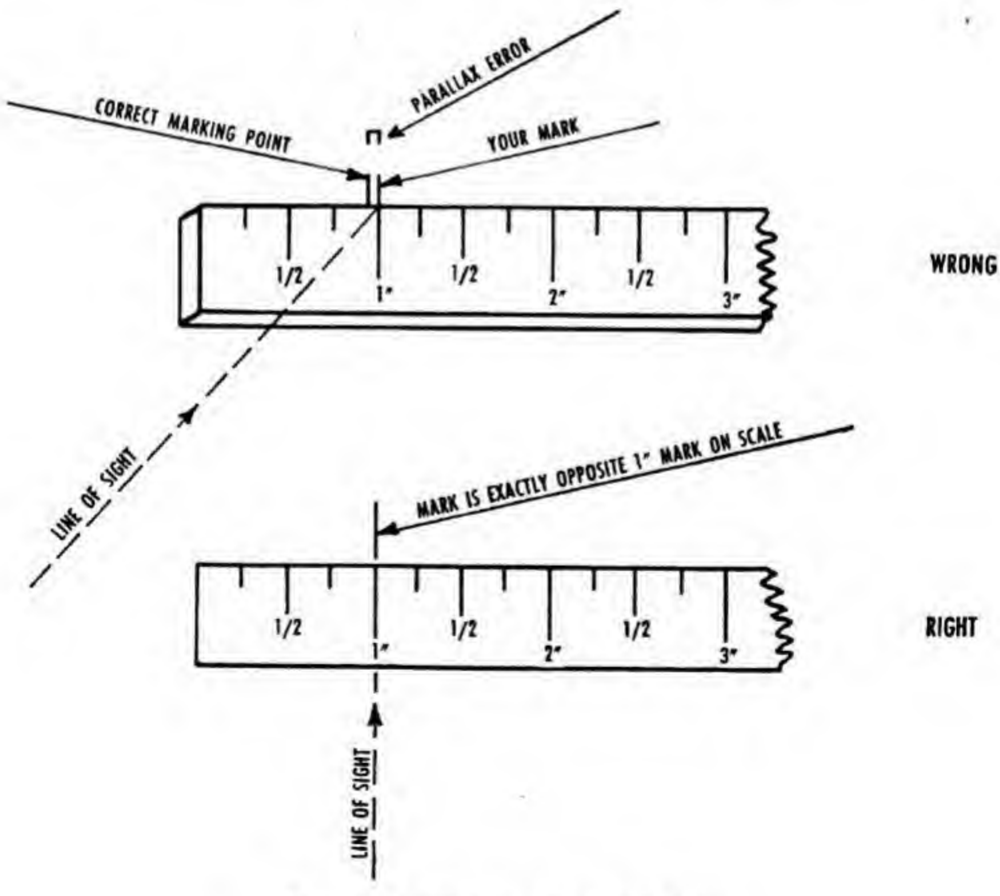


Figure 89.—How to avoid parallax.

of the hole and makes it possible to center large holes correctly and to cut them out easily.

120. The chassis punch is shown in figure 92. The threaded part of the chassis punches used to make the

Figure 90.—The scriber.

holes for most tube sockets is $3/8''$ in diameter. You would, therefore, have to drill a hole that large before you could use the chassis punch. The chassis punch is operated by turning the cutting part that is fixed to a stud, while a nut

Figure 91.—The correct way to hold a drill.

on the opposite side of the chassis is held stationary. This nut is roughened and grips the chassis surface so that it does not turn while the cutting part and stud are rotated. After you have drilled the pilot hole for the stud

of the chassis punch, separate the nut from the stud and put the stud through the pilot hole. Then turn the nut on the stud until it is tightened against the chassis surface; the friction will keep it from turning. Turn the cutting head fixed to the stud, and, as the thread advances, the cutting edge pierces the chassis, thus making the desired hole.

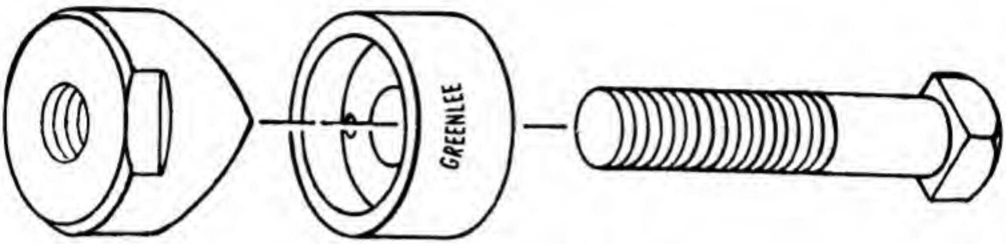


Figure 92.—Chassis punch.

121. When you work with metal stock, remember that these materials come in standard stock sizes. You must consider the size of the standard piece of stock required when planning the layout of the piece you wish to cut. For example, if you have to cut a circular disc 20″ in diameter from sheet steel and if the metal comes in sheets that are 24 by 36 inches in size, you will have to draw from stock a standard 24″ x 36″ sheet, or 864 square inches. You can see that careful planning of the use of stock is important. After a little experience in working with these materials, you will learn to use them with minimum waste.

USE OF TEMPLATES

122. Engineers who design electronic equipment often develop improvements on a piece of equipment after it has been installed on many ships. Sometimes changes must be made on the equipment to adapt it for use with another piece of equipment. Whenever there are electrical changes, there are usually certain mechanical changes to be made as well. For example, an electrical change of a receiver may require an additional switch and tuning

mechanism. To complete the installation, you must drill two new holes on the front panel at certain specified points. Instead of indicating the dimensions, the engineers send out (1) FIELD-CHANGE KITS, which contain

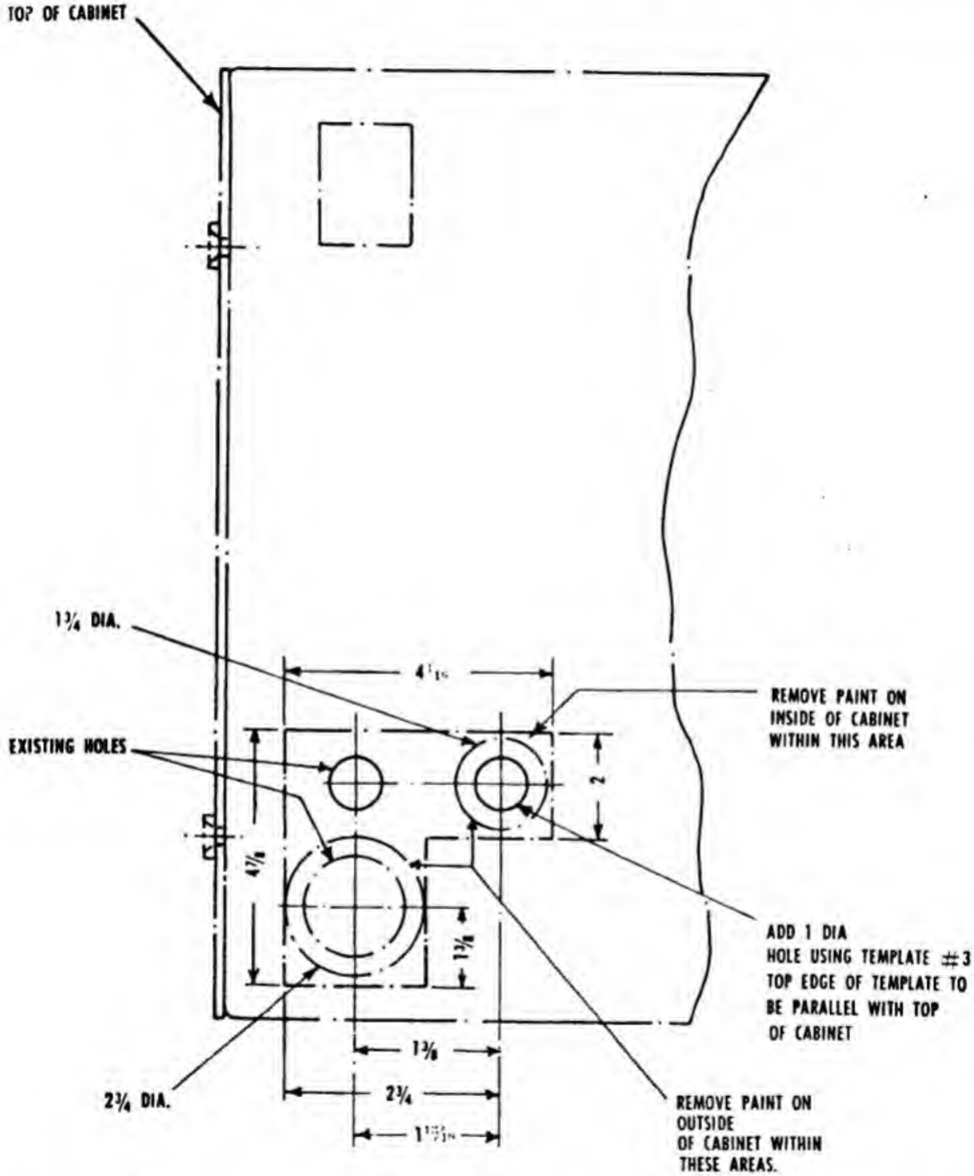


Figure 93.—Examples of a template used in electrical installation work.

the new parts, and (2) a TEMPLATE. A template is a full-sized pattern made of some thin material, either metal or cardboard. You place the template directly over the side of the cabinet on which the changes are to be made.

The template has holes punched in it to coincide with those that are already drilled in the cabinet or chassis; it also has center marks for the new holes to be drilled marked on it. It is then a simple job to scribe the location of the new holes directly from the template. The template also tells you the size of the holes to be drilled.

123. An example of a template used in electrical installation work, is the one shown in figure 93. Another vacuum tube, together with a filter unit, must be added to a Navy receiver. Two holes are already drilled in the back of the receiver cabinet; a third hole must be drilled for an additional cable. To mark the location of this third hole, the template is placed against the cabinet with its top edge coinciding with the top edge of the cabinet. The template is then lined up with the existing holes, and the center of the new hole (1″ diameter)—as indicated on the template—is punched on the cabinet through the template card. The paint is to be removed as indicated, so that the cables may be grounded properly. The reason for this change was to adapt the receiver for use with another piece of equipment. Figure 94 shows a top view of the receiver. The unit to be used with the receiver is to be mounted on the top of it. Two aluminum spacer-strips are provided with the kit. These are to be mounted on top of the receiver; the new unit then is mounted on top of the spacer-strips. As stated on the plan view, the strips may be used as templates to locate the holes to be drilled. Notice that the dimensions are given at the bottom of the drawing, although they are really not needed for locating the holes. The base-line dimension method is used in this drawing.

124. If a template is not furnished in a field-change kit, you may make one on heavy paper or cardboard from scale drawings that are furnished with the kit or that are found in equipment instruction books.

125. Equipment modifications are sometimes developed in the field or at sea. Accurate drawings should always be made to accompany reports prepared for the Bureau of

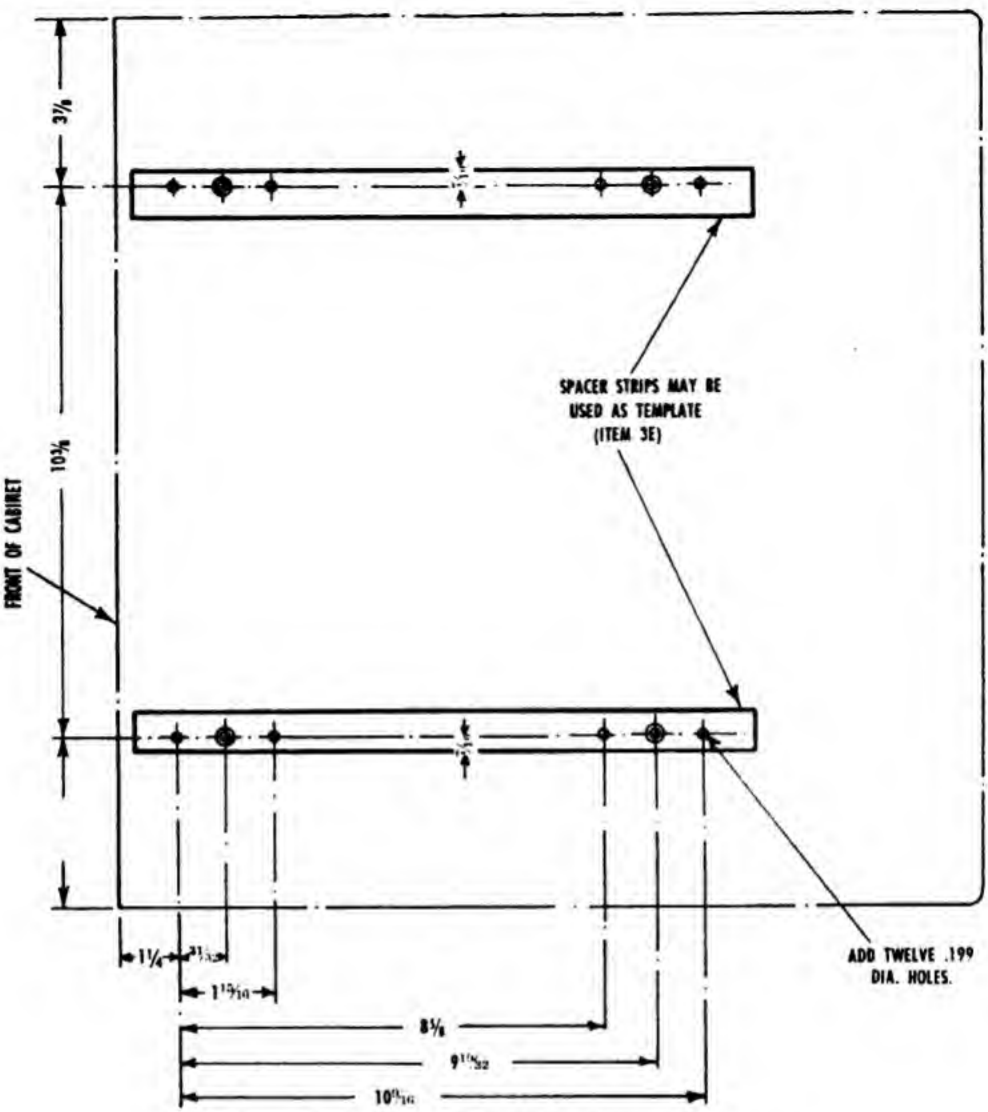


Figure 94.—Top view of the Navy receiver.

Ships. Your ability to do accurate drawing and layout work will be favorably noticed by your superior officers.

If you did not pass the pretest in this chapter, apply what you have learned by working the problems now and also the problems in section IV of the Remedial Work.

CHAPTER 5

GRAPHS

Graphs are widely used in many phases of everyday life. Science and industry, popular magazines, and advertising literature all use graphs of various types to put ideas across and to make facts easily understood. The electronics technician uses many types of graphs to help in solving electronic problems, from those involving a single vacuum tube to checking the operation of a high-powered transmitter.

Why are graphs used so extensively? Because they use a simple picture to illustrate the relations between numbers. Mathematical ratios, long tables of numbers, electronic characteristics, in fact anything that can be measured and may have different values under varying conditions may be shown most clearly on a graph. The graph transforms a confusion of tiresome details and calculations into a picture—something you can see, understand, and use quickly and easily.

In this chapter you will study the principal types of graphs used in electronics. You will learn to read and draw them, and you will study some of their applications. It is important that you understand graphs thoroughly, for graphs may be your best source of information in every part of electronics. In this chapter many examples of electronic graphs are given. These examples use electronic terms with which you are not yet familiar. A complete understanding of these terms is not necessary because in this chapter you are studying only the way in which two or three measurements are related to each other.

PRETEST 5

126. (a) The common graph shows a mathematical .... or relationship.

(b) Refer to figure 95. What was the average cost of light bulbs in 1930?

(c) The three common graphs are ...., ..., and ....

(d) A circle graph shows the relationship of .... to the ....

(e) Draw a circle graph to represent the following: A certain ship has 30 pieces of electronic equipment; of these, 7 pieces are radar, 5 are radio transmitters, 15 are receivers, and 3 are navigation aids.

(f) Refer to figure 98, a. With a load resistance of 7K, what relative power output can be expected?

(g) Refer to figure 98, a. What two values of plate-load resistance will give a power output of 80 percent?

(h) Refer to figure 101. With 125 volts on the plate and –2 volts on the grid, how much plate current is flowing?

(i) Refer to figure 103. What will be the swing in plate current if the swing in grid voltage is from –1 volt to –3 volts?

(j) A nomograph shows the relationship of .... quantities.

(k) Refer to figure 114. With a 15μh. coil and a 130 μμf. capacitor, what output frequency can be expected?

STUDY GUIDE ON HOW TO READ GRAPHS—BAR GRAPHS

127. There are three types of graphs—BAR, CIRCLE, and LINE. The BAR GRAPH, shown in figure 95, is the simplest type and serves as an introduction to graphs. Notice that it shows the relation between two things—the cost of light bulbs (expressed in cents) and the time (plotted in 5-year steps). The cost is indicated on the vertical scale at the left. Usually, when a point is located by its distance from a horizontal line and its distance from a vertical line, the two distances are referred to as RECTANGULAR COORDINATES of that point. Any distance measured on the vertical scale above or below the horizontal reference line or axis is called an ORDINATE. Time, expressed in years, is plotted on the horizontal scale at the

bottom of the graph. A distance measured on the horizontal scale to the right or left of the vertical axis is called an ABSCISSA. The intersection of the horizontal and vertical axes is sometimes called the ORIGIN OF THE COORDINATE SYSTEM.

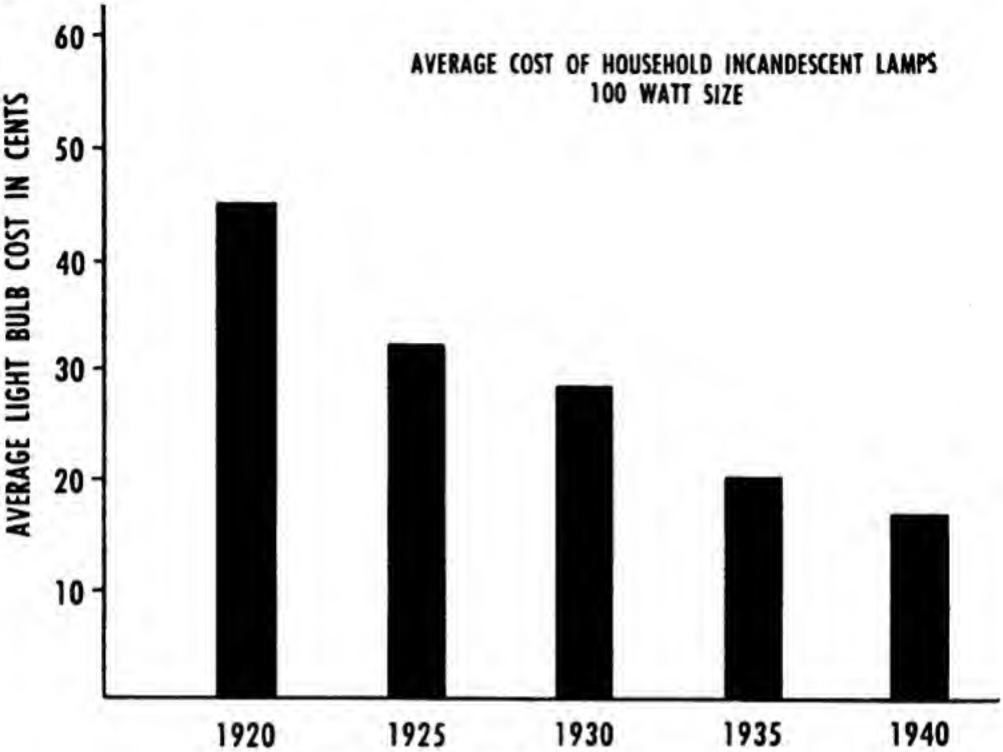


Figure 95.—Bar Graph.

128. Here is how to read the graph. To find the cost of bulbs in 1920, read across the abscissa (horizontal scale) to 1920. Follow the bar marked 1920 to the top and read the cost on the ordinate (vertical scale) at that point. You should read about 44 cents. Following the same procedure, you will find that the cost of bulbs in 1940 was about 16 cents.

129. A graph like the one shown in figure 95 gives two kinds of information. One is exact information, the cost of light bulbs in 1920. You have seen how the graph is used in this way. The second kind of information is an over-all picture of certain conditions. Glancing again at figure 95, you can see the decrease in the unit cost of light bulbs from 1920 to 1940.

130. Often two interrelated graphs are plotted together to present a general relationship more clearly. For example, using the graphs in figure 95, and some additional information, you may draw a graph like the one shown in figure 96. In this figure the added information shows

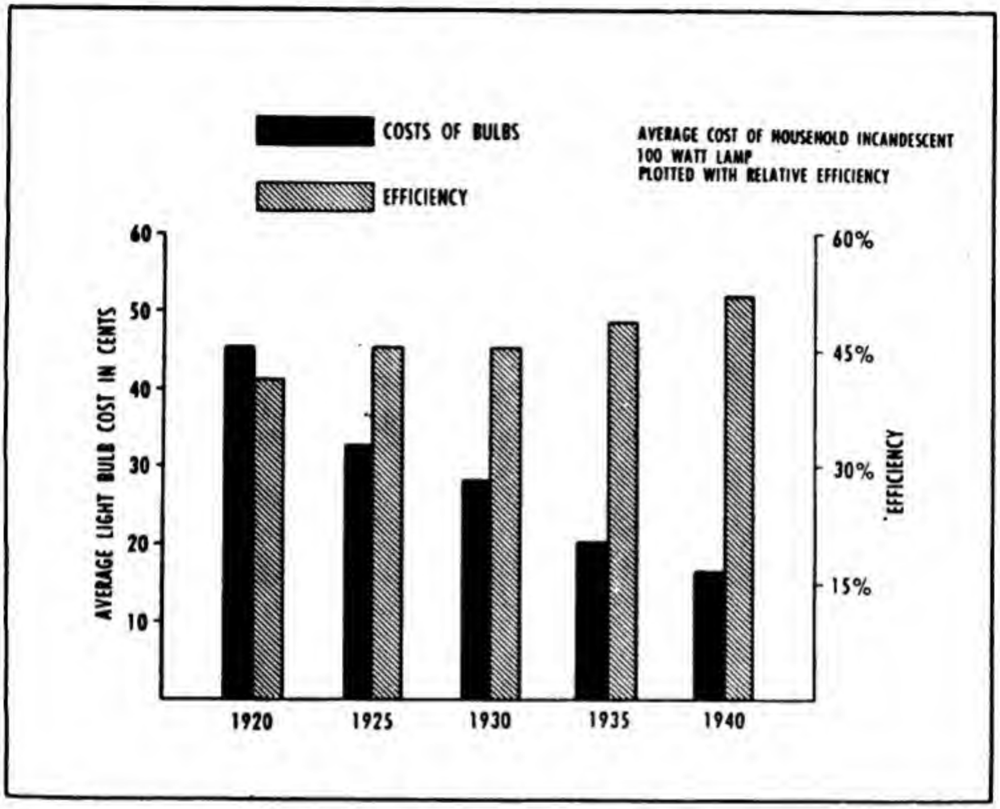


Figure 96.—Two interrelated bar graphs.

light-bulb efficiency plotted against time. Efficiency is plotted on the ordinate at the right. Time is plotted in 5-year steps on the abscissa. The bars are shaded differently so that you can identify them easily. Shading of the bars is explained above the graph. By making use of both vertical scales and remembering that the horizontal scale is common to both, you can show the individual relation of two subjects to a third common value on one graph.

CIRCLE GRAPHS

131. A circle graph shows the relation of parts to the whole, each sector indicating the ratio of each part to the

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total. This ratio is often expressed as a percentage. An example of a circle graph is shown in figure 97. The electronics supply officer of a radio station wished to show graphically how many of each type of transmitting tubes were on hand; he also wished to show what part of the total tube stock each type represented. When an inventory was prepared, he found that there were 100 tubes in stock, grouped as follows: 10 diodes, 14 pentodes, 15 triodes, 45 tetrodes, and 16 miscellaneous types. This information is presented in graphic form in the circle graph shown in figure 97. Expressed as a percentage,

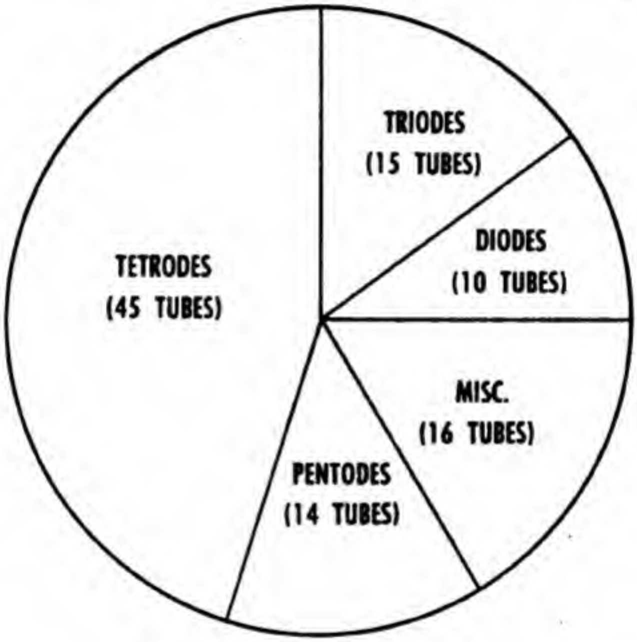


Figure 97.—A circle graph.

10 percent of the tubes were diodes ($^{10}\!/_{100} = 10\%$). What portion of the circle would correctly represent the diode section? The whole circle of 360° will represent the total number of tubes on hand or 100 tubes. Since 10 percent of the total number of tubes on hand are diodes, then 10 percent of the circle should represent them. The actual number of degrees in this section is 10 percent of 360° or 36°. Therefore, the piecut (sector) representing the diodes should be 36°. The other sectors are found in the same manner.

132. When you read a circle graph you can see at a glance the approximate percentage of one part to the total. In this case, if you know the total number of tubes, by looking at figure 97 you can approximate the number of tubes of any one type. The exact number can usually be found by examining the graph more closely, since the accurate information is usually written in each section of the graph.

LINE GRAPHS

133. In electrical and electronics work you will use line graphs more often than any other type. A line graph is used to show the relation between two quantities that are dependent on each other. When you studied simultaneous linear equations in *Essentials of Mathematics for Naval Reserve Electronics* (NavPers 10093) you used line graphs to show the time and distance relation of moving objects. In electronics you will use this type of graph to represent the relations between the electric currents, voltages, and other quantities in vacuum-tube circuits.

134. When you work with electronic equipment, you will often use vacuum-tube amplifiers. The power output of a vacuum tube depends on the electrical resistance placed across its output terminals. Figure 98, a, is a graph showing the relation of power output to load resistance. Notice that the power output is plotted in percentage units; the maximum power output, under the best conditions, is used as the 100-percent value, and all the other values are related to it. That is why the scale is marked "Relative Power Output." The important idea here is not how units of power are obtained, but rather how much of the maximum power output is realized by using any particular resistance. The load resistance is plotted on the abscissa in units of 1,000 ohms. The Navy symbol for 1,000 is "K"; for example, 6K means 6,000.

135. Here is how to read the graph as shown in figure 98. Suppose you know the load resistance to be 5K ohms

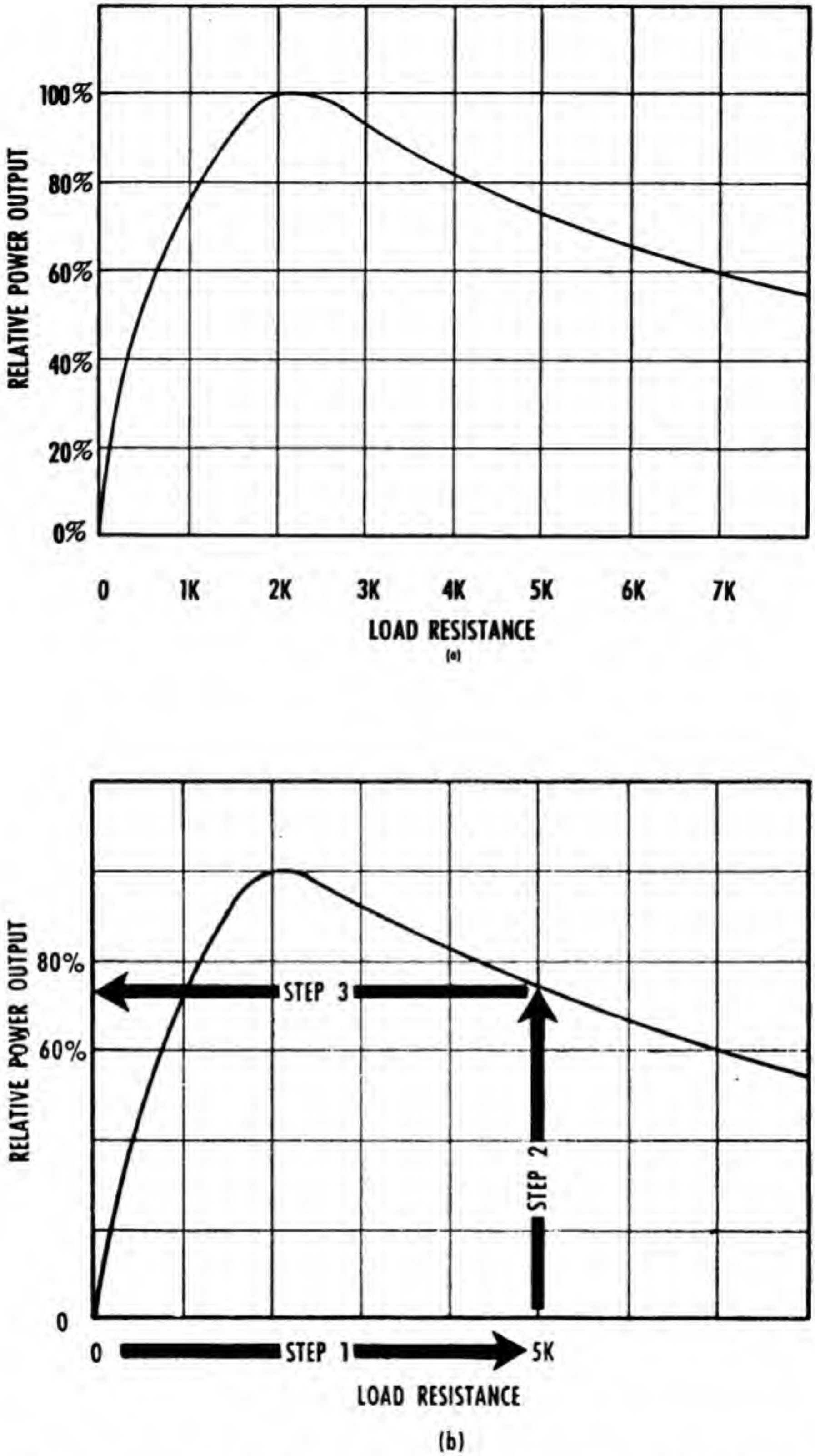


Figure 98.—A line graph showing power output versus load resistance.

(5,000 ohms), and you wish to know the relative power output under that condition. Follow the three steps shown in figure 98, b. Step 1, locate the 5K resistance on the abscissa. Step 2, draw a straight vertical line from the 5K mark until it intersects the curve. Step 3, draw a straight horizontal line from this intersection to the ordinate and read the relative power output at that point on the scale. A 5K resistor results in 75 percent of maximum power output.

136. Here are a few problems that may be solved with this type of graph. They are typical of the practical problems you will find in electronics work. If you have a 3K resistor and a 5K resistor, which will give you the greater relative power output? You can easily see that using the 3K resistor will result in a power output that is about 93 percent of maximum, whereas you saw that the 5K resistor will result in only 75 percent of the maximum power output. What resistor should you use to get maximum power output? The graph tells you at a glance that a 2K resistor is the answer. However, suppose that your 2K resistor burned out and you had only a 0.5K and a 3.5K resistor for replacements, which would you use? Each replacement differs from the desired value by 1.5K, but a glance at the graph tells you that using the 3.5K resistor will result in 87 percent of maximum power output, whereas a 0.5K resistor will result in only 40 percent of maximum power output. From these simple problems you will realize that line graphs have a very important place in electronics work. If any part in this discussion of graphs is not clear to you, review these sections again until you thoroughly understand them.

SPECIAL TYPES OF GRAPHS—THE SINE CURVE

137. Before you read any graph, you should study carefully the units and scales on each axis of reference. With this information clearly in mind, you will avoid errors of hasty work and will not become confused. Determine the

smallest unit given on each scale and the smallest unit you can estimate; often an over-all glance at the graph will give you a rough idea of the answer you are seeking. Then, if the final answer you get doesn't compare with your approximation, you should recheck your work. When you examine the axes of a graph, note how they are laid out, where the zero points are, and whether the scales are linear.

138. The graph shown in figure 99 has a vertical axis somewhat different from those you have studied. The zero of the vertical axis is at the middle of the scale. There are two vertical scales; one is a positive scale, reading from

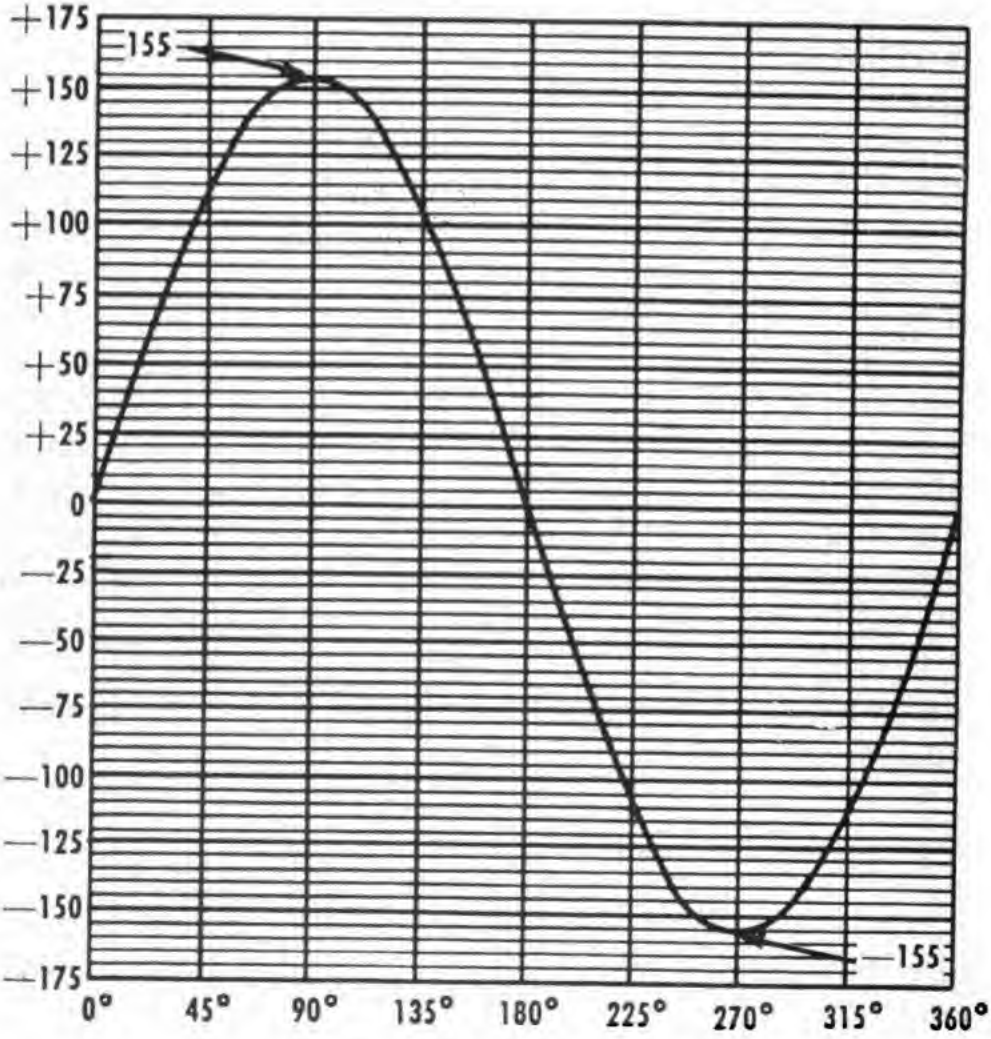


Figure 99.—A sine curve.

the zero upward, and the other is a negative scale, reading from the zero downward. Sometimes the horizontal scale is divided in the same way.

139. The curve shown in figure 99 is a special type of graph used to show the relations of either alternating cur-

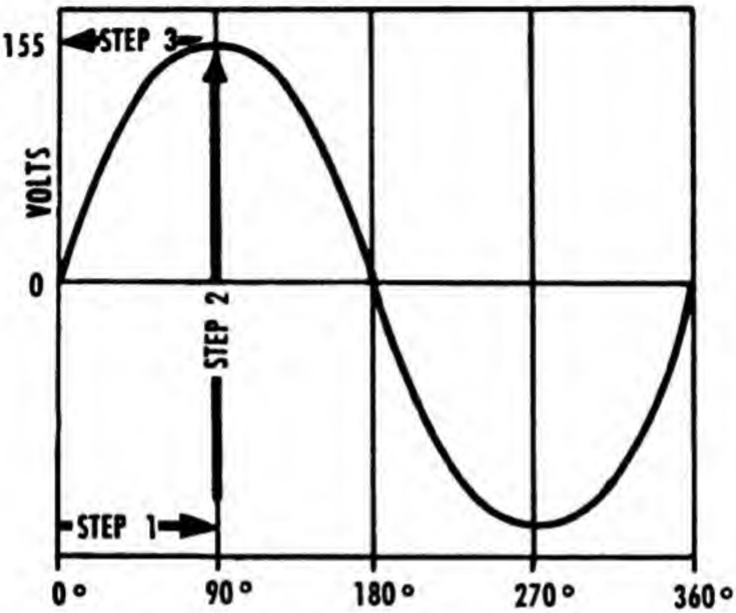


Figure 100.—Reading a sine curve.

rents or voltages to time. Each complete waveform on this graph represents one complete cycle of the voltage normally used in house circuits and for many types of electronic equipment. This type of voltage changes from zero to about 155 volts positive, then reverses to 155 volts negative, and finally returns to zero. This cycle occurs 60 times during each second, and the graph shown in figure 99 is a picture of the voltage value at any instant of time during any cycle. The values given here are peak values. The 110 volts supplied to your home is the average value of this voltage. The 1-cycle interval is usually divided into 360°. One-half of a cycle is 180°, and therefore the voltage at that instant, as shown on the graph, is momentarily zero.

140. You will use SINE CURVE graphs in many phases of electronics work. This particular curve shows the voltage

of house current, but all alternating currents and voltages may be represented in a similar manner. The currents and voltages of radio frequencies used in many electronic devices are often of a similar sine-curve pattern. You should become thoroughly familiar with this curve.

141. The sine curve is read in the same way as any other line graph. (See fig. 100.) For example, find the values of the voltage after 90° or after ¼ of the cycle has elapsed. First, read along the horizontal axis until you come to 90° (step 1). Then draw a line vertically upward to intersect the curve (step 2). Reading over to the left on the vertical scale you will find the voltage to be 155 volts (step 3). The same process may be followed in reverse to find the time in any cycle when a particular voltage occurs. For instance, you can locate the point or points on the curve where a voltage of –100 volts occurs. Read down the vertical scale to –100 volts. Then read across until you intersect the curve. Notice that there are two intersections in this case, which means that there are two points on the curve, during any one cycle, when the voltage has a value of –100 volts. You will find that the voltage will have a value of –100 at about 220° and 320°.

FAMILIES OF CURVES

142. Vacuum-tube handbooks and electronic-equipment instructions often contain many pages of graphs like the ones shown in figure 101. These groups of similar graphs are called FAMILIES OF CURVES. They represent the relationship of three interdependent values. The families of curves shown in figure 101 show the relationship of grid voltage, plate voltage, and plate current for a single vacuum tube. Figure 102 shows what these three quantities are. The plate voltage is the voltage difference between the positive terminal or plate of the tube, and the negative terminal or cathode. The plate current is a measure of the number of electrons that flow across the gap between the plate and the cathode, and through the wires connecting

them outside the tube. The grid voltage is the voltage difference between the grid (a wire screen between the plate and the cathode) and the cathode. For any given constant value of the grid voltage E_g, the relationship of the plate voltage E_p, and the plate current I_p is shown by a single curve. A separate curve is required to show this

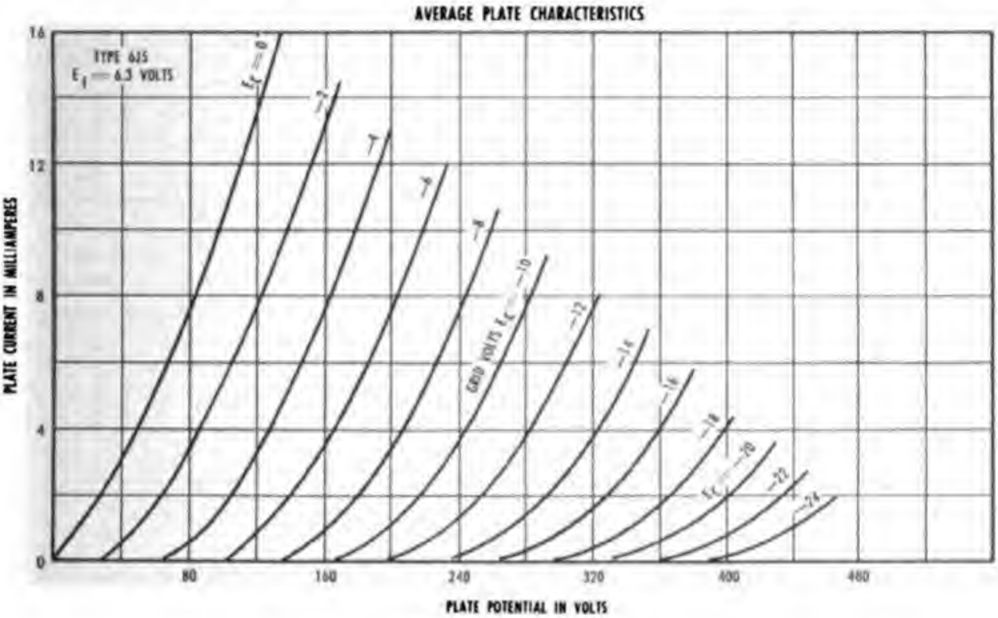


Figure 101.—A family of curves.

E_p—I_p relationship for each value of E_g. Since all these curves are similar and may be plotted to the same scale, they are grouped on a single graph to save space and for easy reference. This group of curves is known as the PLATE CHARACTERISTIC (fig. 101). Each curve is marked at its upper end by the grid voltage at the time it was made. In addition to the information given by the curves themselves, there may be other conditions under which those curves were drawn. For example, in the family of curves shown in figure 101, the note $E_f = 6.3$ volts in the upper left corner indicates that the filament voltage was 6.3 volts at the time these curves were made. The filament is the heating element, usually located inside the cathode of a tube.

143. Here are two problems involving the use of the family of curves shown in figure 101. You know that the grid voltage is –2 volts, and you wish to find the plate current obtainable with a plate voltage of 125 volts. First, read along the horizontal scale to 125 volts. Then read

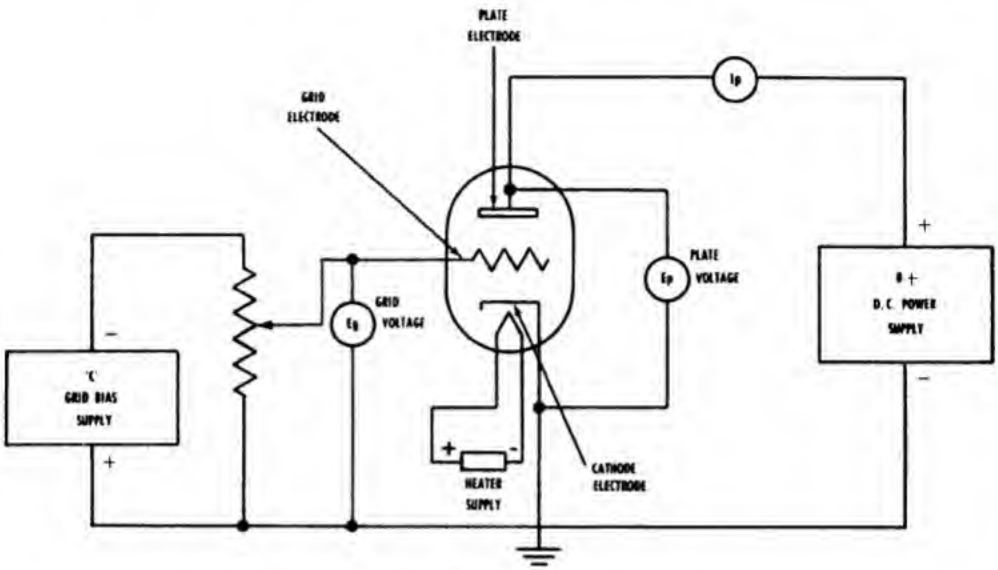


Figure 102.—Symbols used in a vacuum-tube circuit.

straight up the 125 volt line to the intersection of that line and the –2 grid volts curve. Notice that you will cross a number of other curves at various grid voltages, which are not applicable in this particular problem. From the intersection of the 125 plate volts line with the –2 grid volts curve, draw a line horizontally across to the plate-current scale. You should read a plate current of about 8.4 milliamperes. Suppose, however, that you wanted a current of 8 milliamperes at a plate voltage of 200 volts, what grid voltage would you require? In this case draw a line horizontally from the 8-milliampere mark on the vertical scale, and draw a second line vertically from the 200-volt line on the horizontal scale. The intersection of these two lines is near the curve $E_g = -6$. You would, therefore, require a grid voltage of about –6 volts to obtain the conditions you required.

OTHER TUBE CURVES

144. In addition to the plate characteristic graphs, there are a number of other curves of tube characteristics with which you should be familiar. One of these shows the relation between the grid voltage and the plate current. It is called the grid voltage—plate current curve and is illustrated in figure 103. It is particularly important because the operation of most amplifiers depends on this

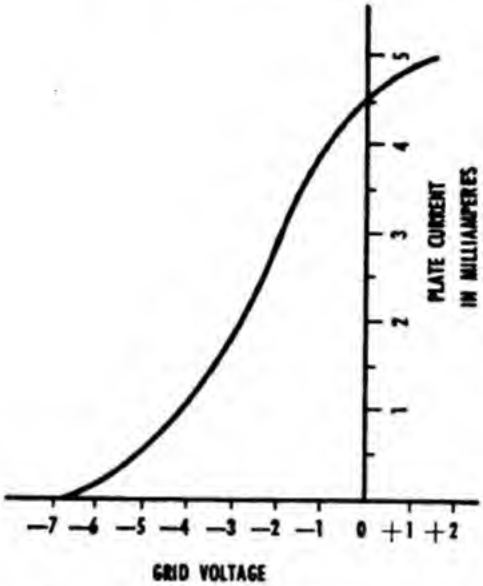


Figure 103.—Grid voltage—Plate current curve.

relationship. Input signals are fed into a vacuum tube by way of the grid, as shown in figure 104. This input signal is usually an alternating voltage and therefore causes the voltage of the grid to vary. The variation of the grid voltage causes the plate current to vary in a similar manner, as you will learn here. The manner in which the signal input to the grid is reproduced or amplified depends on the shape of the E_g—I_p curve of the particular tube used. Figure 103 is one such curve.

145. Until now you have worked with graphs when you were concerned with only one point, or a pair of values, at a time. When you work with the E_g—I_p curve you

will use a series of points, because the information you are seeking is the manner in which the plate current varies with respect to the input signal at the grid. For example, you may wish to know what swing or change in plate current is caused by a change from –1 volt to –3 volts on the

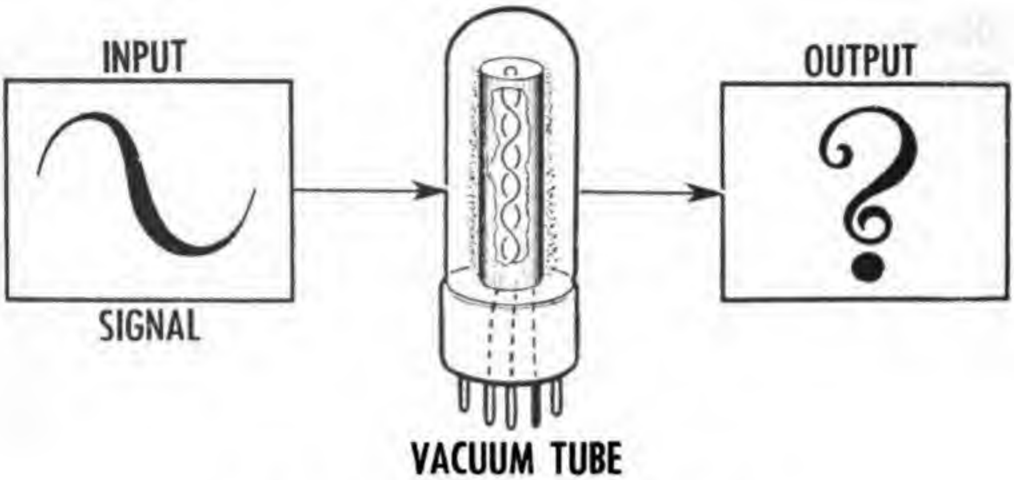


Figure 104.—What will the output be?

grid of a tube. You would project two vertical lines upward from the two points, –1 and –3 volts, on the horizontal scales, to intersect the curve. By drawing horizontal lines from the two points of intersection to the vertical or plate-current scale you could find the values of the plate current for those two grid voltage values. You would then know the swing in plate current that would result from a swing from –1 to –3 volts on the grid (fig. 105).

146. The input to the grid of a vacuum tube usually is an alternating voltage which varies with time. It can be represented by a sine curve similar to the one shown in figure 99. The plate current, following the relation given by the E_g—I_p curve, swings back and forth as a function of this alternating voltage input to the grid. By plotting several points during one cycle of the input voltage and by finding the corresponding plate-current values, the pattern or curve of the plate current with respect to time may be found. The result is something like the illustration

shown in figure 106. It provides a curve of the I_p values. This problem presents one of the fundamental principles of vacuum-tube amplifiers. Since the frequency of the grid-voltage input can be reproduced in this manner and

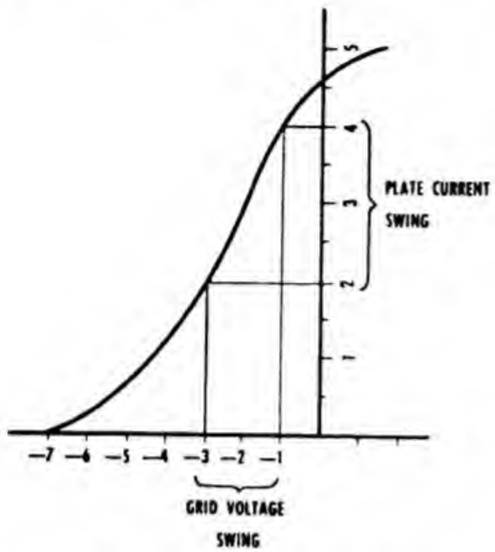


Figure 105.—Using the plate current —Grid voltage curve.

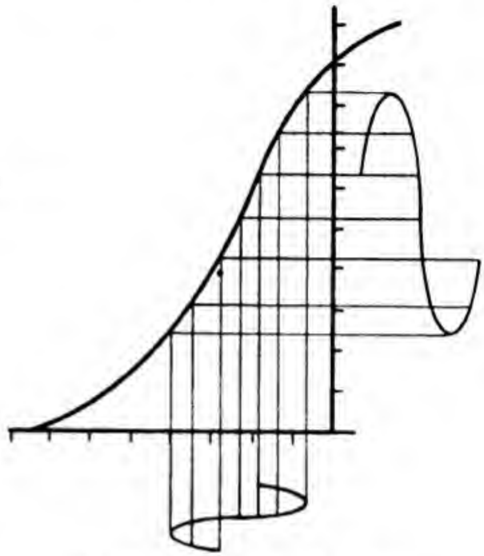


Figure 106.—E_g—I_p curve reproduces the signal.

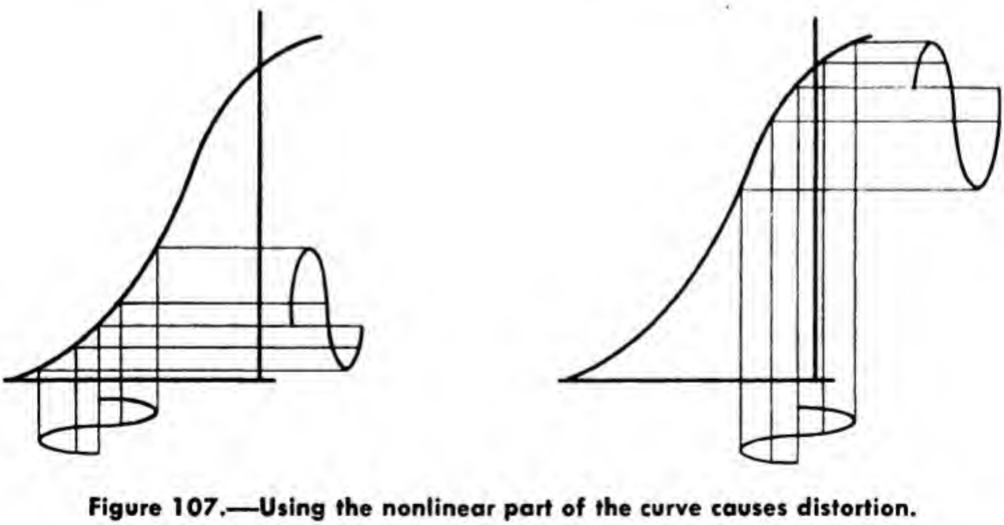


Figure 107.—Using the nonlinear part of the curve causes distortion.

since the strength of the signal can be increased or amplified by applying a high plate voltage to the tube, the use of the vacuum tube for this purpose is a most important one. However, if the part of the E_g—I_p curve over which the

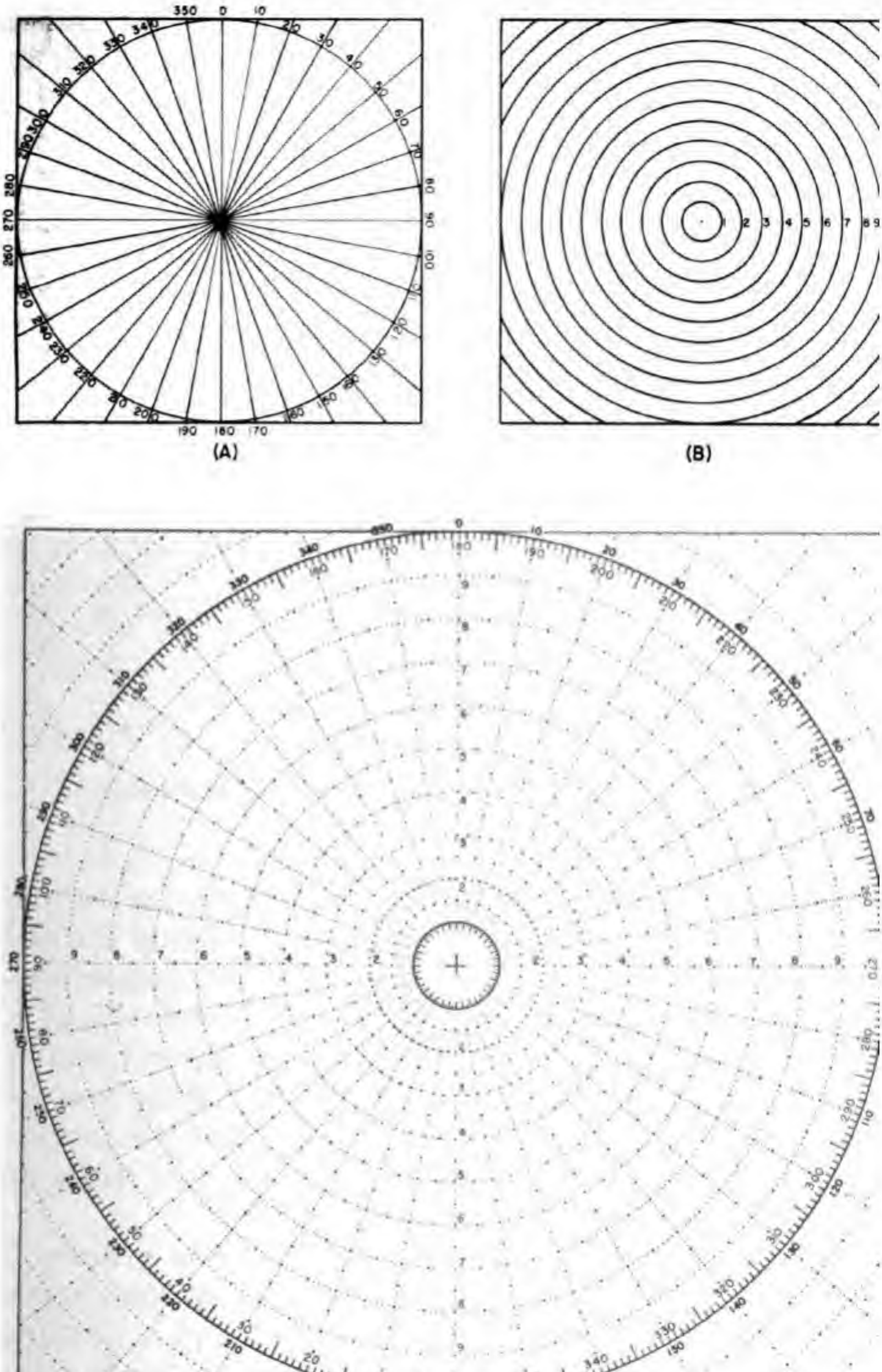
tube is used is not reasonably straight, the sine-curve shape of the signal is changed or distorted in the amplification process. It is important that you use only the straight part of the curve when you wish to reproduce the input signal accurately. There may be times when you want a distorted reproduction of the input signal. Distortion can be produced by adjusting the grid bias so that the input signal swings the grid over a curved portion of the E_g—I_p curve. (See fig. 107.) All of these problems require a thorough understanding of the manner in which tube characteristic curves are plotted, how they are read, and what they mean.

POLAR GRAPHS

147. The graphs you have studied so far have shown the relation between two quantities. One quantity is laid out on a horizontal line and the other on a vertical line. Many times it is desirable to change the arrangement of the scales to show the best relation between two quantities. One graph that does this is called a POLAR GRAPH.

148. In a polar graph one scale gives direction and is laid out as shown in figure 108, a. The second scale can show any quantity which varies with direction or is dependent on it (fig. 108, b). Putting the two scales together, you obtain a POLAR CHART, upon which you may plot your graph (fig. 108, c). Since one scale shows direction, all polar graphs have this scale in common. This scale usually is marked off in degrees from 0 to 360 (fig. 108, c). The units for the second scale vary for different graphs, but usually the center is taken as the zero point.

149. Polar graphs are used widely in showing antenna radiation patterns. An example of such a graph is shown in figure 109. A radiation pattern shows (1) the relative amount of electrical energy that leaves an antenna and (2) its direction with respect to the antenna. In the example shown in figure 109 the relative strength of the emitted energy is given in db (the abbreviation for decibel,



(A) (B)

(C)

Figure 108.

the unit used in electrical work for measuring relative signal strengths). The position of the antenna is shown by the heavy line in the center of the graph.

150. Refer again to figure 109. In which direction from the antenna is the maximum signal radiated? With a little studying of the graph you will see that the maximum

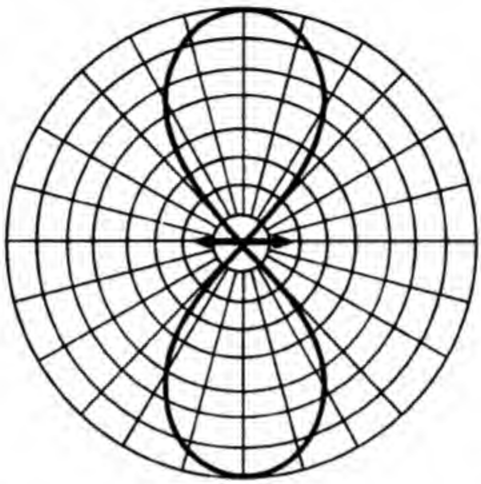


Figure 109.—A polar graph showing an antenna radiation pattern.

signal is emitted perpendicular to the antenna (at 90° and 270°). What is the relative strength of the signal emitted in the 45° direction? To find the answer to this question, start from the center and follow the 45° line out until it intersects the curve. Notice that you cut the concentric rings which mark the relative power output. When you cut the curve, you read the relative strength from the concentric rings; or, if you are in between rings, you make an approximation.

151. Another example in the use of polar graphs is shown in figure 110, which shows a TYPICAL MANEUVERING BOARD. A chart of this type is used aboard ship in plotting the position of aircraft or of other ships in relation to your own ship. When you plot aircraft, which have speeds much greater than those of a ship, you consider the plotting ship to be stationary and located in the center of the board.


300
290
280
270
260
250
240

Consider the graph that results from plotting the following reports—

Report	Bearing	Range (yards)
A	080°	14,000
B	090°	13,000
C	100°	12,000
D	108°	13,000
E	118°	14,000

The range scale is marked by the concentric rings, of which there are 10. By letting each concentric ring represent 2,000 yards of range, you can use the whole chart to plot ranges up to 20,000 yards. Since the farthest range report is 14,000 yards, you can plot all reports on the chart. Report A places the target at the intersection of the bearing 080°, and the range marker of 14,000 yards (on your scale, a distance of 14,000 yards is 14,000 ÷ 2,000 = 7 range rings from the center). To plot this point, you follow the 080° bearing line until it crosses the seventh circle, and then place a small mark to indicate the position. The next report, B, locates the target at the intersection of the bearing line 090°, and 13,000 ÷ 2,000 = 6½ range circles from the center. The rest of the reports are plotted in the same manner. By using the proper instruments and procedure, you can obtain the course and speed of the other vessel on the polar graph.

CALIBRATION CURVES

152. Many electronic instruments are provided with a large number of knobs for adjusting various parts and circuits within the piece of equipment. Often these knobs are simply numbered 1, 2, 3, 4, 5, or A, B, C, D, E. Although you may know what they control, you have no direct way of measuring how much to turn each knob to get the desired results. Usually graphs called CALIBRATION CURVES are provided, from which you can learn how much and which way to turn the knobs to obtain the results you wish. Figure 111 shows such a curve or series of curves. The

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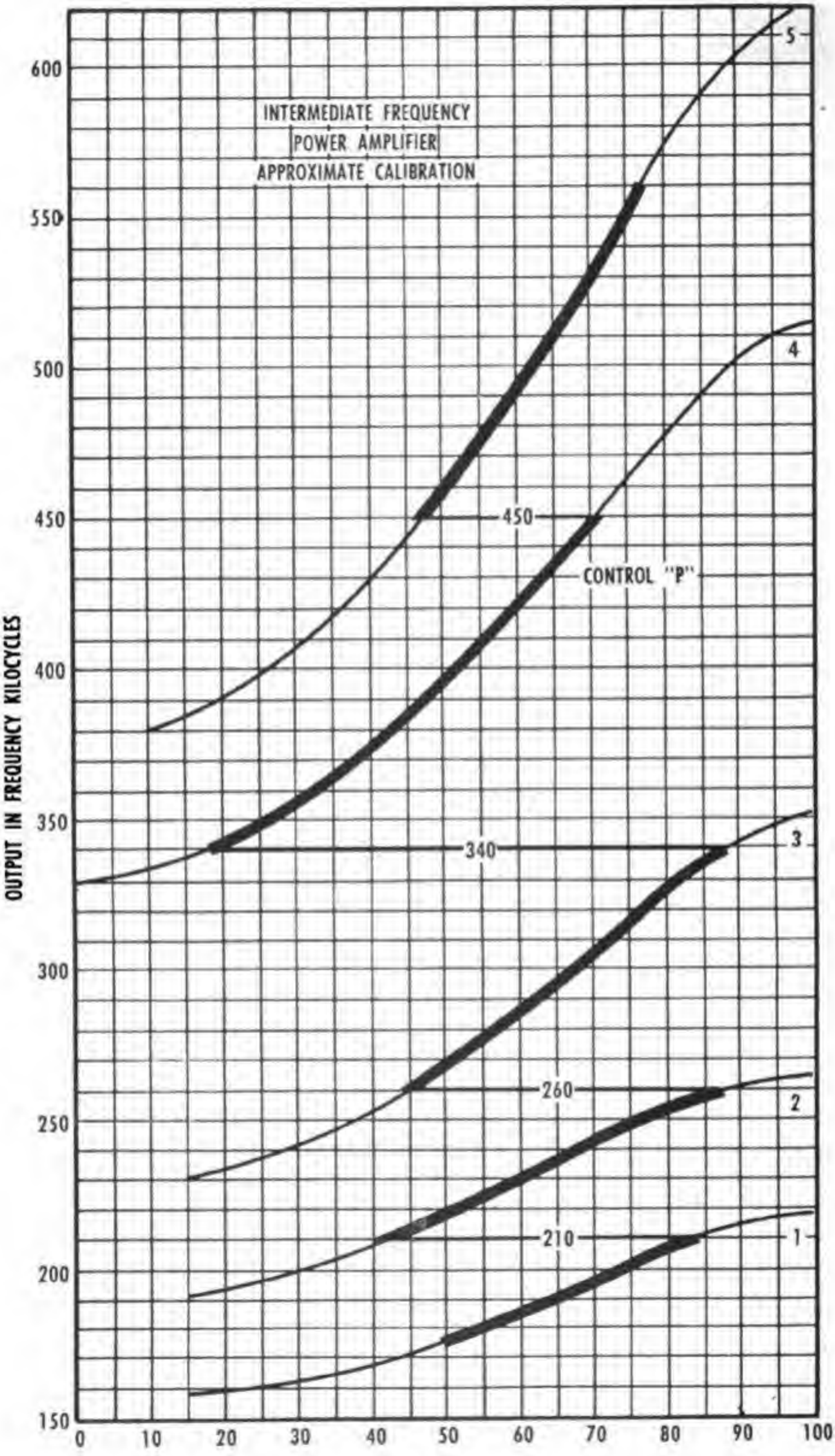


Figure 111.—A calibration curve.

controls involved are shown in figure 112. Knob *A* has five positions, for which the graph has five corresponding curves. Each curve is numbered at the top and corresponds to the same numbered position of knob *A*. The numbers on the horizontal scale of the graph correspond to the numbers on the dial of the small knob *B*. Knob *A* is for coarse

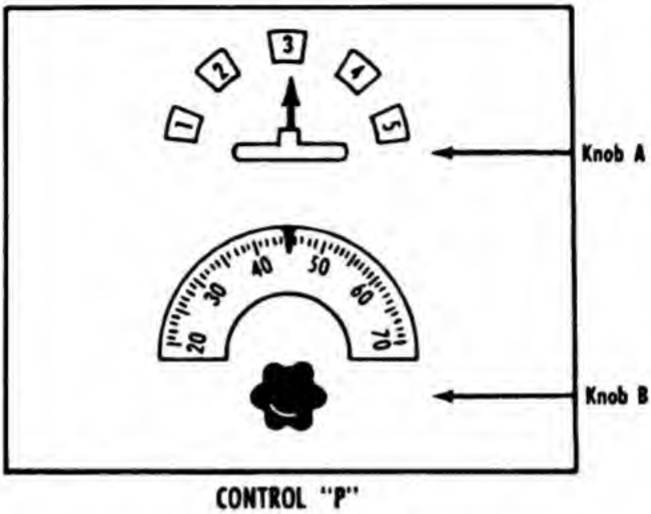


Figure 112.—Control knobs.

adjustments; knob *B* is for fine adjustments. The calibration curves tell you how to set these knobs to adjust the output to any particular frequency.

153. Suppose you wish to set the controls for an output frequency of 500 kilocycles (500,000 cycles per second). You read up the vertical scale until you come to the point marked 500 kc. Then read horizontally across the graph from that point until you intersect one of the five curves. You should cross curve 5, which tells you that the coarse control should be set to that number. Now read vertically downward from that intersection to the horizontal scale and find the number to which the fine control, knob *B*, should be set. The answer is 62. Therefore, to get an output frequency of 500 kc. you should set the coarse control to position 5 and the fine control to position 62. Sometimes a calibration graph has only one knob to adjust. Whatever

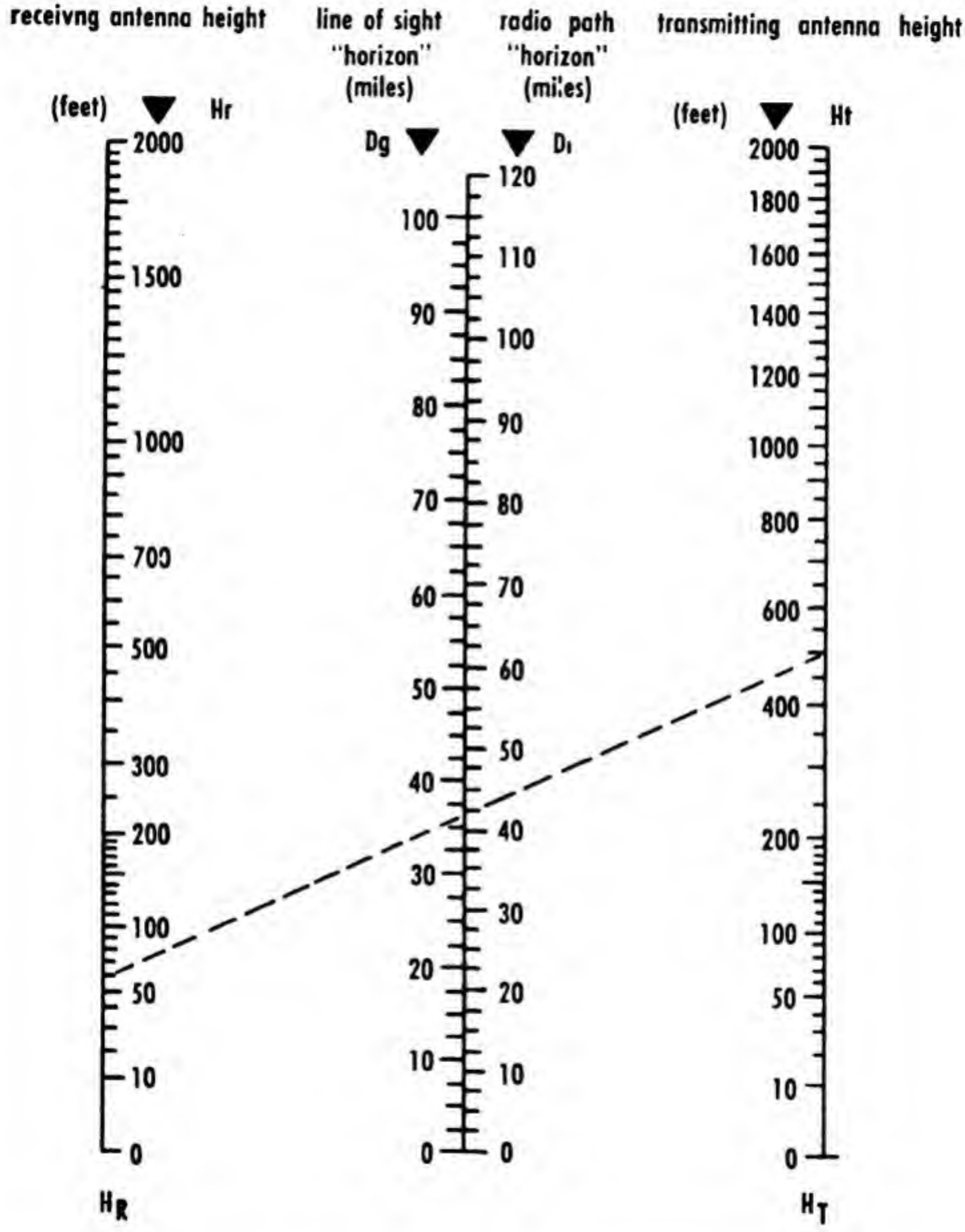
the calibration problem, this is the correct procedure for adjusting the controls. On a large transmitter you may have more than 12 knobs to adjust.

NOMOGRAPHS

154. Most graphs express a relation between two quantities or conditions. Although ordinary graphs show a relation between only two variables, one type of graph may be used to express the relationship of three interdependent quantities. Such a graph, called a NOMOGRAPH or NOMOGRAM, is particularly useful for solving many practical problems in electronics. Figure 113 is a nomograph used to determine the maximum distance at which an ultrahigh-frequency radio wave can be detected under normal conditions. Ultrahigh-frequencies are waves with frequencies of 300 to 3,000 megacycles per second. (A megacycle is 1,000,000 vibrations.) The distance depends on the height above ground of both the receiving and the transmitting antennas. Theoretically, it should be the same as the maximum distance you could see from the top of one antenna to the other, but certain atmospheric conditions increase the range of ultrahigh-frequency radio waves. The two antenna height scales are clearly indicated on the nomograph. The center scales show (1) the theoretical maximum path (line of sight) on the left scale, and (2) the maximum path of the radio waves on the right scale. To read the nomograph, place a straightedge across the known values on two scales, and you will find the answer you are seeking at the point where the straightedge intersects the third scale.

155. If you know that a UHF (ultrahigh-frequency) transmitter antenna is 130 feet high and the antenna of a receiver is 50 feet high, what would be the maximum theoretical path (line of sight) of the radio waves? Place a straightedge on a line joining the 130-foot mark on the transmitter antenna scale (scale H_t), and the 50-foot mark on the receiver antenna scale (scale H_r). You will

U-H-F path length and optical line-of-sight distance range of radio waves



Example shown: Height of receiving antenna 60 feet, height of transmitting antenna 500 feet, and maximum radio path length 41.5 miles.

Figure 113.—A nomograph.

find the answer on the left center scale. It is 22.5 miles. Without moving the straightedge, read the right center scale, which indicates the ACTUAL path length under these conditions to be about 25 miles.

156. Here is another problem. A transmitter with an output frequency of 600 megacycles per second has an antenna 200 feet high. How high an antenna must you use to receive from this transmitter at a distance of 20 miles? Lining up your straightedge as shown by the line *CD* on figure 113, you will find that the receiver antenna height must be at least 18 feet. This nomograph is really a device for finding graphic solutions for the equations, as follows:

$$D_l = 1.41\ (\sqrt{H_t} + \sqrt{H_r}) \text{ statute miles,}$$

$$D_g = 1.23\ (\sqrt{H_t} + \sqrt{H_r}) \text{ statute miles,}$$

where—

D_l is the radio path length in statute miles,
D_g is the line of sight "horizon" in statute miles,
H_t is the transmitter antenna height in feet, and
H_r is the receiver antenna height in feet.

157. Another nomograph you will often use in electronics work is the inductance, capacity, and frequency chart, shown in figure 114. Inductance is an electrical characteristic of wire coils; capacitance is an electrical characteristic of capacitors—devices which consist of two electrical conductors separated by a nonconducting material. When any given inductor and capacitor are connected in an electrical circuit in a certain manner, they are resonant (particularly sensitive) to a single frequency, which depends on their inductance and capacitance. By adjusting variable inductors and capacitors you can tune such a circuit to any desired frequency. The relationship of the inductance (L), the capacitance (C), and the resonant frequency (F) is expressed by the equation

$$F = \frac{1}{2\pi\sqrt{LC}}.$$

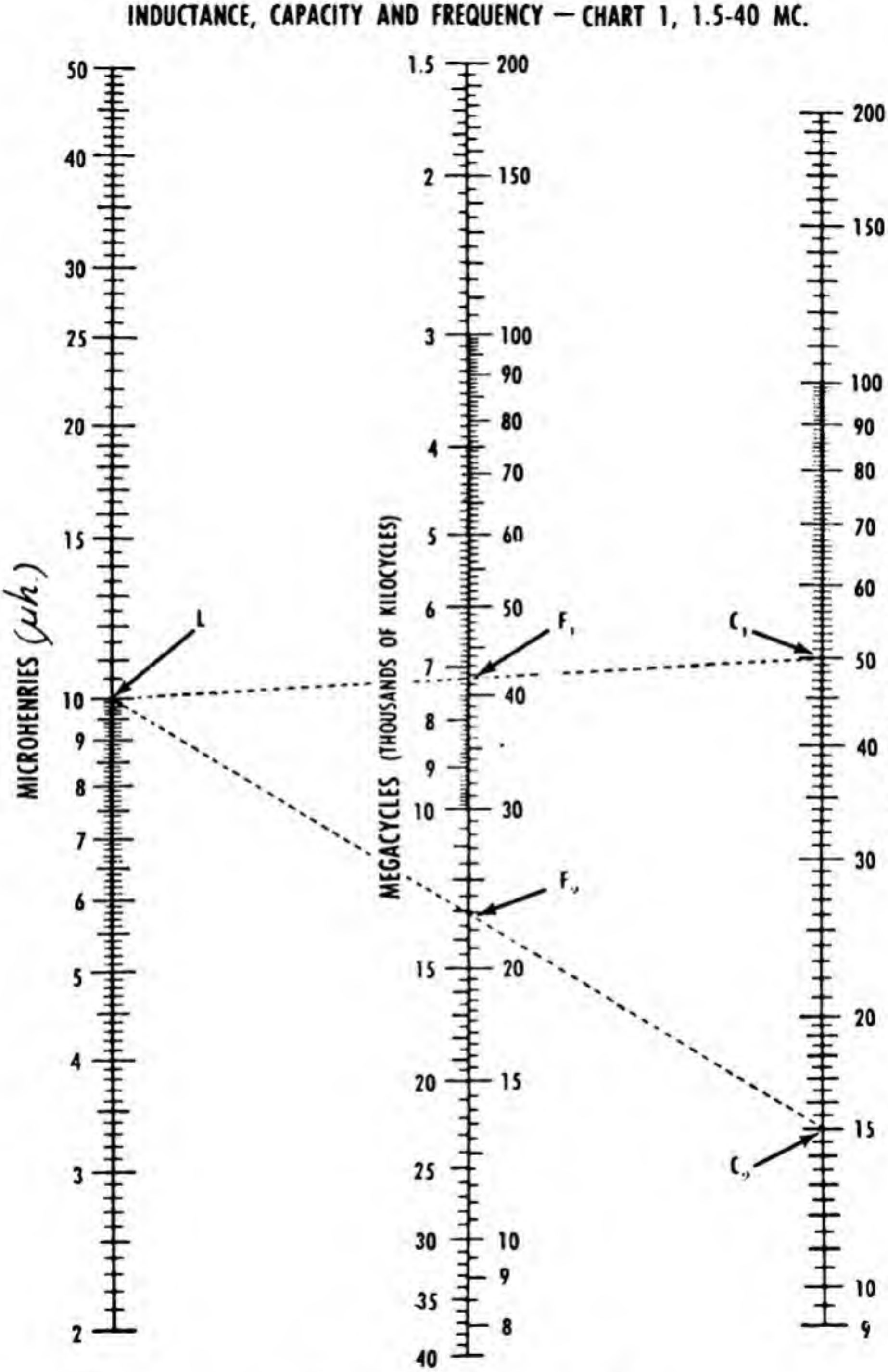


Figure 114.—A nomograph.

The nomograph of figure 114 permits you to read the solution of this equation directly.

158. Suppose you wish to construct a tuner for a maximum frequency of 13 megacycles. You have an inductance of 10 microhenries (0.000,010 henries). What minimum capacitance would you require? Place your straightedge so that it connects the 13-megacycle mark, F_2, on the frequency scale with the 10-microhenry mark, L, on the inductance scale. You will then be able to read the correct minimum capacitance, C_2, (where your straightedge intersects), on the capacity scale. The answer is 15 μμf. equivalent to 0.000,000,000,015 farads. If the capacitor to be used is variable and may be tuned from 15 μμf. to 50 μμf., over what range of frequencies can you tune the combination of this capacitor and the 10 μh. inductor? The line LC_1 shows how the nomograph is used to find the frequency you will obtain with the 10 μh. inductor and the capacitor set for 50 μμf. That frequency, F_1, is 7.2 megacycles. Therefore, with the 10 μh. inductor and a capacitor that may be varied from 15 μμf. to 50 μμf. you can tune over the range from 7.2 to 13 megacycles.

If you did not pass the pretest in this chapter, apply what you have learned by working the problems now and also the problems in section V of the Remedial Work.

CHAPTER 6

FORMULAS USED IN PRACTICAL GEOMETRY

In electronics, as in everyday life, you need to deal with the areas of common plane figures, such as triangles, circles, and squares; and with the volumes of common solids, such as cylinders and cones. In this chapter you will deal with the formulas for finding areas and volumes of common plane figures and solids.

Formulas by themselves may look complicated but in most cases their use is very simple. You can use a formula in a purely mechanical fashion, that is, just substitute given values and arrive at the right answer. It is more satisfying, however, if you know what you are doing. Knowing what the formula means makes you feel that you are master of the situation.

PRETEST 6

159. In this pretest a geometric figure accompanies each exercise. In each exercise (1) name the figure, (2) write the formula called for, and (3) use the formula to solve for the quantity asked.

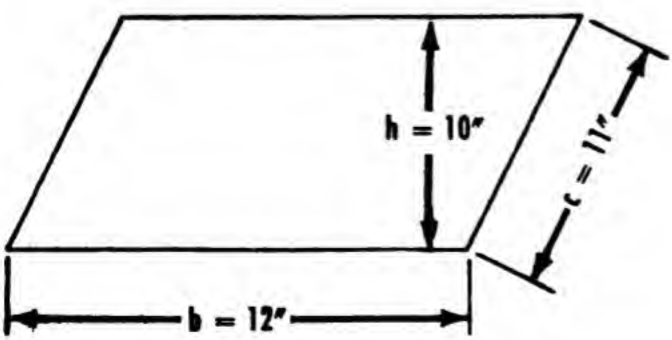


Figure 115.

(a) (1) This is a .....

(2) The formula for its area is ....

(3) The area of this figure is ....

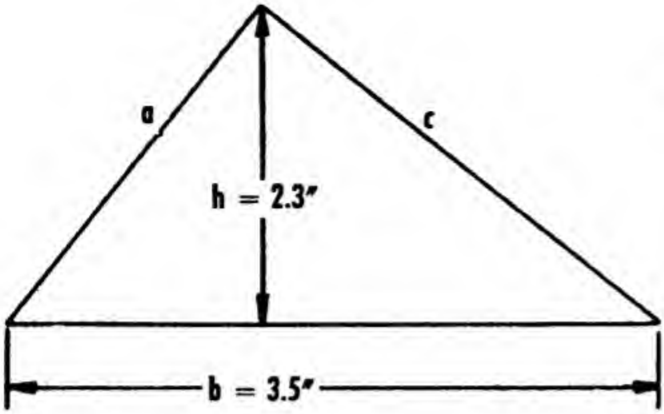


Figure 116.

(b) (1) This is a ......

(2) The formula for its area is ....

(3) The area of this figure is ....

(c) (1) This is a ....

(2) The formula for its circumference is ... ., and for its area is ....

(3) The circumference of this figure is ...., and the area is ....

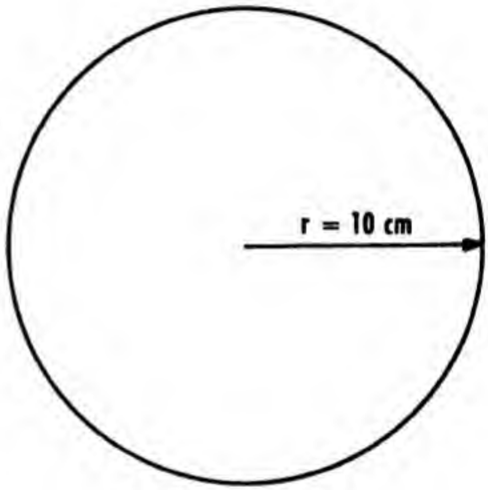


Figure 117.

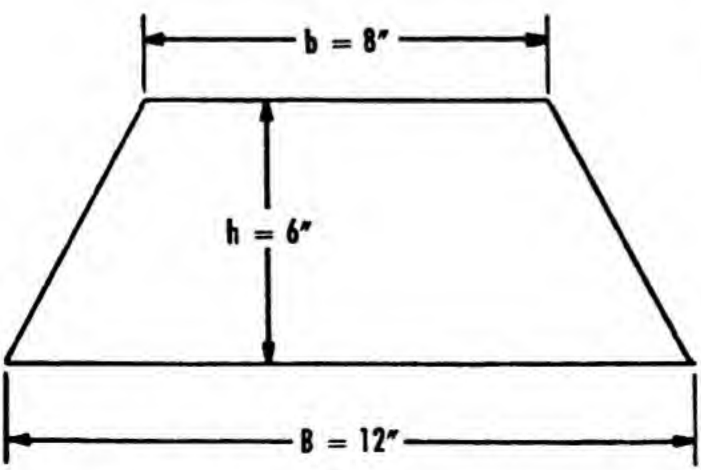


Figure 118.

(d) (1) This is a ....

(2) The formula for its area is ....

(3) The area of this figure is ....

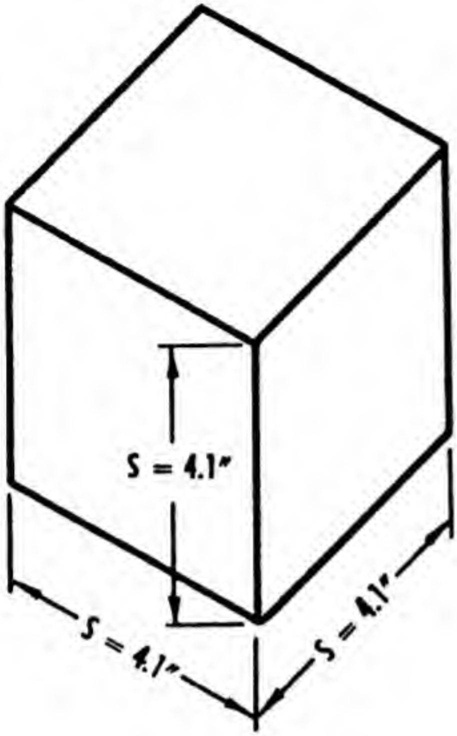


Figure 119.

(e) (1) This is a ....

(2) The formula for its volume is ....

(3) The volume of this figure is ....

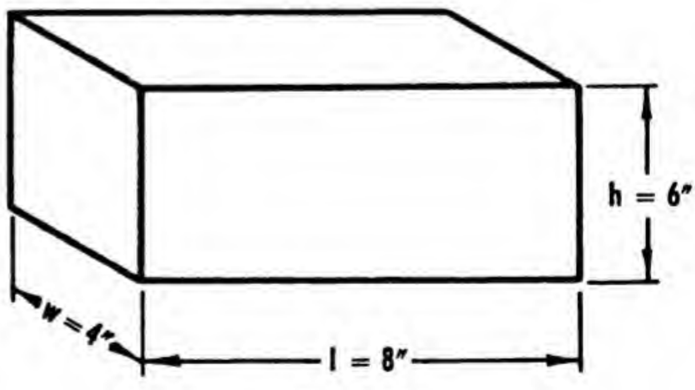


Figure 120.

(f) (1) This is a ....

(2) The formula for its volume is............ .

(3) The volume of this figure is.................. .

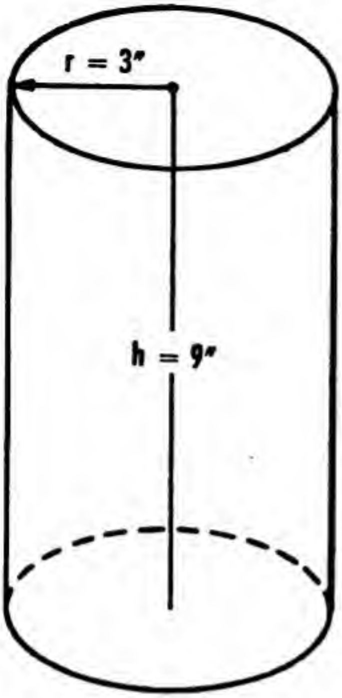


Figure 121.

(g) (1) This is a ....

(2) The formula for its volume is ....

(3) The volume of this figure is ....

(h) (1) This is a ....

(2) The formula for its volume is ....

(3) The volume of this figure is ....

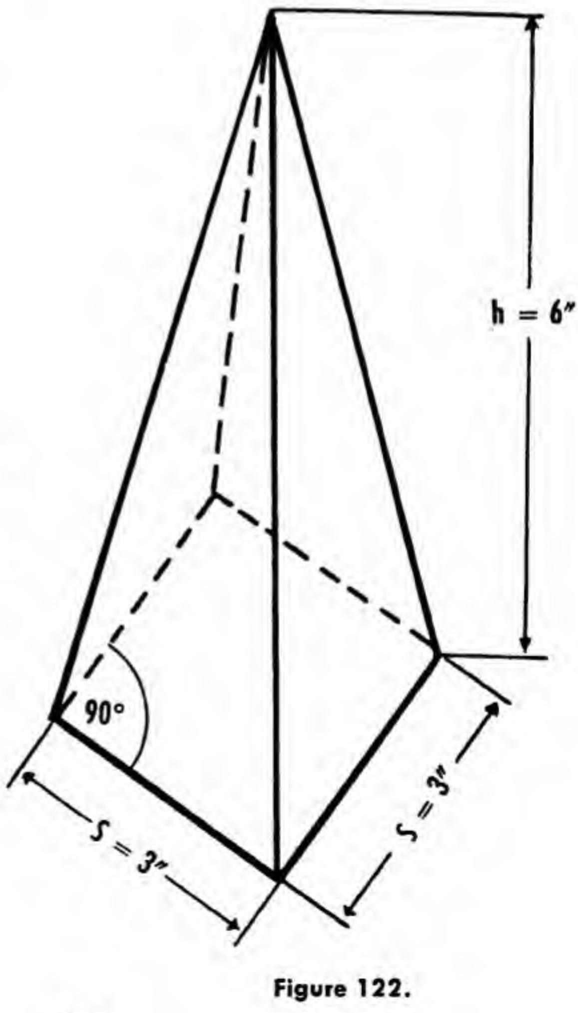


Figure 122.

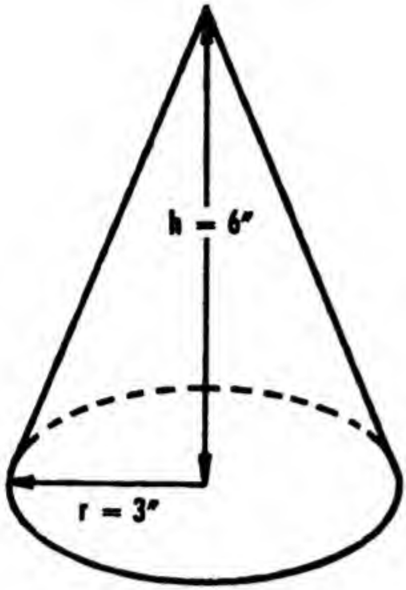


Figure 123.

(i) (1) This is a ....

(2) The formula for its volume is ....

(3) The volume of this figure is ....

(j) (1) This is a ....

(2) The formula for the area of its surface is .... , and for its volume ....

(3) The area of the surface of this figure is .... , and its volume is ....

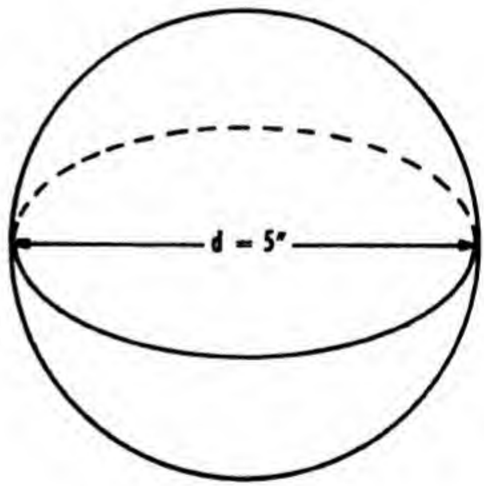


Figure 124.

STUDY GUIDE ON FORMULAS

160. In your dealings with formulas there are three things that you should keep in mind—

(a) A formula is an abbreviation of a practical rule indicated by letters and symbols of a mathematical operation. The formula for finding the area, A, of a rectangle when the length of the base, b, and the height, h, are given, is area = base × height. Stated briefly the formula

is $A = bh$. This is a general formula because it is used to solve not merely a particular problem, but ALL problems of THAT kind.

(b) Always keep the units of the formula straight. In applying the formula $A = bh$, b and h have to be expressed in the same unit. They can be in feet, inches, or anything you please, but they both must be in the same unit. If you choose inches, then A will be in square inches. Always label the units you are dealing with. It will prevent many errors.

(c) When a letter is used in a formula, that letter always means the NUMBER of units. Never say "Let x equal men," but say "Let x equal the number of men." Never write "Let x equal inches," but write "Let x equal the number of inches."

AREA

161. Figure 125 shows that the AREA OF A SQUARE is the product of its base and height. Since the sides are

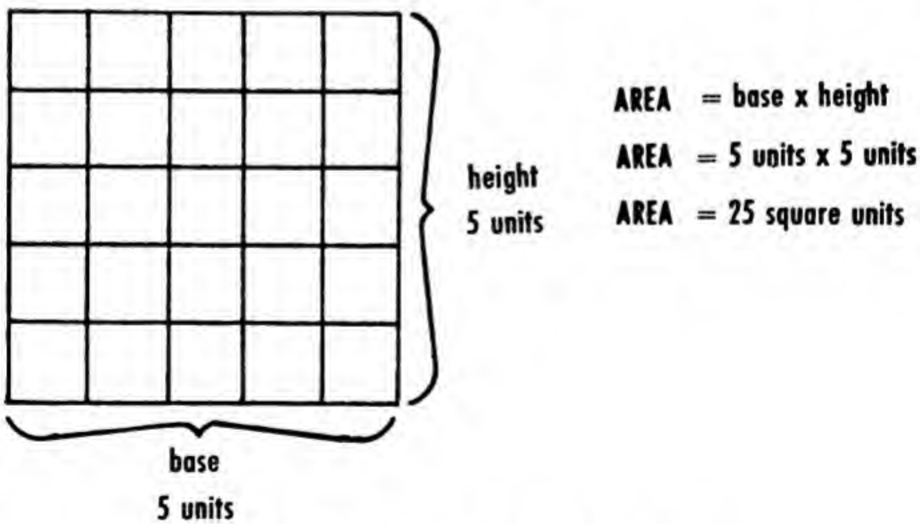


Figure 125.—Finding the area of a square.

equal in any square, the formula may be simplified in the following way: Area of square = the square of any side, or $A = s^2$, where s is the number of units in the length of one side. Reversing the procedure, when the area of a

square is given to find a side, take the square root of the area, thus: SIDE OF SQUARE = $\sqrt{\text{AREA OF SQUARE.}}$

162. Figure 126 shows that the AREA OF A RECTANGLE is the product of its base and height. Suppose that a rectangle 6″ long and 4″ wide is drawn on squared paper (fig. 126), how many square inches does it contain?

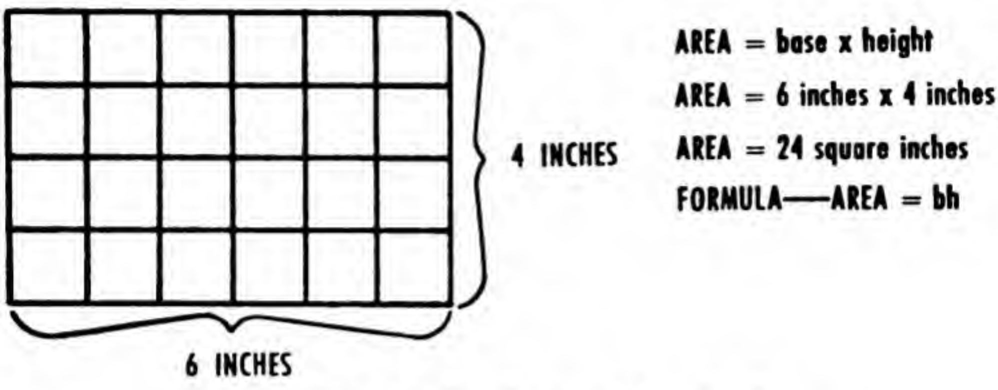


Figure 126.—Finding the area of a rectangle.

Evidently there are four rows of squares, each containing six squares. Hence, the number of square inches the rectangle contains is 4×6, or 24. That is, the area is 24 square inches. This method can be used in the case of ANY rectangle.

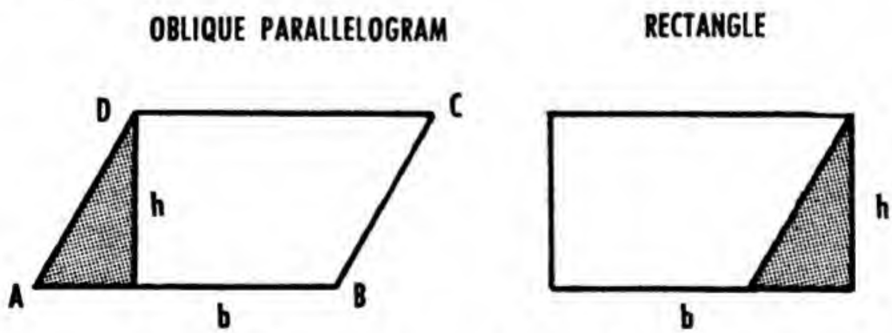


Figure 127.—Changing a parallelogram to a rectangle.

163. A parallelogram is a four-sided plane figure having both pair of opposite sides parallel. Squares and rectangles are parallelograms containing only right angles. Figure 127 shows a rectangle that has been pushed into a slanted position. The angles are no longer right

angles, but the figure is still a parallelogram as long as opposite sides remain parallel. The formula for the area is always $A = bh$. In a rectangle the height is the same as the length of one side. A figure that does not contain any right angles is sometimes referred to as oblique. An oblique parallelogram is readily changed to a rectangle in the following manner: A line is drawn from D perpendicular to AB. Then the shaded portion is cut off and placed on the opposite end (fig. 127), making a rectangle whose area is equal to the product of the base and the

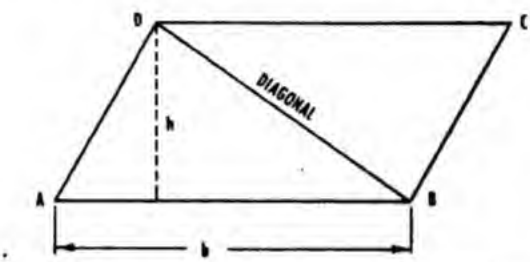


Figure 128.—Changing a parallelogram into two similar triangles.

height. Therefore, the area of any parallelogram may be found by this formula—

$$\text{area of parallelogram} = \text{base} \times \text{height},$$

or

$$A = bh.$$

164. The formula for finding the AREA OF A TRIANGLE may be found by starting with a parallelogram, as shown in figure 128. By drawing a diagonal as DB, you can see that the parallelogram is divided into two equal triangles. The area of triangle ABD is, therefore, one-half of the area of the parallelogram $ABCD$. Since the formula for the area of a parallelogram is $A = bh$, the formula for the area of a triangle is—

$$A = \frac{bh}{2}.$$

This formula applies for finding the area of ALL triangles.

165. Figure 129 is a picture of a trapezoid. A trapezoid is a four-sided plane figure with only one pair of op-

posite sides parallel. The two parallel sides (CD and EF in the figure) are the bases. The distance h between them is the altitude, or height. You'll find a good many things in electronic shops that are made in the form of a trapezoid, and you'll need to know how to find the AREA OF A TRAPEZOID. The formula is easy to use once you understand it.

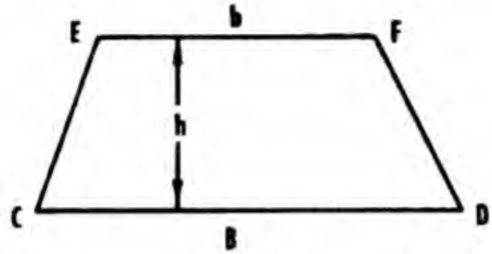


Figure 129.—A trapezoid.

166. Looking again at this figure of a trapezoid, you can see that it looks something like a rectangle, but the two bases are unequal. Remember the formula for the area of a rectangle, $A = bh$. You can use the rectangle formula if you can find the average of the two bases. You can do this by taking the sum of the two bases and dividing by 2, $\frac{b + B}{2}$. Using this value for the base, substitute in the rectangle formula and you have—

$$\text{area of a trapezoid} = \frac{b + B}{2} \times h,$$

or

$$A = \frac{h}{2}(b + B).$$

The rule to find the area of any trapezoid is to take half of the sum of the bases and multiply by the altitude.

167. A relation that is encountered many times in mathematics is the one that exists between the diameter and circumference of a circle. In every circle, whether it is large or small, the relation or ratio between the di-

921364—51——10

ameter and circumference is always the same. (See fig. 130.)

168. For hundreds of years mathematicians tried to find an exact value for the relation between the circumference and the diameter of a circle. In fact, an English mathematician worked out the ratio to 707 decimal places, but the result still didn't come out even. Since an exact fraction or decimal cannot be used to express the ratio of

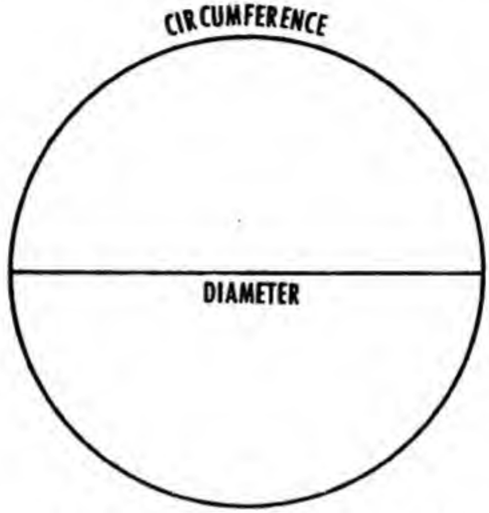


Figure 130.—Circle—circumference and diameter.

the diameter to the circumference of a circle, the ratio is represented by the Greek letter π (spelled pi, pronounced pie).

$$\frac{\text{circumference}}{\text{diameter}} = \pi,$$

$$\pi \times \text{diameter} = \text{circumference}.$$

169. For solving problems the ratio of diameter to circumference may be approximated by the fraction $^{22}/_{7}$, or in decimal form $\pi = 3.14159$. For most calculations π can be taken as being equal to 3.1416. You can readily show that 3.14 is an approximate value of the ratio of the circumference to the diameter by doing the following simple experiment—

(a) Take any circular object such as a 50-cent coin, and scratch a mark on its edge. Then place the coin on a flat

surface with the scratch mark touching the flat surface. Mark the flat surface at this point. Now carefully roll the coin until the scratch mark again touches the flat surface (fig. 131). Measuring the distance between these two points on the flat surface will give you the circumference

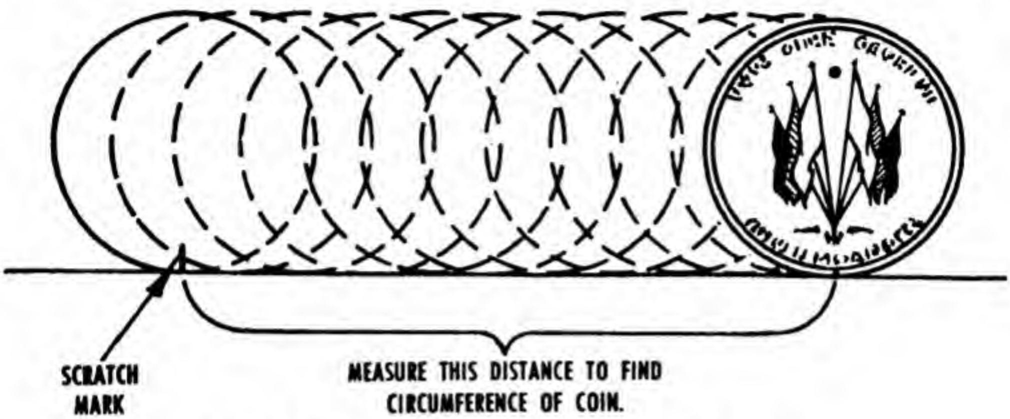


Figure 131.—Finding the circumference of a coin.

of the coin. Measure the diameter of the coin. Now dividing the circumference by the diameter, you will find the result to be 3.14, or π.

170. To find the circumference, C, of a circle, multiply the diameter, d, by 3.14. The exact formula is $C = \pi\, d$. You may find the formula written as $C = {}^{22}\!/_{7}\, d$ in some books, and in others as $C = 3.1416\, d$. For ordinary computations, you will find the formula $C = 3.14\, d$ accurate enough.

171. In figure 132, the radius, which is half the diameter of the circle, is 10 units long. If you are careful and estimate the fractions of squares accurately you will find that there are approximately 314 squares in the circle. Taking the radius (10) and squaring it will give you $10 \times 10 = 100$. Dividing the area (314) by the radius squared will give you ${}^{314}\!/_{100} = 3.14$. This is your old friend pi again. Note that 3.14 (or π) plays a part in both circumference and area formulas for a circle. The exact formula for the AREA OF A CIRCLE is —

$$A = \pi\, r^2.$$

For solving problems, the formula $A = 3.14\, r^2$ may be

used. Since the diameter of a circle is equal to $2r$, that is, $d = 2r$, the radius of a circle may be written in terms of the diameter as $r = \frac{d}{2}$. Substituting this value for r in the formula $A = \pi r^2$, you get the alternate formula—

$$A = \frac{\pi}{4} d^2.$$

Both the formulas are used to find the area of a circle. If the area of a circle is known and you wish to know the radius, use the following formula—

$$\text{radius} = \frac{\sqrt{\text{area}}}{\pi}.$$

172. A unit of area which finds particular importance in electronics is the CIRCULAR MIL. The circular mil is used in the measurement of a cross-sectional area of wire. Be-

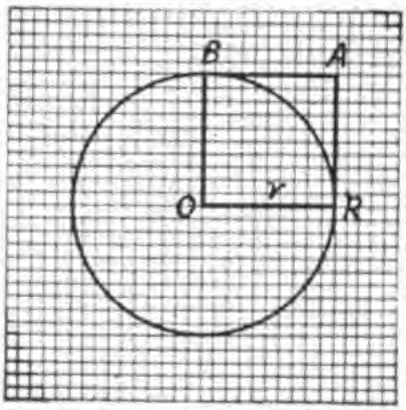


Figure 132.—Finding the area of a circle the hard way.

cause the diameter of wire is usually only a small fraction of an inch it is convenient to use a small unit of measurement. Therefore, the diameter of a wire is expressed in terms of a unit called the MIL, which is $\frac{1}{1000}$ of an inch. That is, there are 1,000 mils in an inch.

173. The circular mil, abbreviated cir mil or c.m., is the AREA of a circle whose diameter is 1 mil. You should note here that the circular mil is a unit of area which bears a fixed relation to the square mil. Except when it is

desired to compare round wires with rectangular conductors, it is seldom necessary to convert wire cross-sectional areas in circular mils into any other unit. (See fig. 133.)

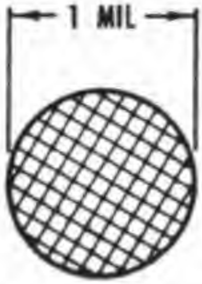
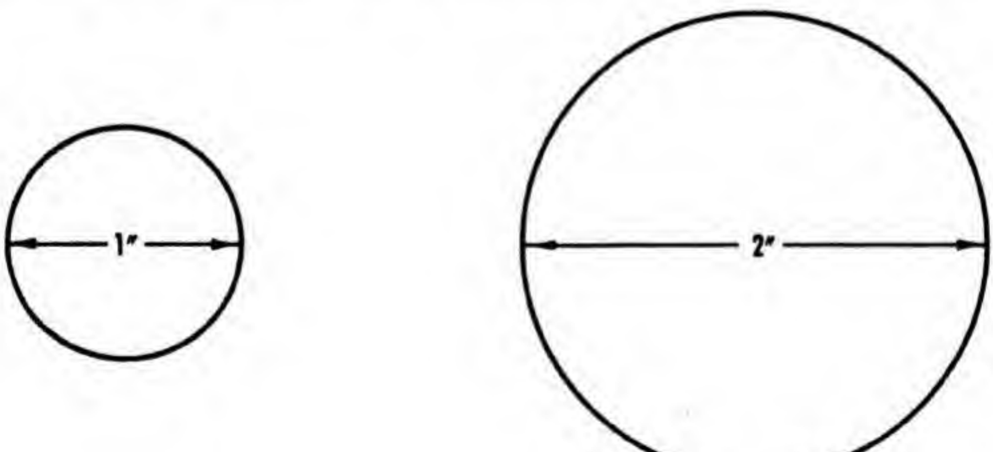


Figure 133.—A circular mil.

174. Referring to figure 134 you can see that the areas of circles vary as the squares of their diameters. The example given shows that a circle whose diameter is 2 inches has four times the area of a circle having a diameter of 1 inch.

175. Likewise, the area of a circle whose diameter is 2 mils (0.002 inch) has four times the area of a circle



$$A = \pi \frac{D^2}{4}$$

$$A = 0.785 \text{ sq. in.}$$

$$A = \pi \frac{D^2}{4}$$

$$A = 3.14 \text{ sq. in.}$$

$$0.785 \times 4 = 3.14$$

Figure 134.—Areas of circles vary as square of diameter.

having a diameter of 1 mil (0.001 inch). From the definition given previously, you know that one circular mil is the area of a circle whose diameter is 1 mil. Therefore, a circle whose diameter is 2 mils must have an area of 4 circular mils. Hence, the area of a circle can be expressed in circular mils by squaring the diameter, provided that the diameter is expressed in mils. (See fig. 135.)

176. By comparing the methods used to find the areas in figures 134 and 135, you can see how much easier it is to express cross-sectional areas of wire in circular mils. In the circular-mil system it is unnecessary to use continually the factor pi (3.14). When the area of a circle is given in circular mils, you can find the diameter (in mils) by taking the square root of the area. Consider the following examples—

(a) No. 20 wire has a diameter of 0.032 inch. What is its diameter in mils? What is its circular mil area?

$$\text{diameter in inches} \times 1000 = \text{diameter in mils}$$

$$0.032 \times 1000 = 32 \text{ mils}$$

$$\text{circular mil area} = (\text{diameter in mils})^2$$

$$\text{cir mil area} = (32)^2 = 1{,}024 \text{ circular mils.}$$

(b) No. 30 wire has a cross-sectional area of 100 circular mils. What is its diameter in mils? in inches?

$$\text{diameter in mils} = \sqrt{\text{area in cir mil}} = \sqrt{100} = 10 \text{ mils,}$$

$$\text{diameter in inches} = \frac{\text{diameter in mils}}{1000} = \frac{10}{1000} = 0.01 \text{ inch.}$$

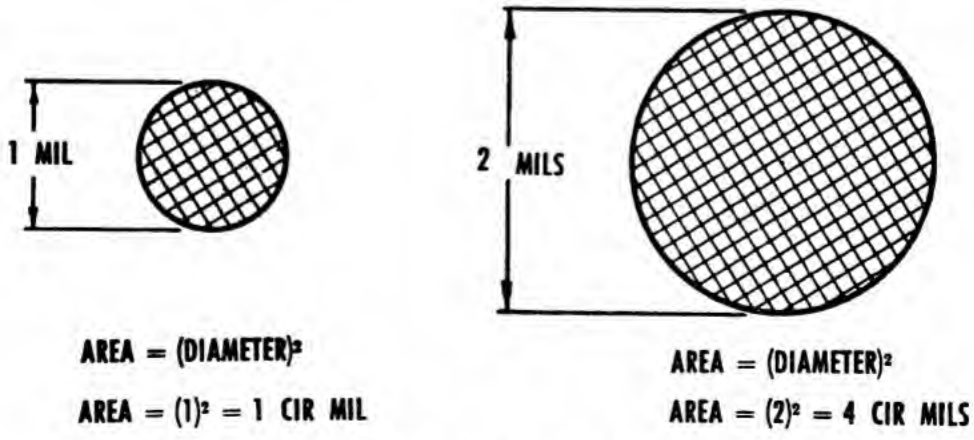


Figure 135.—Find circular mil area by squaring diameter.

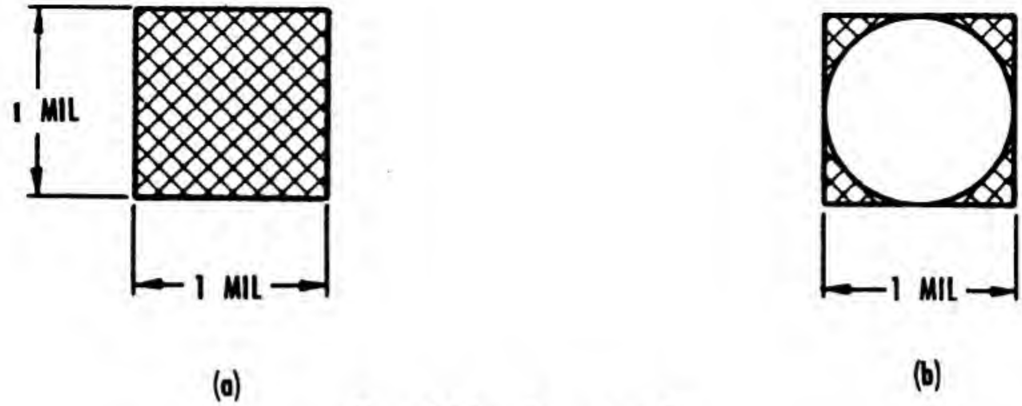


Figure 136.—A square mil.

177. Areas of square or rectangular conductors used in electronics can be expressed by use of a unit called the SQUARE MIL (fig. 136, a). To compare the square mil and the circular mil, look at figure 136, b. Since the area of a circle is—

$$A = \frac{\pi}{4}d^2,$$

or $$A = 0.7854d^2 \text{ square units,}$$

then it follows that—

$$\text{sq. mils} = \text{cir mil} \times 0.7854$$

$$\text{cir mils} = \frac{\text{sq. mils}}{0.7854}.$$

Work through the following example: A bus bar is ½-inch wide and ¼-inch thick. What is its cross-sectional area in square mils? in circular mils?

Given:

$$\text{width} = \tfrac{1}{2} \text{ in.} = 500 \text{ mils,}$$
$$\text{thickness} = \tfrac{1}{4} \text{ in.} = 250 \text{ mils,}$$
$$\text{area} = \text{width} \times \text{thickness} = 500 \times 250 = 125{,}000 \text{ sq.mils,}$$

$$\text{cir mils} = \frac{\text{sq. mils}}{0.7854} = \frac{125{,}000}{0.7854} = 159{,}150 \text{ cir mils approx.}$$

178. Wires are manufactured in sizes numbered according to what was formerly known as the Brown and

Sharpe (B & S) gage but what is now known as the American wire gage (AWG). (See fig. 137.) Notice that the numbers progress in reverse order — that is, the smaller size wires are denoted by larger numbers. Also

gauge no	diameter, mils	cross section circular mils	cross section square inches	ohms per 1,000 ft at 20° C (68° F)	lb per 1,000 ft	ft per lb	ft per ohm at 20° C (68° F)	ohms per lb at 20° C (68° F)
0000	460.0	211,600	0.1662	0.04901	640.5	1.561	20,400	0.00007652
000	409.6	167,800	0.1318	0.06180	507.9	1.968	16,180	0.0001217
00	364.8	133,100	0.1045	0.07793	402.8	2.482	12,830	0.0001935
0	324.9	105,500	0.08289	0.09827	319.5	3.130	10,180	0.0003076
1	289.3	83,690	0.06573	0.1239	253.3	3.947	8,070	0.0004891
2	257.6	66,370	0.05213	0.1563	200.9	4.977	6,400	0.0007778
3	229.4	52,640	0.04134	0.1970	159.3	6.276	5,075	0.001237
4	204.3	41,740	0.03278	0.2485	126.4	7.914	4,025	0.001966
5	181.9	33,100	0.02600	0.3133	100.2	9.980	3,192	0.003127
6	162.0	26,250	0.02062	0.3951	79.46	12.58	2,531	0.004972
7	144.3	20,820	0.01635	0.4982	63.02	15.87	2,007	0.007905
8	128.5	16,510	0.01297	0.6282	49.98	20.01	1,592	0.01257
9	114.4	13,090	0.01028	0.7921	39.63	25.23	1,262	0.01999
10	101.9	10,380	0.008155	0.9989	31.43	31.82	1,001	0.03178
11	90.74	8,234	0.006467	1.260	24.92	40.12	794	0.05053
12	80.81	6,530	0.005129	1.588	19.77	50.59	629.6	0.08035
13	71.96	5,178	0.004067	2.003	15.68	63.80	499.3	0.1278
14	64.08	4,107	0.003225	2.525	12.43	80.44	396.0	0.2032
15	57.07	3,257	0.002558	3.184	9.858	101.4	314.0	0.3230
16	50.82	2,583	0.002028	4.016	7.818	127.9	249.0	0.5136
17	45.26	2,048	0.001609	5.064	6.200	161.3	197.5	0.8167
18	40.30	1,624	0.001276	6.385	4.917	203.4	156.6	1.299
19	35.89	1,288	0.001012	8.051	3.899	256.5	124.2	2.065
20	31.96	1,022	0.0008023	10.15	3.092	323.4	98.50	3.283
21	28.46	810.1	0.0006363	12.80	2.452	407.8	78.11	5.221
22	25.35	642.4	0.0005046	16.14	1.945	514.2	61.95	8.301
23	22.57	509.5	0.0004002	20.36	1.542	648.4	49.13	13.20
24	20.10	404.0	0.0003173	25.67	1.223	817.7	38.96	20.99
25	17.90	320.4	0.0002517	32.37	0.9699	1,031.0	30.90	33.37
26	15.94	254.1	0.0001996	40.81	0.7692	1,300	24.50	53.06
27	14.20	201.5	0.0001583	51.47	0.6100	1,639	19.43	84.37
28	12.64	159.8	0.0001255	64.90	0.4837	2,067	15.41	134.2
29	11.26	126.7	0.00009953	81.83	0.3836	2,607	12.22	213.3
30	10.03	100.5	0.00007894	103.2	0.3042	3,287	9.691	339.2
31	8.928	79.70	0.00006260	130.1	0.2413	4,145	7.685	539.3
32	7.950	63.21	0.00004964	164.1	0.1913	5,227	6.095	857.6
33	7.080	50.13	0.00003937	206.9	0.1517	6,591	4.833	1,364
34	6.305	39.75	0.00003122	260.9	0.1203	8,310	3.833	2,168
35	5.615	31.52	0.00002476	329.0	0.09542	10,480	3.040	3,448
36	5.000	25.00	0.00001964	414.8	0.07568	13,210	2.411	5,482
37	4.453	19.83	0.00001557	523.1	0.06001	16,660	1.912	8,717
38	3.965	15.72	0.00001235	659.6	0.04759	21,010	1.516	13,860
39	3.531	12.47	0.000009793	831.8	0.03774	26,500	1.202	22,040
40	3.145	9.888	0.000007766	1,049.0	0.02993	33,410	0.9534	35,040

Temperature coefficient of resistance:

The resistance of a conductor at temperature t °C is given by

$R_t = R_{20}[1 + \alpha_{20}(t - 20)]$

where R_{20} is the resistance at 20° C and α_{20} is the temperature coefficient of resistance at 20° C. For copper, $\alpha_{20} = 0.00393$. That is, the resistance of a copper conductor increases approximately 4/10 of 1 percent per degree centigrade rise in temperature.

Figure 137.

note that for every third gage of smaller size, the wire halves in cross-sectional area and doubles in resistance. Therefore, the ratio between the cross-sectional areas of any two consecutive sizes is $\sqrt[3]{2}$, or 1.26 to 1. Furthermore, a No. 10 wire is practically $\frac{1}{10}$ of an inch in diameter; it has a cross section of 10,000 circular mils and a resistance of about one ohm per 1,000 feet. By remembering these few facts, you can estimate quickly the area and resistance of any size and length of wire without the use of tables. Consider the following example: It is desired to estimate the cross-sectional area and resistance of 1,000

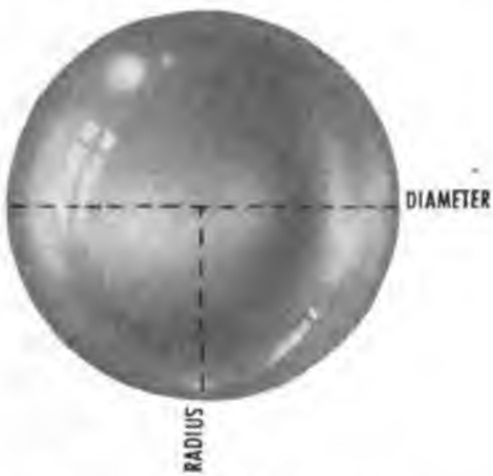


Figure 138.—Dimensions used for finding the area of a circle.

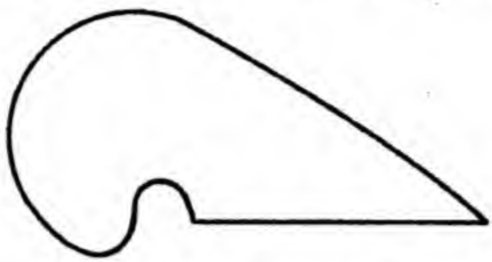
feet of No. 17 wire. Count from No. 10 to No. 16 by threes. This requires two steps, indicating an area one-fourth that of No. 10, or 2,500 circular mils, and a resistance of four times that of No. 10, or 4 ohms. Since the ratio of one size to the next is 1.26, the area of No. 17 is $2{,}500 \div 1.26 = 1{,}984$ circular mils, and its resistance is 4×1.26 or 5.04 ohms.

179. Many things take the form of a sphere. It would be helpful in your work to know how to compute both the area of the surface and the volume of a sphere. (See fig. 138.) Strangely enough, the AREA OF THE SURFACE (OUTSIDE AREA) OF A SPHERE is exactly four times the area of a

circle that has the same radius. The formula for the area of the surface of a sphere is—

$$S = 4\,\pi\,r^2, \text{ or } S = 4 \times 3.14\,r^2.$$

180. In shop work there often arises a need for measuring the AREA OF AN IRREGULAR OBJECT. Refer to figure 139, which shows the shape of a capacitor plate used in a radio



CAPACITOR PLATE OF IRREGULAR SHAPE

Figure 139.—Condenser plate of irregular shape.

circuit. Suppose that you wish to find the area of such a plate? Although mathematical means are available for solving for the area of irregular objects, they are beyond the scope of this book.

181. One method for finding the area involves using only mathematics that you already know. It is as follows—

(a) Carefully cut out a square or rectangular piece of thin metal with a pair of tin snips. Accurately calculate its area. Now weigh the piece of metal carefully (accurate scales can be found in the sick bay). With these figures you can write one ratio of a proportion, as follows—

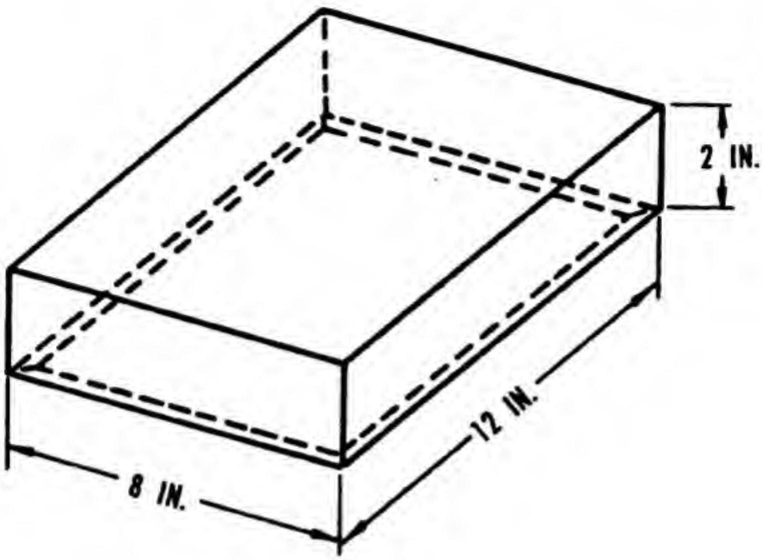
$$\frac{\text{Area of whole plate}}{\text{Weight of whole plate}}.$$

Next take the irregular object whose area is desired and trace its outline on the metal plate. Cut out this figure with the tin snips and weigh the metal. With this figure you can complete your proportion as follows—

$$\frac{\text{area of whole plate}}{\text{weight of whole plate}} = \frac{\text{area of irregular object}}{\text{weight of metal cut to shape of irregular object}}$$

By solving this equation, you can find the area of the irregular object.

182. Another method for finding formulas you have studied is shown in the following problem. A radio chassis is to be built to the dimensions given in figure 140. The problem is to find the over-all dimensions of the metal



RADIO CHASSIS

Figure 140.—A radio chassis.

stock sheet so that a requisition may be made for ordering the material. The area may also be required to enable the supply department to figure the cost of the metal that is to be used. If the material you use costs $\frac{1}{4}$¢ per square inch, what will be the cost of the metal to construct the chassis?

183. In solving a problem of this type, first mentally unfold the chassis pan and flatten it out. Then make a sketch showing the dimensions of the unfolded layout, such as in figure 141. Once the chassis pan is mentally unfolded, you can readily obtain the over-all dimensions of the stock sheet. In the example given, the length is made up of these dimensions: $\frac{1}{4}'' + 2'' + 12'' + 2'' + \frac{1}{4}''$, which gives an over-all length of $16\frac{1}{2}''$. The width is made up of the following dimensions: $\frac{1}{4}'' + 2'' + 8'' + 2'' + \frac{1}{4}''$, which gives an over-all width of $12\frac{1}{2}''$.

184. To construct a chassis as shown in figure 141, you need a piece of metal $16\frac{1}{2}''$ by $12\frac{1}{2}''$. You can find the area by applying the rectangle formula, $A = bh$—

$$A = 16\tfrac{1}{2}'' \times 12\tfrac{1}{2}'' = 206.2 \text{ sq. in.}$$

The four corners of the chassis are cut out from the $16\frac{1}{2}''$ by $12\frac{1}{2}''$ sheets. Since each edge of these corners is 2 inches, the area of one corner is $2 \times 2 = 4$ sq. in. The area for four corners is $4 \times 4 = 16$ sq. in. Therefore, the area of the metal used for the chassis is—

206.2 sq. in. –16 sq. in., or 190.2 sq. in.

If the metal costs $\frac{1}{4}$c per square inch, the cost of material in the chassis is—

190.2 sq. in. × $\frac{1}{4}$¢ = 47.5¢, or 48¢.

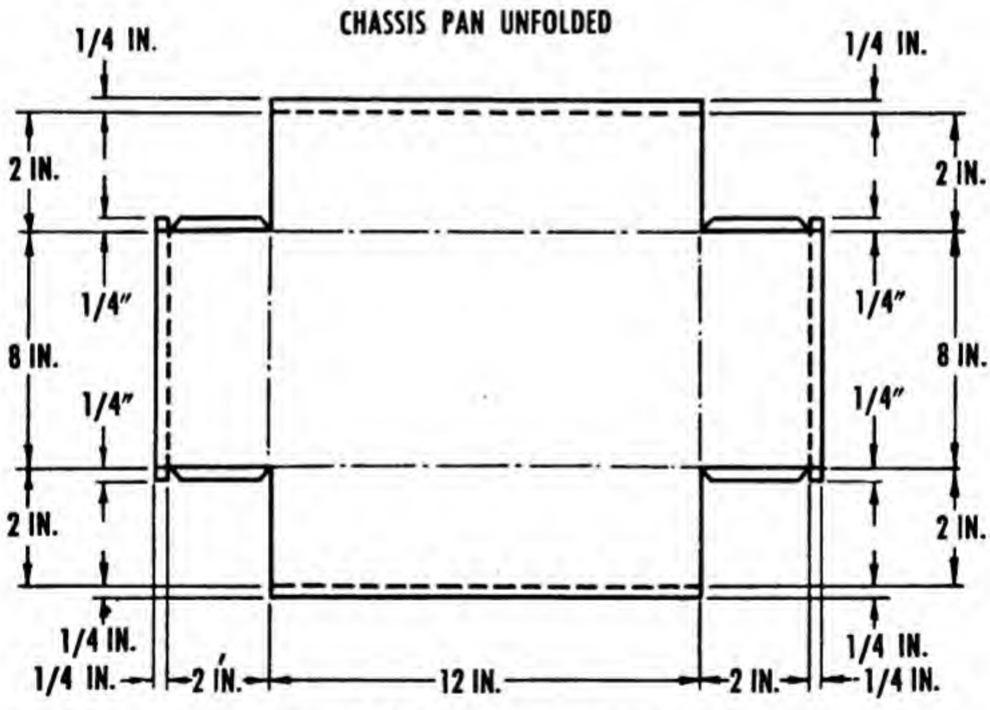


Figure 141.—Chassis pan unfolded.

185. If the only size of stock sheets available measure $14'' \times 18''$, how much metal is wasted? What is the cost of the wasted metal? (See fig. 142.) First, find the area of the metal that the chassis is to be cut from by using the rectangle formula, $A = bh$—

$$A = 18 \times 14 = 252 \text{ sq. in.}$$

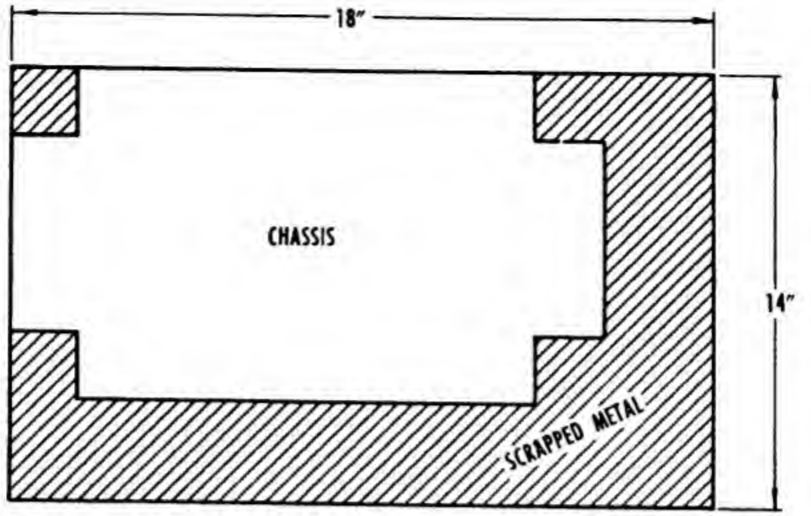


Figure 142.—Chassis layout on stock sheet.

Subtracting the 190.2 sq. in. needed for the chassis from 252 sq. in. gives you the area of the metal to be scrapped, or 61.8 sq. in.—

$$252 \text{ sq. in.} - 190.2 \text{ sq. in.} = 61.8 \text{ sq. in.}$$

Cost of the scrapped metal can be found by multiplying 61.8 sq. in. by ¼¢—

$$61.8 \text{ sq. in.} \times \tfrac{1}{4}¢ = 15.4¢, \text{ or } 15¢.$$

VOLUMES

186. In setting up the formulas for finding the volumes of solids, you use simple algebra again. In figure 143 the most common solids are shown. You have undoubtedly observed many familiar objects illustrated by these geometric figures, such as boxes, rooms, balls, and shafts.

187. In measuring volume, keep in mind that three dimensions are involved. This differs from the finding of areas where only two dimensions are used. Before you

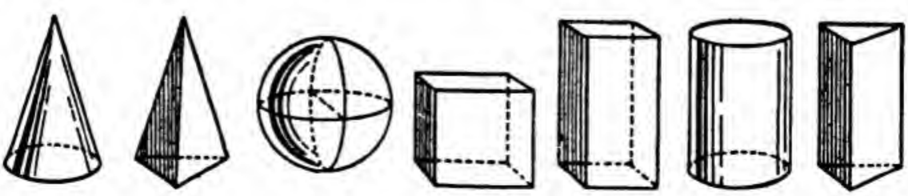


Figure 143.—Common solids.

can measure volume you need a standard unit as a reference. The unit used as a standard for measuring volume is a cube, each of whose edges is a standard unit of length (fig. 144). A common unit of volume is a cubic inch. Other units used for measuring volume are the cubic centimeter, cubic foot, and cubic yard. The number of cubic units in a solid is its volume.

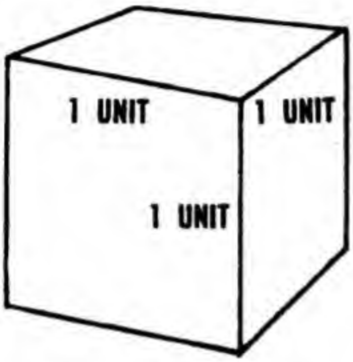


Figure 144.—A cube.

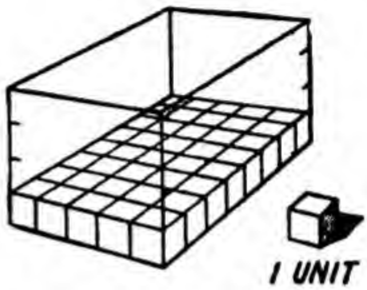


Figure 145.—Rectangular prism.

188. By far the simplest way to find the volume of each geometric solid in figure 143 is to use the right formula. The formulas will be given on the following pages. For a rough experimental check of the formulas, consider the example of the rectangular prism in figure 145. Suppose that you wish to fill the box in figure 145 with unit cubes, like the one shown in figure 144. If the box is 8 units long and 5 units wide, how many blocks could you put on the bottom layer? If the box is 4 units high, how many layers would it hold? What is the volume of a box 8 units by 5 units by 4 units? In answering these questions you should know that there are 40 cubic units on the bottom layer. Since the rectangle is 4 units high, there are 4 layers in the box and each layer contains 40 cubic units. Thus, there are 160 cubic units in the box.

189. In finding the answer, you first multiplied the number of units in the width by the number of units in

the length. This gave you the number of units in one layer; then, multiplying by the number of units in the height, you found the total cubic content of the box. What then is the formula for the volume of a box? Let V represent the number of cubic units in the volume, and L, W, and H represent the number of units in the length, width, and height (or depth) of the prism in figure 145. Then the VOLUME OF A RECTANGULAR PRISM equals $L \times W \times H$, and the formula may be written—

$$V = LWH.$$

Remember when the formula $V = LWH$ is used L, W, and H must be given in the same unit of measure.

190. A cube is a special form of rectangular prism (fig. 144). Its length, width, and height are equal—that is, all its edges are equal. You can use the formula $V = LWH$ to find the VOLUME OF A CUBE. Since the length, width, and height are all equal in a cube, you can let S represent the length of any side or edge. The volume, V, can be expressed by the formula, VOLUME OF CUBE $= S \times S \times S$, or—

$$V = S^3.$$

191. If a circle (A in fig. 146) is moved at right angles to its plane, it generates a right circular cylinder. A circular cylinder always has two circular ends that are the same size. The VOLUME OF A CYLINDER is found by multiplying the area of the base (one end) by the height, or altitude. Since the base of a cylinder is a circle of radius, r, and the formula for the area of a circle is πr^2, then if the height of a cylinder is h, the volume becomes $\pi r^2 h$. The formula for the volume of a cylinder may be written—

$$V = \pi r^2 h, \text{ or } V = 3.14\, r^2 h.$$

Remember that r and h must be in the same units.

192. The objects shown in figure 147 are pyramids. The sides or faces of ANY pyramid are triangles, and all faces meet in a point called the APEX. The base of a pyramid may be a square, a triangle, a rectangle, a hexagon, or any other straight-line figure. The shape of the base gives a pyramid its particular name. Refer to the names

of the three pyramids shown in figure 147. The perpendicular distance from the apex to the base is the height or altitude of a pyramid.

Figure 146.—A cylinder.

Figure 147.—Pyramids.

193. For experimental proof of the formula for the VOLUME OF A PYRAMID, you can try the following experiment—

(a) On cardboard make a drawing similar to figure 148, a. Let $w = 3$ in., $l = 4$ in., and $e = 6.5$ in.

(b) Construct a prism $3'' \times 4'' \times 6''$ (fig. 148, b).

(c) The pyramid and prism have equal bases and altitudes. Compare their volumes by filling the pyramid with any substance that will pour (water, sand, sugar, and so forth), and pour the contents into the prism. Repeat this

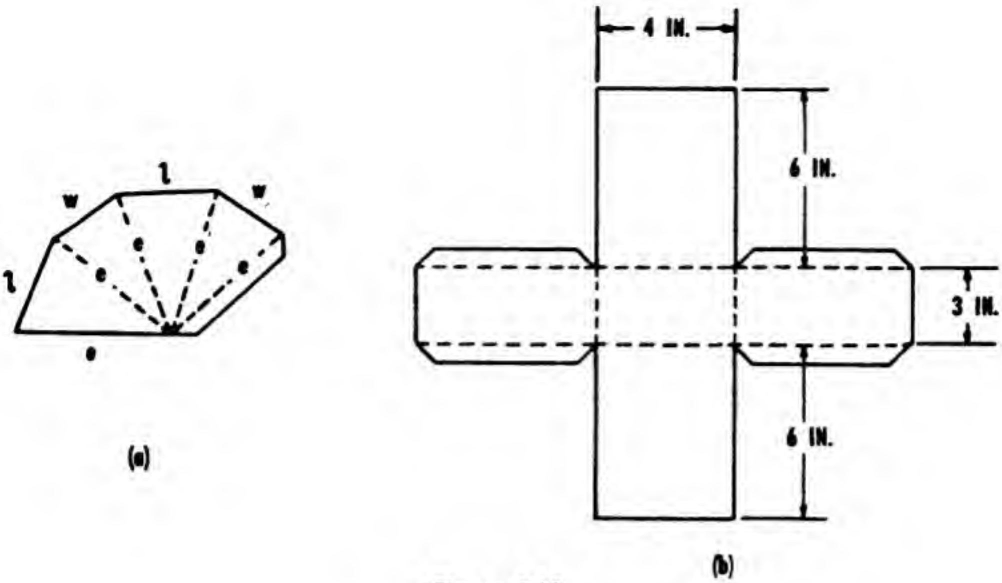


Figure 148.

process until the prism is filled. If your construction is accurate, you will find that the volume of the pyramid is exactly one-third the volume of the prism. (See fig. 149.) The volume of any pyramid is one-third the area of the base times the altitude. If B represents the area of the

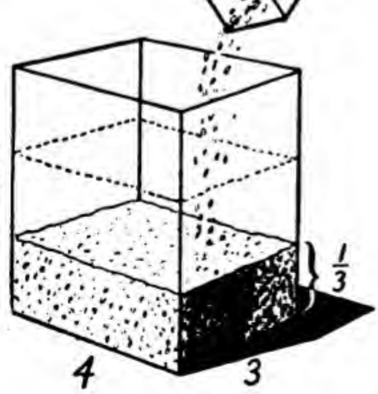


Figure 149.—Finding volume of pyramid.

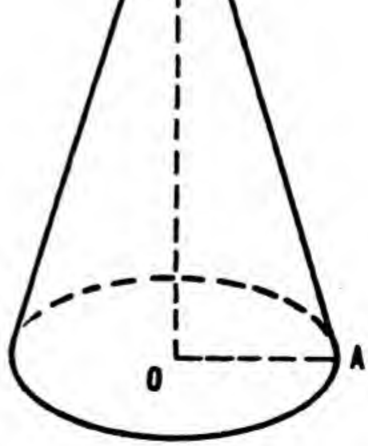


Figure 150.—A cone.

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base (regardless of its shape), the formula for volume is—

$$V = \frac{1}{3} Bh, \text{ or } V = \frac{Bh}{3}.$$

194. A cone is similar to a pyramid. (See fig. 150.) The VOLUME OF A CONE is one-third the product of the base and altitude. If the materials are not at hand, reading through the experiment will help you to remember the formula. Experimental proof of this formula may be obtained by performing the following experiment—

(a) Construct a cone by cutting out a part of a circle, as shown in figure 151, a. Place OA on OB and paste together.

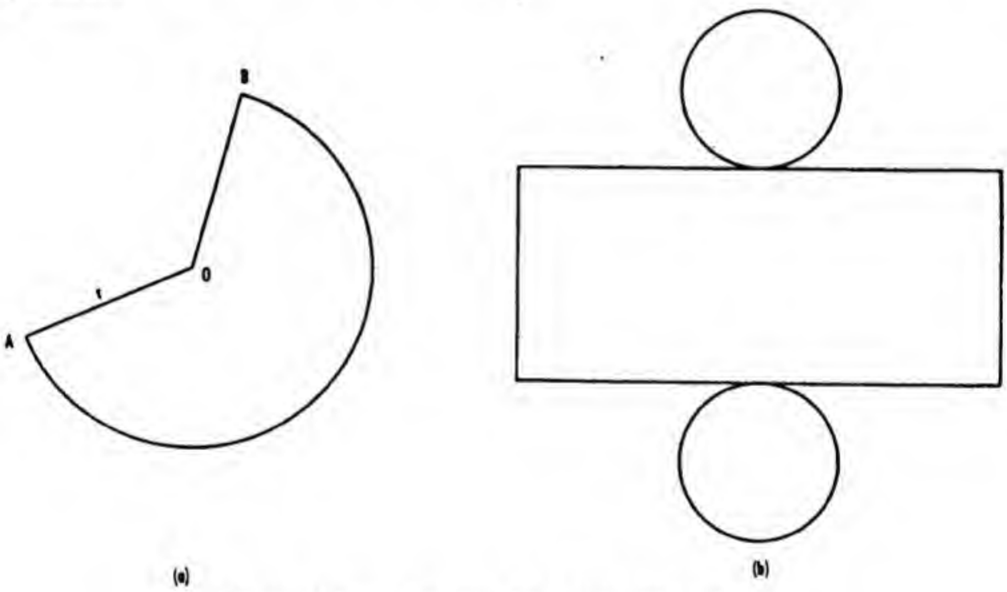

Figure 151.—Constructing a cone and cylinder.

(b) Construct a cylinder so that it has a base and altitude equal to the base and altitude of the cone (fig. 151, b).

(c) Compare the volumes by filling the cone with a substance that readily pours (water, sand, sugar, and so forth). Now pour the contents into the cylinder (fig. 152). Repeat the process until the cylinder is filled. You will find that the volume of the cone is one-third the volume of the circular cylinder. Therefore—

$$\text{volume of cone} = \frac{1}{3} \pi r^2 h.$$

195. The VOLUME OF A SPHERE is one-third the area of the surface times the radius. The formula is—

$$V = \tfrac{4}{3}\pi r^3.$$

196. When an occasion arises to measure the VOLUME OF AN IRREGULAR OBJECT, the following practical method may sometimes be used:

(a) Fill a container full of water. Carefully drop the irregular object, whose volume is desired, into the water until it is submerged. Collect all the water that overflows

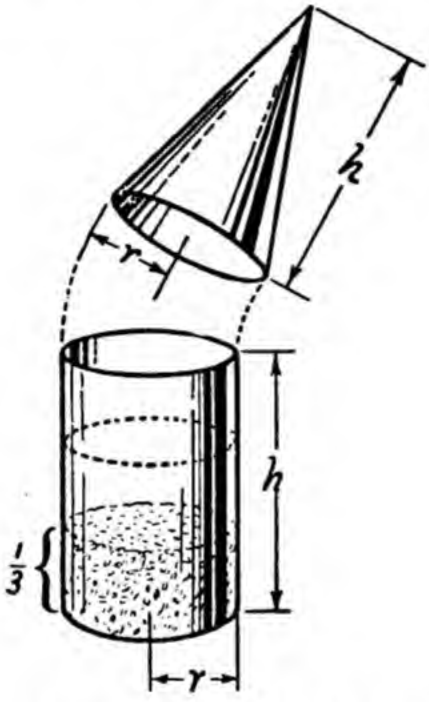

Figure 152.—Finding volume of a cone.

in a graduated container. Measuring the volume of the water gives you the exact volume of the irregular object. (See fig. 153.) This method can be used only with objects that will not be damaged by immersion. Always be alert to devise practical solutions on problems that may involve complicated calculations if handled on a strictly mathematical basis. If allowances are made for the approximate values that result from these methods, they can serve as a valuable check on theoretical calculations.

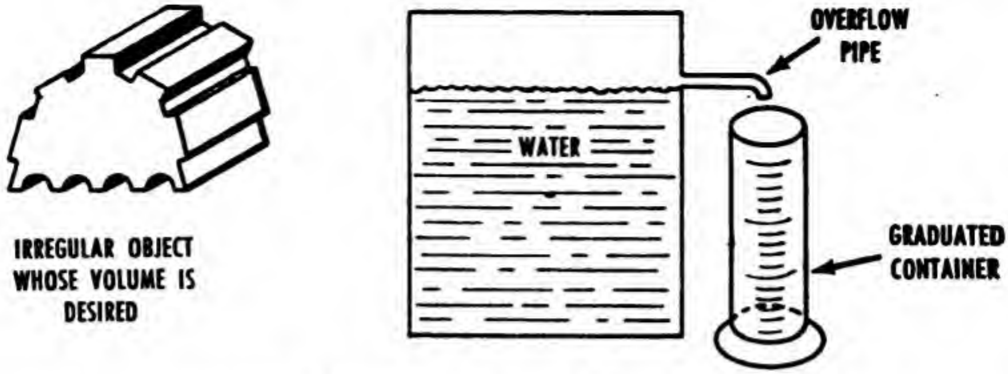

Figure 153.—Finding volume of irregular object.

If you did not pass the pretest in this chapter, apply what you have learned by working the problems now and also the problems in section VI of the Remedial Work.

CHAPTER 7

THE PYTHAGOREAN THEOREM

Of all the geometric figures covered so far, the right triangle is the most important for future work in electronics. When you understand how to find squares and square roots by the use of the PYTHAGOREAN THEOREM you can find the length of the third side of a right triangle if you know the lengths of the other two sides. You will discover that in electrical alternating-current circuits the voltage, current, and impedance of components other than resistors lead or lag the corresponding measurements in resistors by 90° and are thereby related to each other in the same fashion as are the sides and hypotenuse of a right triangle. The Pythagorean theorem can be used in these cases to find the resultant or the value of the third side of the right triangle, which is the over-all voltage, total current, or total impedance.

PRETEST 7

197. (a) Without use of the tables, solve the following—

(1) $\sqrt{841}$ (2) $\sqrt{1{,}444}$ (3) $\sqrt{26{,}569}$ (4) $\sqrt{48{,}620}$

(b) Given $Z = \sqrt{R^2 + X^2}$, solve for X in terms of R and Z.

(c) The square of a side of a right triangle equals the square of the hypotenuse .... the square of the .... side.

(d) The sides of a right triangle are designated a, b, and c, with c as the hypotenuse. Solve for the value of the missing side in the following problems. Use the table of square roots in the appendix.

(1) $a = 8, b = 10$. (4) $a = 12, b = 20$.
(2) $a = 9, c = 20$. (5) $a = 5, b = 10$.
(3) $b = 10, c = 26$. (6) $b = 8, c = 17$.

(e) By propulsion of its own screws, a ship heads due north at 30 knots. There is a current to the east running at 5 knots. Use

the Pythagorean theorem (hypotenuse rule) to find the speed made good.

(f) An insulator is mounted 27 feet up the side of a building. What length ladder must be used to reach the wall beside it, if the foot of the ladder is placed 8 feet from the bottom of the building?

(g) What is the diagonal of a square whose sides are 20 feet? (See fig. 154.)

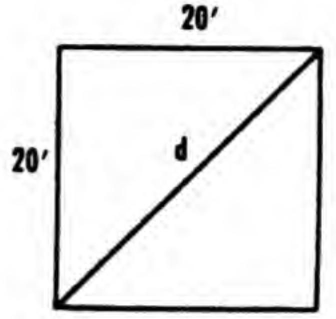

Figure 154.

(h) The sides of a triangle are 9, 12, and 15. Is the triangle a right triangle?

STUDY GUIDE ON PYTHAGOREAN THEOREM

198. A simple triangle is shown in figure 155. The side opposite the 90° angle is called the HYPOTENUSE, side b is the base, and side a is the altitude.

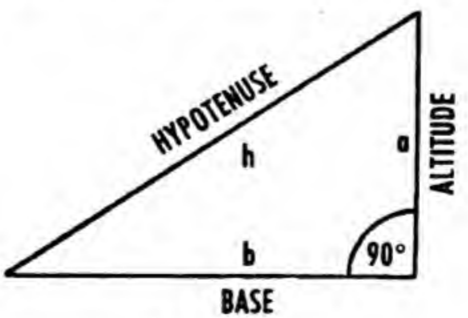

Figure 155.

199. Early Egyptian surveyors learned that a knotted line in the form of a closed loop could be used to construct a square which, as you know, has 90° corners. This line was stretched around three stakes so that a line 3 units

long formed one side of a right triangle, a line 4 units long formed the other side of the right triangle, and a line 5 units long formed the hypotenuse. This method (fig. 156, a) is still useful for rough construction when modern surveying instruments are not available. Later, a Greek

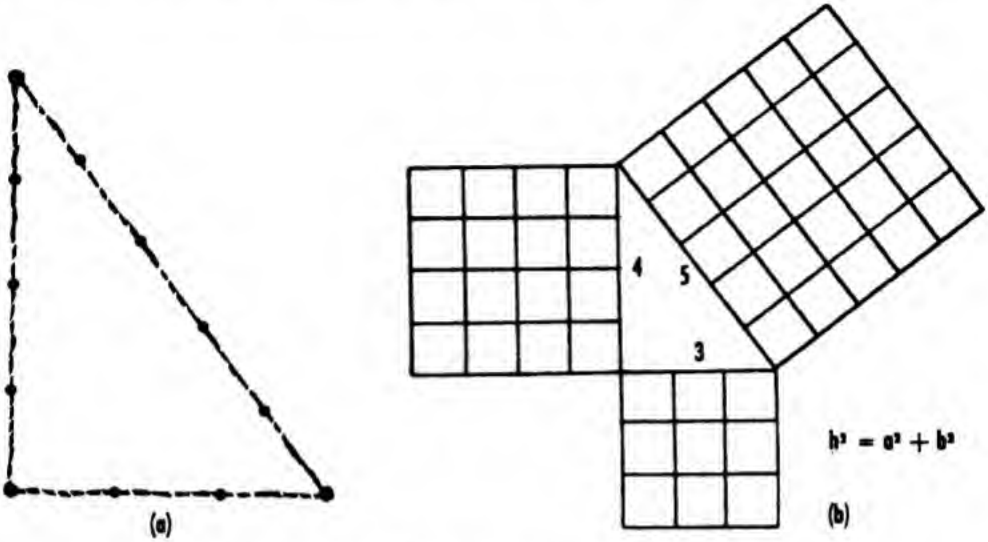

Figure 156.—The magic Egyptian rope and Pythagorean theorem.

mathematician named Pythagoras proved that "IN ANY RIGHT TRIANGLE THE SQUARE ON THE HYPOTENUSE IS EQUAL TO THE SUM OF THE SQUARES ON THE OTHER TWO SIDES." In figure 156, b, count the small squares on both the base and the altitude. You will find that the sum of their squares equals the number of squares on the hypotenuse. Putting the Pythagorean rule in equation form, you have $h^2 = a^2 + b^2$. Referring again to figure 156, b, $3^2 + 4^2 = 5^2$, or $9 + 16 = 25$. Notice that $a^2 + b^2$ does NOT equal $(a + b)^2$.

PROBLEMS IN EVERYDAY LIFE

200. Now for a practical example using the Pythagorean rule: A baseball field measures 90 feet from home plate to third base, and 90 feet from home to first base. The corners of the "diamond" are all right angles. What is the distance from third base to first base? Figure 157 is a

diagram of the problem. From chapter 2, you should remember that this line, h, is called a diagonal since it joins two opposite vertices of a four-sided figure. Therefore—

$$h^2 = a^2 + b^2, \text{ or since } b = a.$$
$$h^2 = 2a^2,$$
$$h^2 = 2(90^2) = 2(8{,}100),$$
$$h^2 = 16{,}200.$$

Taking the square root of both sides—

$$h = \sqrt{16{,}200},$$
$$h = 127.3.$$

The distance between first base and third base is 127.3 feet. Since the infield of the baseball field is a square,

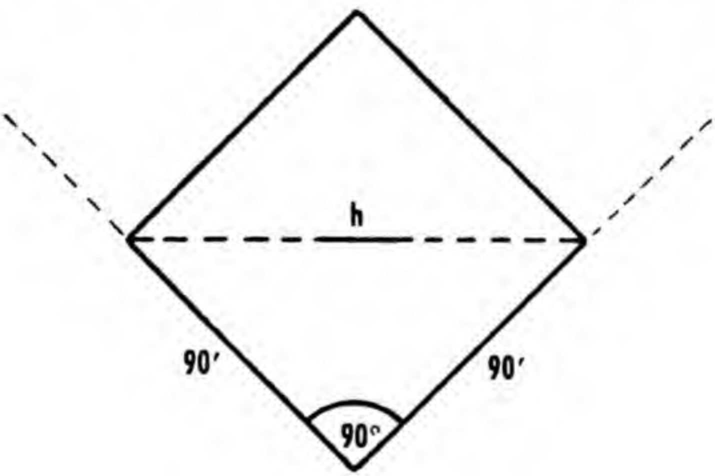

Figure 157.

by reasoning you can see that the distance from home plate to second is also 127.3 feet because the diagonals of a square are equal. The diagonal of a square also can be found quickly by multiplying the length of one side by $\sqrt{2}$, or 1.414 +. This follows from $h^2 = 2a^2$, or $h = a\sqrt{2}$. By use of this formula the diagonal of a 90-foot square is—

$$h = 90\sqrt{2} = 90\ (1.414) = 127.3 \text{ feet.}$$

201. Since it is necessary to extract square roots of numbers to solve right-angle triangle problems using the hypotenuse rule (Pythagorean rule), review this subject

in *Essentials of Mathematics for Naval Reserve Electronics,* NavPers 10093, if you do not remember how to find square roots. In the appendix to this text is a table of squares and square roots of whole numbers. Also the slide rule can be used to extract the square root.

202. Can you see the possibilities of solving many problems in everyday life by using the Pythagorean theorem? For instance, consider the problem: How far up the side of a building will a 40-foot ladder reach, if it is set 7 feet from the base of the building? Your first step is to draw a diagram for the problem. Your figure does not have to be drawn to scale; it can be a rough sketch with the given dimensions put in. (See fig. 158.) Since the hypotenuse of the right triangle is given, turn the formula around to solve for the unknown. Thus—

$$h^2 = a^2 + b^2,$$
$$a^2 = h^2 - b^2,$$
$$a^2 = 40^2 - 7^2,$$
$$a^2 = 1{,}600 - 49,$$
$$a^2 = 1{,}551.$$

Taking the square root of both sides—

$$a = \sqrt{1{,}551},$$
$$a = 39.4.$$

The ladder will extend approximately 39.4 feet, or 39 feet 5 inches, up the side of the building.

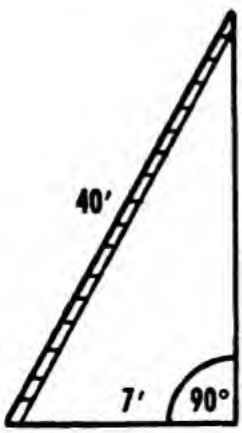

Figure 158.

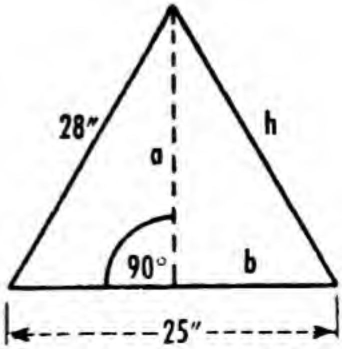

Figure 159.

203. Here is another way of looking at problems involving the Pythagorean theorem. The three sides of a given triangle are 8", 10", and 12". Is it a right triangle? Use the two sides given as 8" and 10" and go through the regular procedure to find the hypotenuse of a right triangle. Thus—

$$h^2 = a^2 + b^2,$$
$$h^2 = 8^2 + 10^2,$$
$$h^2 = 164.$$

Taking the square root of both sides—

$$h = \sqrt{164},$$
$$h = 12.8.$$

Since the third side of the triangle is given as 12" and the hypotenuse rule shows that in order for a right triangle to exist the third side must measure 12.8", the given triangle is not a right triangle.

204. Remember that by drawing a rough sketch of the problem given, your work is half completed. Problem: The base of an isosceles triangle (a triangle having two equal sides) is 25 inches and one of the equal sides is 28 inches. (See fig. 159.) Find its altitude. The altitude divides the isosceles triangle into two equal right triangles. What two values do you now have with which to find the altitude of the isosceles triangle? The base of each of the right triangles is 12½", and the hypotenuse is 28". Thus—

$$h^2 = a^2 + b^2,$$
$$a^2 = h^2 - b^2,$$
$$a^2 = 28^2 - 12\tfrac{1}{2}^2,$$
$$a^2 = 627.75.$$

Taking the square root of both sides—

$$a^2 = \sqrt{627.75},$$
$$a = 25.1.$$

The altitude of the given isosceles triangle is 25.1 inches.

ALTERNATING-CURRENT PROBLEMS

205. It will now be pointed out to you how the Pythagorean theorem can be used to solve practical problems

encountered in the study of alternating-current circuits. In addition to vacuum tubes, the three fundamental components of any piece of electronic equipment are resistors, capacitors, and inductances. The three are shown by their symbols and letter designations in figure 160. You will find that the symbol of each element roughly describes

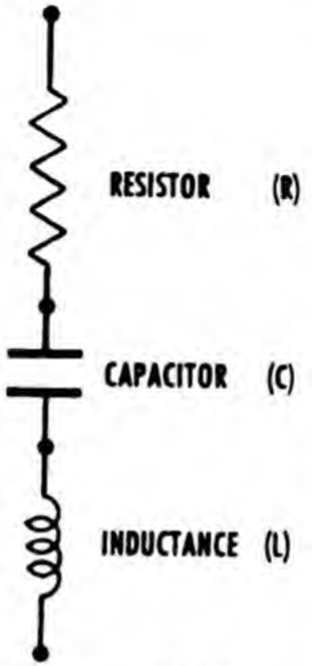

Figure 160.

its physical structure. A RESISTOR is a device that offers opposition to the flow of electric current. A straight piece of wire offers some resistance to current flow. The zigzag line symbolizing a resistor simply means that wire or a combination of other materials is formed purposely to retard current flow. A mechanical parallel to resistance is the action of a friction-producing device, such as the brakes on automobile wheels. A resistance or frictional device does not store up energy; it only changes it to heat.

206. Again, by the symbol, a CAPACITOR is an electrical device consisting of two metal plates separated by an insulating material such as air, glass, paper, mica, or oil. A capacitor in an electrical circuit acts somewhat like a

spring in a mechanical system. An INDUCTOR is a wire wound in the shape of a coil. An inductor in an electrical circuit acts somewhat like a flywheel in a mechanical system. The opposition to current flow in a capacitor or an inductor is called REACTANCE and is designated by the letter X. In a mechanical system, both the spring and the flywheel can store up energy and give it back, thus REACTING to an applied force.

207. When an alternating-current generator has for its load a pure resistance, the voltage and current in the circuit are IN PHASE. That is, they both pass through their zero and maximum values at the same time. This is pictured on the graph shown in figure 161. This graph shows one complete cycle of alternating current and voltage. By alternating is meant that the current or voltage varies

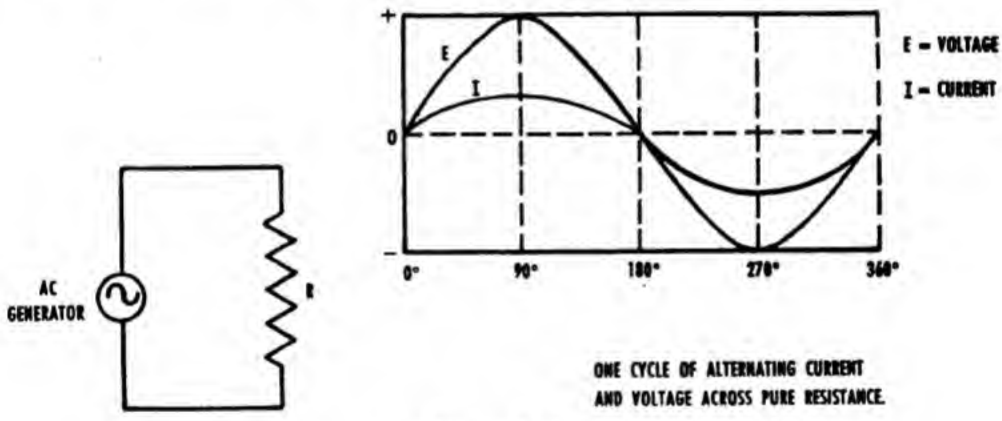

Figure 161.

continually in value and reverses its direction of flow at regular intervals. Each repetition from zero to a maximum in one direction, and then to a maximum in the other direction, and back to zero is called a CYCLE (as shown on the graph in figure 161, one cycle contains 360°). These two directions of flow are called POSITIVE and NEGATIVE in order to distinguish one direction from the other. Don't get the idea that the negative current is any weaker than the positive current—they are absolutely equal in strength. Notwithstanding its reversing action,

alternating current has as much power as direct current. The number of cycles occurring in 1 second is called FREQUENCY. The frequency of the a.c. voltage supplied to your home is 60 cycles per second.

208. In a direct-current circuit, the power consumed is equal to the current times the voltage ($P = EI$). At any instant the power in an alternating-current circuit can be computed by the same formula. Thus, the instantaneous power in an a.c. circuit may be obtained by multiplying the instantaneous value of the voltage by the instantaneous value of the current.

209. Again take a look at figure 161. If you take instantaneous values of power at every point on the cycle, a power curve is developed. (See fig. 162.) The voltage

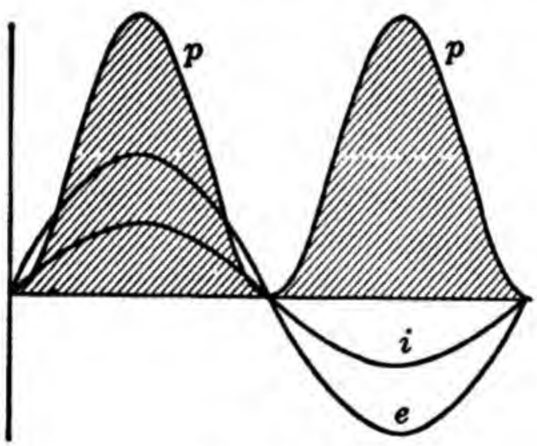

Figure 162.

and current waveforms are plotted for one cycle, and also the resultant power curve. Because the voltage and current across a resistor are always in phase, both have either positive or negative values at any instant so their product (the power) always is positive. Thus, in an a.c. circuit power is always consumed across a resistor:

$$(+I_1) \times (+E_1) = +P_1 \text{ and } (-I_2) \times (-E_2) = +P_2.$$

210. In an a.c. circuit containing pure inductance only as the load of the generator, the reaction of the magnetism of the coil (or inductor) upon the turns of the coil causes

the current to LAG the applied voltage by 90°. (A pure inductance or capacitor is a theoretical condition that can never exist in practice since every element has some resistance. However, by considering them as pure elements without resistance, the phenomena of a.c. theory can better be understood.) That is, the current passes through its maximum and minimum values 90° behind the maximum and minimum values of the voltage. (See fig. 163.)

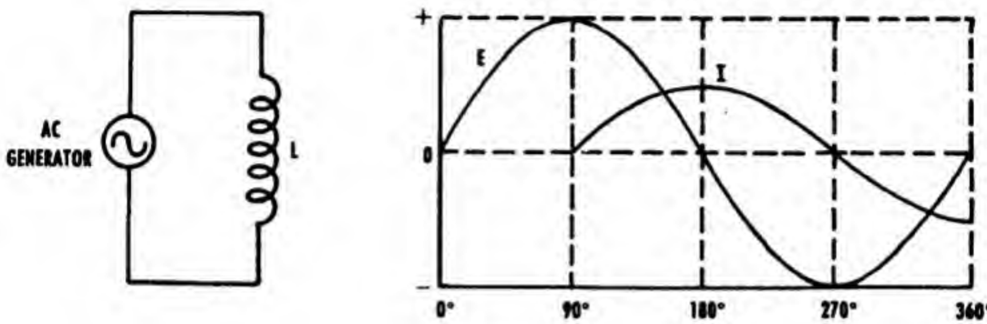

Figure 163.

The voltage curve is at 90° in a maximum positive position when the current curve is at zero level just beginning its cycle. When the voltage and current do not begin their cycles together (fig. 163), they are said to be OUT OF PHASE. The opposite of inductance takes place when the load across the circuit consists of a pure capacitor (sometimes called a CONDENSER). The effect of the capacitor is exactly opposite to the effect of the inductor; the capacitor causes the current to LEAD the applied voltage by 90°.

211. Since it is important for you to have clearly fixed in your mind the phase relations between the current and voltage in alternating-current circuits containing coils and capacitors, the following paragraphs will show you the "how and why" of this strange phenomena. Then, like Houdini's best friend, after a little behind-the-scenes explanation, the mystery will fade away.

212. First, consider the relation between the current and voltage in a direct-current circuit. If you connect a battery and a resistor in series and then close a switch

completing the circuit, the current starts flowing and continues as long as the circuit is complete. But what about the amount of current? In a circuit containing only resistance the current rises immediately to the value $I = \frac{E}{R}$ since the current and voltage are in phase across a resistor. Now, what happens when a coil or a capacitor is connected in the circuit?

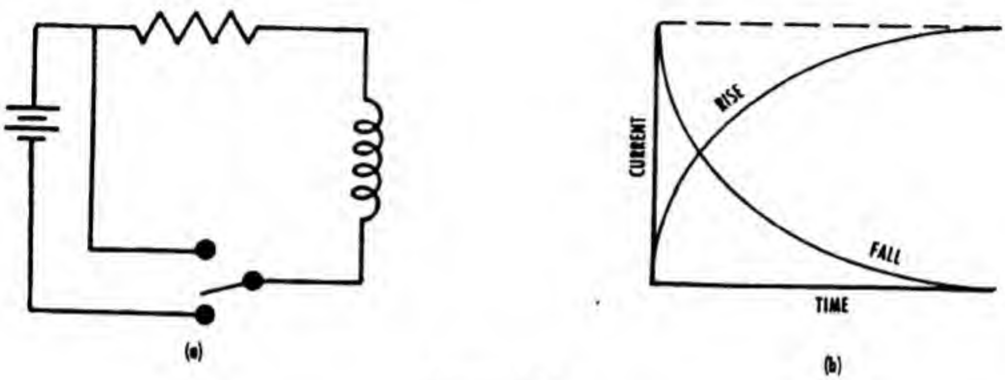

Figure 164.

213. Figure 164, a, shows a d.c. circuit consisting of a battery, a resistor, and an inductance in series. (The series resistor is the internal resistance of the coil, which for our explanation is taken out of the coil and shown as a separate component of the circuit.) When the switch is thrown down, the current does not rise immediately to a value $I = \frac{E}{R}$ but slowly rises to this final value, as shown by the "rise" current curve plotted against time in figure 164, b. At the instant the switch is closed, current begins to flow through the circuit and builds up a magnetic field around the coil. This magnetic field induces a back voltage in the coil, which opposes the flow of current. This back voltage (called COUNTERELECTROMOTIVE FORCE) is maximum at the first instant and then slowly decreases, allowing the current gradually to reach

the value $I = \frac{E}{R}$. Thus, the voltage of CEMF leads the circuit current by 90°. After the current has reached its steady maximum value, suppose the switch is suddenly thrown to the "up" position. This removes the battery from the circuit, and the current through the coil starts to decrease. However, the magnetic field around the coil now collapses, and again a back voltage is induced in the coil, which tends to keep the current flowing. Thus, the current does not immediately drop to zero but decreases, as shown in the current "fall" curve in figure 164, b. Thus, the property of inductance is to OPPOSE ANY CHANGE in the current flow.

214. The reaction of an inductance is similar to that of a flywheel in a mechanical circuit. For instance, a one-cylinder gasoline engine does not produce uniform power because the power pulses come from the piston only during the explosion stroke. To provide an even flow of power, a heavy-rimmed flywheel is fitted to the engine shaft. The flywheel OPPOSES ANY CHANGE in the speed of rotation. On starting, the engine must overcome the added drag of the heavy flywheel, which is opposing the start. (Compare this action with that which occurs in the circuit of figure 164 when the switch is thrown down and current first begins to flow.) However, after the engine has been brought up to running speed, each power stroke of the piston tends to accelerate the speed of the engine. This is opposed by the weight of the flywheel so that the speed accelerates only slightly on each stroke. Also, the inertia of the flywheel carries the piston back and forth through the idle strokes between power strokes, while the engine tends to slow down. In other words, the flywheel, after taking energy from the piston to gain speed, gives back this energy during idle periods, thus giving a smooth output. In a similar manner an inductance in an electrical circuit can absorb and give up energy over short intervals.

215. Now let's get back to alternating-current circuits again. In figure 165 the voltage and current waveforms

across a large inductance, which has practically no resistance, are plotted along with the resulting power curve, which is the product of current and voltage at each instant. Whenever the current and voltage are not in phase

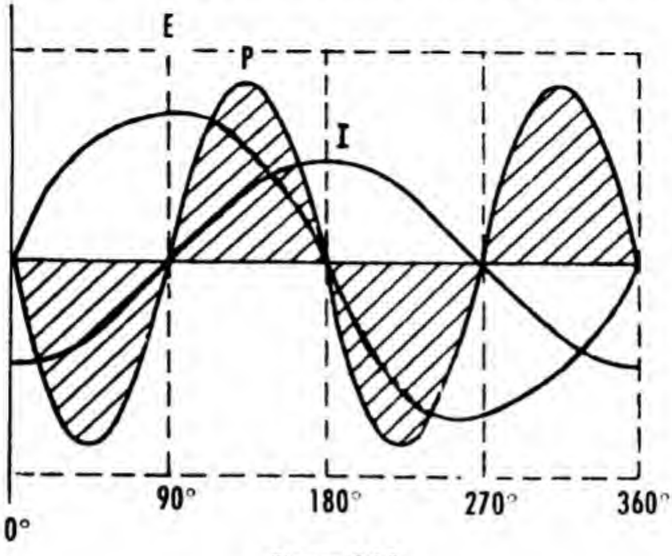

Figure 165.

they are of opposite sign during some part of the cycle, thus resulting in a period of "negative" power, as shown by the power loops extending below the zero axis. When the voltage and current are 90° out of phase the power curve has equal positive and negative loops, and it has double the frequency of either the current or voltage waveforms. The negative-power loop results from the use of energy from the generator in building up the magnetic field around the coil as the current increases in the coil. As stated before, the magnetic field produces a back voltage or counter-electromotive force, which opposes the increase in current. The period of negative power is equivalent to the period in which the flywheel is being brought up to speed. This building up of a magnetic field takes energy from the source and stores it. Then as soon as the current begins to decrease, the collapsing magnetic field again induces a back voltage to keep the current flowing, and thus returns energy to the source. The negative-power curve represents the period when energy is being stored

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in the coil; the positive-power curve represents the period when energy is being restored to the source. The average power consumed by a pure inductance is zero since as much energy is restored as is taken from the source. In practical inductances the ratio of ohms resistance to ohms reactance is kept very low but is never zero.

216. Although the action of a capacitor in a circuit also results in out-of-phase relations between voltage and current, its reaction is exactly opposite to that of the coil (inductance). As pointed out, the reaction of an inductance causes a back voltage to exist when the current varies, and the greatest opposition to the flow of current is at the first instant the voltage is applied. The back voltage always opposes the change that produces it. The effect of a capacitor is to PERMIT a current to flow when the voltage is varied. When a capacitor is connected to a source of voltage, the first surge of current is the greatest.

217. Let's consider the action of a capacitor in a d. c. circuit. In figure 166, a, we have the same set-up as the d. c. inductive circuit. Again the series resistor is the internal resistance of the capacitor set-up as a separate component. At the instant the switch is thrown down, the capacitor offers no reactance and is considered a short circuit. The only opposition to current flow is the resistor, and therefore the current rises immediately to the value $I = \frac{E}{R}$. However, as current continues to flow a voltage

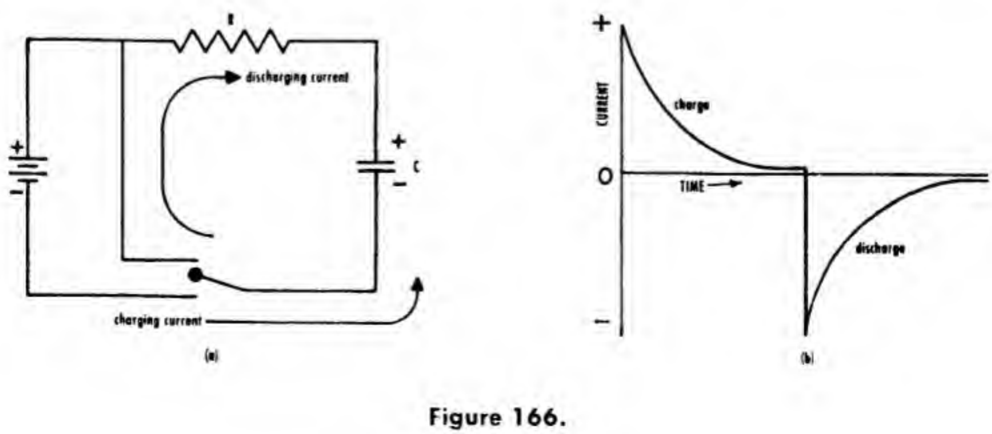

Figure 166.

is gradually built up across the capacitor. This voltage opposes the battery voltage, thereby resisting current flow, which gradually decreases. The voltage across the capacitor continues to build up (capacitor charging), and when this voltage equals the value of the battery voltage, current stops. (See fig. 166, b.) Now, if the switch is suddenly thrown to the "up" position, since the capacitor is fully charged to the battery voltage, the current again immediately reaches the value of $I = \frac{E}{R}$ flowing in the opposite direction. As current continues to flow the voltage across the capacitor gradually drops (capacitor discharging), and therefore the current gradually falls to zero.

218. The reaction of a capacitor can be likened to the compliance of a spring in a mechanical circuit. Consider a spring at rest. If you apply a mechanical force the spring complies or yields. When a spring is compressed practically no resistance is encountered at the first instant. The spring compresses very easily at this point. As the spring is compressed closer and closer together, it offers more and more resistance to the force applied, until finally it resists with a force equal to the compressing force. At this point compression stops. This is similar to the effect of a capacitor, which, having become fully charged to the battery voltage, causes the current to fall to zero. The spring was absorbing energy during the compression cycle. Now, suppose you release the spring. At the first instant, the decompression occurs very fast because of the force built up in the spring. As the spring decompresses more and more, the force that was built up in it gradually decreases, so that the discharge curve is a reflection of the charging curve. Notice that in a capacitor the current must flow in first before voltage builds up. In other words, current leads the voltage.

219. If you plot the alternating-current curve, the voltage curve, and the resulting power curve across a pure

capacitor (one having little or no resistance) you will find that no average power is consumed. The power curve has equal positive and negative loops, which effectively cancel out. In other words, the capacitor takes energy from the source in charging and gives back this energy to the circuit in discharging when the voltage applied, varies.

220. An easy way to remember the phase relation between the current and voltage across capacitors and inductances is to repeat the phrase, "ELI the ICE man."

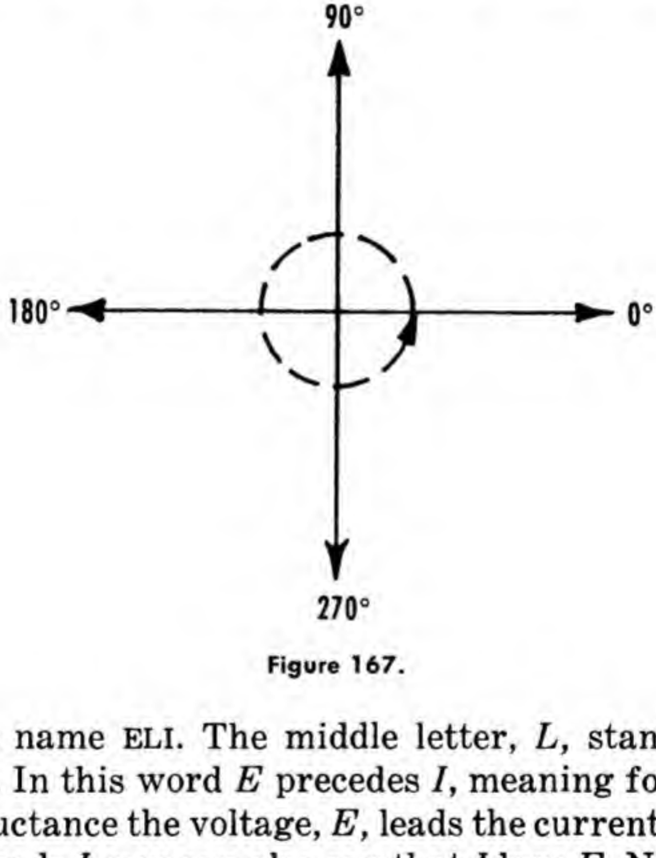

Figure 167.

Take the name ELI. The middle letter, *L*, stands for inductance. In this word *E* precedes *I*, meaning for you that in an inductance the voltage, *E*, leads the current, *I*, by 90°. Since *E* leads *I*, you can also say that *I* lags *E*. Now for the word ICE. The middle letter, *C*, stands for capacitor. The *I* comes before (leads) *E*, which as far as you are concerned means that in a capacitor the current, *I*, leads the voltage, *E*, by 90°. Here again, since *I* leads *E*, *E* lags *I* by 90°.

221. As was mentioned previously, in a.c. theory angles in electricity are assumed to be generated in a counterclockwise (CCW) direction. This can be illustrated easily in diagram form. (See fig. 167.) A horizontal line repre-

sents the zero, or reference, line. A positive or leading angle of 90° is represented by a one-quarter CCW turn from the positive end of the zero line. The terminal (end) side of a 90° angle is a line perpendicular to the reference line and extending upward. An angle of 270° is a three-quarter CCW turn from the zero line but extending downward. Note that the terminal sides of the 90° and 270° angles form one and the same line. However, for the 90° angle you must use the part of the line that extends above the zero line and for the 270° angle, the part of the line that extends below the zero line. Likewise, the zero line and the terminal side of a positive 180° angle are one and

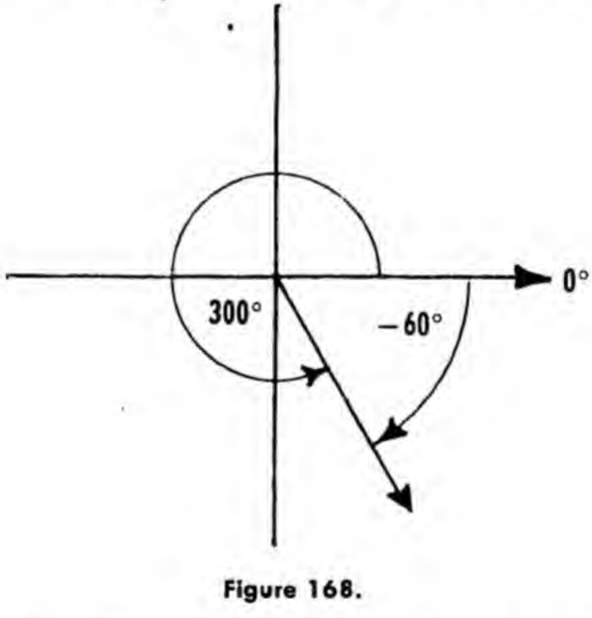

Figure 168.

the same line, but for the zero line, or a 0° angle, you must think of the part of the line extending to the right of the perpendicular and for the 180° angle, the part of the line extending to the left of the perpendicular. A negative or lagging angle is generated by a line turning in a clockwise direction. Angles between 270° and 360° are sometimes converted to negative angles in computation to keep the numerical size of the angle less than 90°. For instance, in figure 168, an angle of 300° can be referred to as an angle of –60°. A current is not usually considered leading or lagging its reference voltage by more than 180°.

222. Before we go any further, it would be a good idea to review Kirchoff's law which you covered in *Essentials of Mathematics for Naval Reserve Electronics,* NavPers 10093. Figure 169 shows a simple direct-current circuit,

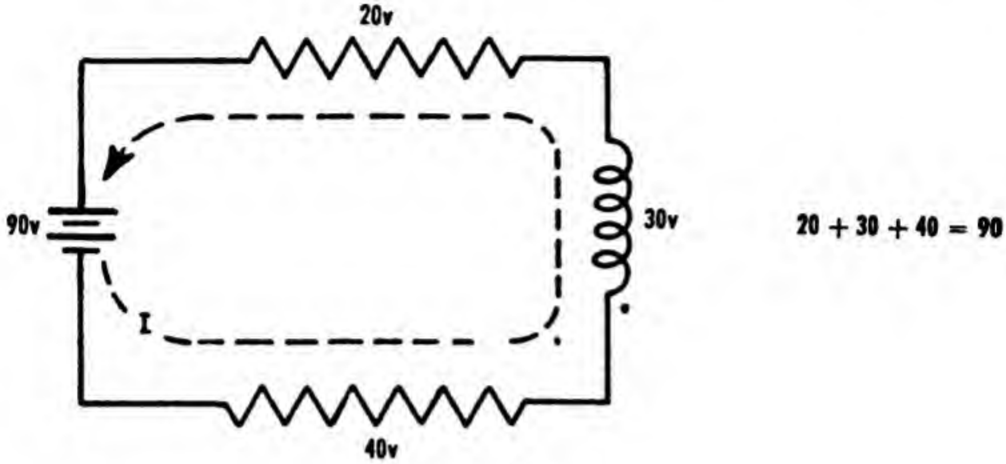

Figure 169.

which consists of a battery, two resistors, and an inductance in series. In a series circuit the same current flows through each component in the circuit. A capacitor is eliminated here because it will not pass direct current, and one of its main purposes in a circuit is to block the flow of direct current. This diagram points out that the individual voltage drops across each component and battery e.m.f.'s add algebraically to zero. The following paragraphs are concerned only with series circuits, and the examples given do not hold true for parallel circuits.

223. Now get ready for some fast curve balls. The drawing in figure 170 is a series a. c. circuit. The generator is supplying 90 volts a. c. to the load, which consists of (1) a capacitor and (2) a resistor in series. A voltmeter placed directly across the resistor reads 54 volts. Now, placing the voltmeter across the capacitor, you get a reading of 72 volts. Why? Because $72 + 54 \neq 90$, ($\neq$ means IS NOT EQUAL TO.) Remember, however, that the voltage across the capacitor is out of phase with the current through the capacitor; and the voltage across a resistor

is always in phase with the current through it. Since this is a series circuit the same current flows throughout the circuit, but the voltages are out of phase and cannot be added algebraically as in the d.c. circuit shown in figure 169. They must be added by a method that takes into account the out-of-phase relations.

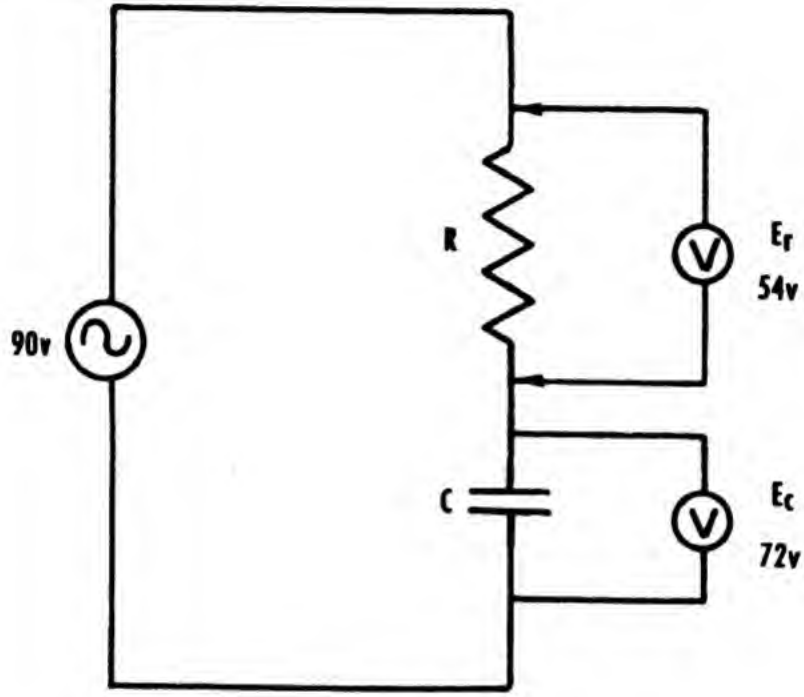

Figure 170.

224. With the help of the Pythagorean theorem let us clear up this mystery. Since the current is the same throughout a series circuit, it is used as the reference and plotted at 0°. Now as the voltages are plotted, E_R is in phase with the current and so is plotted at 0°. (See fig. 171). A check with "ELI the ICE man" tells you the voltage across the capacitor lags behind the current by 90°, so it is plotted on the graph at a clockwise turn of 90°, or at an angle of −90°. Considering the voltages as forces, you can see that the direction of the resultant force (voltage) lies at an angle somewhere between the two. You get the direction by completing a parallelogram, as shown in

figure 171. Now you can find the magnitude of this resultant voltage (which is the generator supply voltage) or the third side of the right triangle by using the Pythagorean theorem.

$$E \text{ generator} = \sqrt{E_R^2 + E_C^2},$$
$$E \text{ generator} = \sqrt{72^2 + 54^2},$$
$$E \text{ generator} = \sqrt{8{,}100},$$
$$E \text{ generator} = 90.$$

Using the hypotenuse rule you determined the supply voltage to be 90 volts at a negative angle between 0 and $-90°$. If you were given the supply voltage and only the

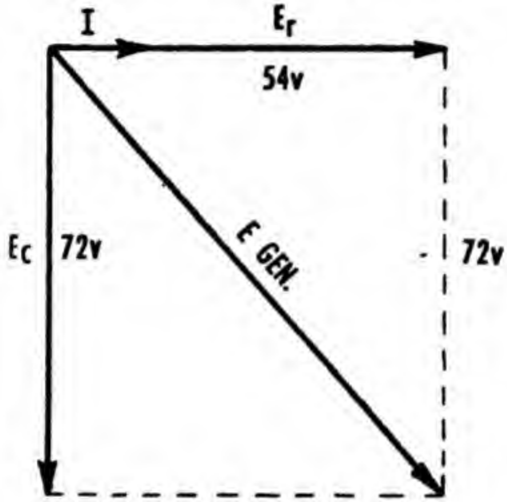

Figure 171.

voltage drop across the resistor, you could solve for the voltage dropped across the capacitor, by changing the formula around.

225. This chapter may seem a little deep for you, but remember that the purpose of the chapter is not to answer all questions on a. c. or d. c. theory. Rather it is to provide sufficient familiarity with electrical terms so that you can associate them with the mathematical methods you will needs to use later. The chapters on trigonometry and vectors will use the same terms, so it is a good idea to go over this chapter carefully for an easier understanding of what follows.

226. Let us now see what happens in an a. c. circuit with all three components in series as the load. (See fig. 172.) Again the voltage drop across each part is marked on the circuit drawing. However, the supply voltage is unknown and you are to solve for it. The best way is to

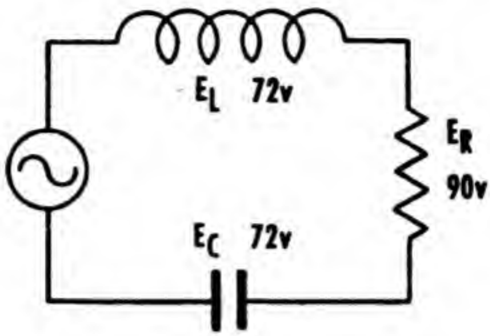

Figure 172.

plot the voltages on a diagram again. Using the same principles as before, you plot E_R on the zero reference line in phase with the current, as shown in figure 173, a. The

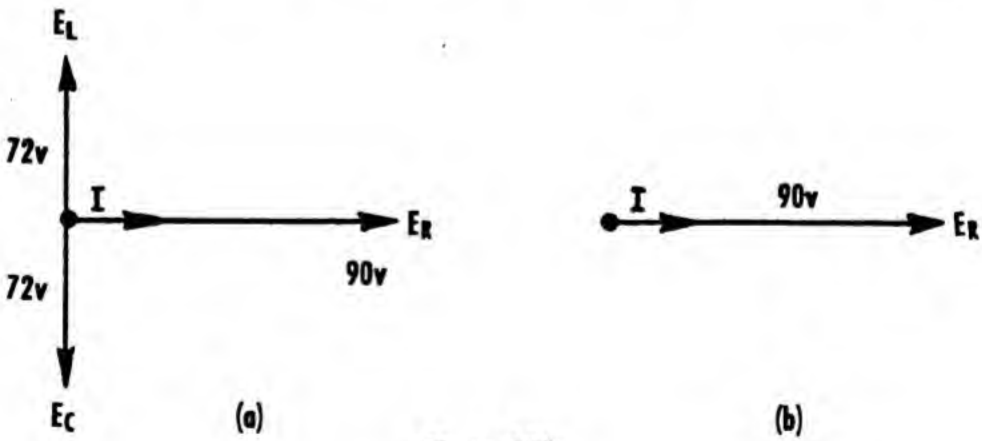

Figure 173.

reactance of the capacitor causes the voltage across it to lag the current through it by 90°, so the voltage is plotted at an angle of –90°, a *CW* turn of 90° from the current reference line. The reactance of the inductor causes the voltage across it to lead the current through it by 90°.

This voltage is then plotted at a positive angle of 90° from the zero or current reference line. Notice that E_L is plotted exactly opposite E_C making them 180° out of phase. Since they are two equal forces (each 72 volts) acting in opposite directions they cancel each other. After the reactive voltages cancel, the resulting diagram consists of just E_R as shown in figure 173, b. You can see that if one of the voltages E_C or E_L had been a larger value than the other, a complete cancellation would not take place, and you again would have to use the Pythagorean theorem to determine the amount of the supply voltage.

227. From Ohm's law you learned that the resistance of a circuit is equal to the voltage divided by the current, $R_{\text{total}} = \frac{E_{\text{total}}}{I_{\text{total}}}$. However, in alternating-current circuits, besides the resistance in the circuit, you have the reactance offered by the coils and capacitors. (The symbol for reactance is X and is measured in ohms. Inductive reactance then is X_L and capacitive reactance X_C.) But you may ask how this ties in with the Pythagorean theorem. The effects of resistance and reactance differ in phase by 90°, and you will see that again right triangles are formed with the total opposition to current flow in the a. c. circuit as the hypotenuse.

228. Inductive reactance is plotted at +90° from the current reference line, and capacity reactance is plotted as −90° from the reference line. (See fig. 174.) From the diagram you can see X_L and X_C oppose each other, and they tend to cancel. When the two reactances are equal and cancel completely, the circuit contains only resistance and is said to be resonant. The effect of resonance on the action of the circuit is explained later in ET rating texts. However, if X_L and X_C are not equal, simply subtract the smaller from the larger to find the resulting reactance of the circuit.

229. There may be some doubt in your mind as to why X_L and X_C are plotted as they are. The following should clear the relationship for you—a vector is a straight line

representing a quantity that has both direction and magnitude. Voltage and current are vector quantities. For instance, the voltage across a pure inductance leads the current by 90°. This voltage then has a magnitude of a definite number of volts, and its direction is at an angle of +90° with the current. In chapter 5 on graphs, you

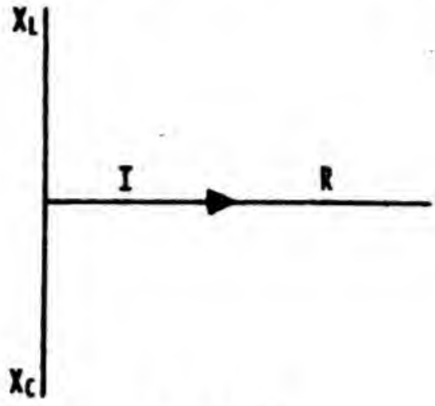

Figure 174.

learned that a quantity expressed by its vertical and horizontal components is in rectangular form, and, when the same quantity is expressed by its magnitude and direction, it is in polar form. Thus, the voltage across a pure inductance in a series a.c. circuit, expressed in polar vector form, is $E_L \underline{/90^\circ}$. When solving for the value of resistance and reactance in alternating-current circuits, you must include the angles of the voltage and current in the division and multiplication. Thus, the rule for multiplying polar vectors is to multiply the magnitudes and add the angles algebraically.

Example: $40\underline{/50^\circ} \times 2\underline{/10^\circ} = 80\underline{/60^\circ}$

The rule for dividing polar vectors is to divide one magnitude by the other and subtract the angles algebraically.

Example: $$\frac{90\underline{/70^\circ}}{30\underline{/-40^\circ}} = 3\underline{/110^\circ}$$

Now following these rules, solve for R, X_L, and X_C of a series a. c. circuit.

Solving for R: $\dfrac{E_R\underline{/0^\circ}}{I\underline{/0^\circ}} = R\underline{/0^\circ}$.

Therefore R is plotted at 0° in phase with the current vector.

Solving for X_L: $\dfrac{E_L\underline{/90^\circ}}{I\underline{/0^\circ}} = X_L\underline{/90^\circ}$ plotted at $+90^\circ$.

Solving for X_C: $\dfrac{E_C\underline{/-90^\circ}}{I\underline{/0^\circ}} = X_C\underline{/-90^\circ}$,

so this value is plotted at -90° behind the current vector on a diagram.

230. The combined effects of resistance and reactance is termed IMPEDANCE (symbol Z) and is also measured in ohms. The impedance of a circuit is not a vector, although it does have in-phase and out-of-phase components and can be expressed as a polar value. Impedance is at an angle with respect to the zero or current reference line. The angle is found by the parallelogram method and may be either positive or negative depending on whether the circuit is inductive or capacitive. (See fig. 175, a and b.) As stated before, impedance forms the third side (hypote-

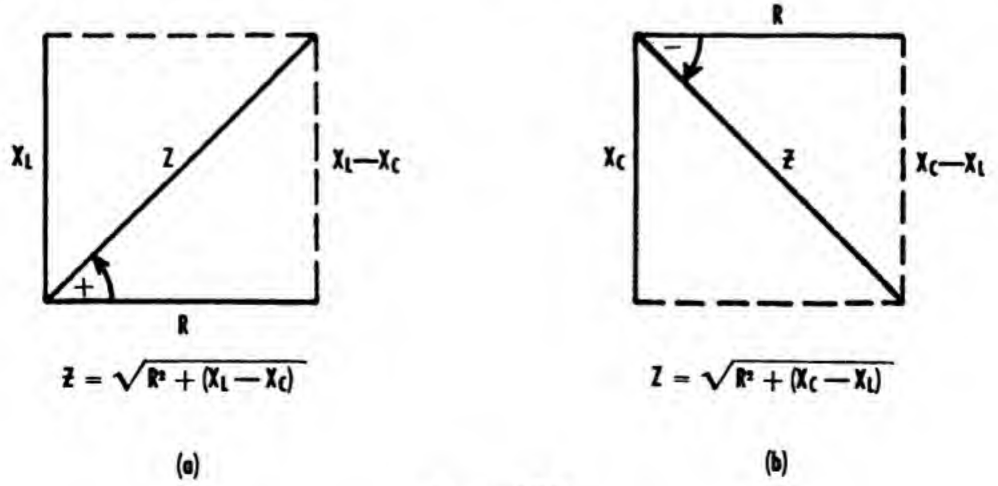

Figure 175.

nuse) of a right triangle along with the resistance and reactance as the other two sides.

231. Consider the problem: An electrical appliance has a resistance of 40 ohms, an inductive reactance of 50 ohms, and a capacitive reactance equal to 30 ohms. Plot the values on a diagram as in figure 176, a, and solve for

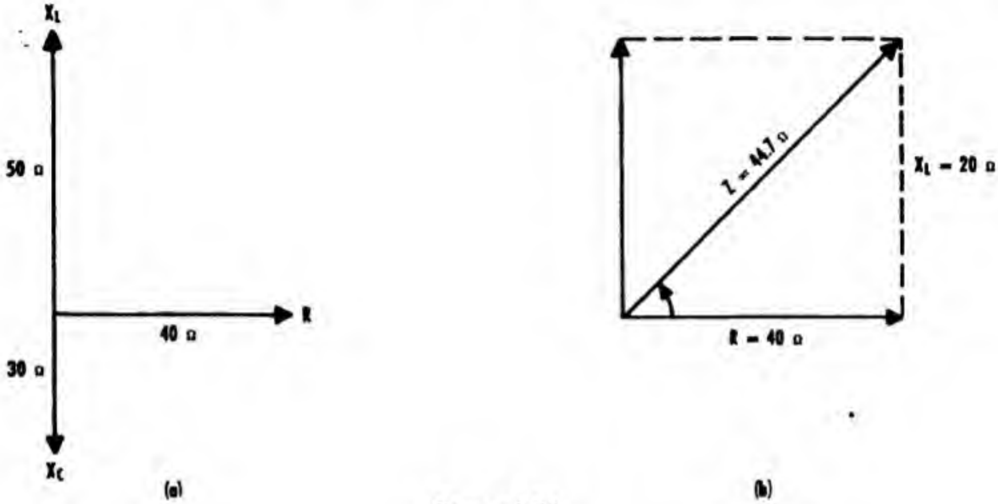

Figure 176.

the impedance. The opposite effects of X_L and X_C tend to cancel, but you can see that by simple subtraction 20 ohms of inductive reactance still exist. The resulting triangle is as in figure 176, b.

Solution:

$$Z = \sqrt{R^2 + (X_L - X_C),}$$
$$Z = \sqrt{40^2 + 20^2,}$$
$$Z = \sqrt{2{,}000,}$$
$$Z = 44.72 \text{ ohms.}$$

If you did not pass the pretest in this chapter, apply what you have learned by working the problems now, and also the problems in section VII of the Remedial Work.

CHAPTER 8

ESSENTIALS OF TRIGONOMETRY

We now come to that branch of mathematics which you, as an electronics technician, will use nearly as much as your right hand. Trigonometry, meaning TRIANGLE MEASUREMENT, is concerned with finding the missing parts of a triangle when certain sides and angles of the triangle are known. Being able to use angles and their trigonometrical relationships in electrical computations is especially important when you study alternating-current circuits because the a.c. cycle is measured in degrees. Most effects of alternating-current circuit components can be studied or described only in terms of the part of a cycle by which a current lags behind the corresponding voltage, or vice versa. Since most of the problems dealing with alternating-current circuits can be solved by the solution of right triangles, they alone will be covered in this chapter.

Trigonometry deals with the relations existing between the sides and angles of triangles. Through the use of trigonometry, problems can be solved easily by indirect measurements. In plane geometry you learned methods of solving right-triangle problems by indirect measurement —for example, the construction of similar triangles and scale drawings. You are already familiar with certain facts about right triangles, namely:

1. The square of the hypotenuse is equal to the sum of the squares of the other two sides ($h^2 = a^2 + b^2$).
2. The sum of the angles of any triangle is 180°. The sum of the acute angles of any right triangle is 90°.

However, it would be impossible to solve certain problems with only this information. Also the same degree

of accuracy could not be obtained for all triangles by the previous methods. After learning other relations of the right triangle, you will find that trigonometry offers easier and more accurate methods of solving many alternating-current problems.

PRETEST 8

232. (a) With the aid of the trigonometric table, find the following values:

(1) $\sin 47.3°$	(4) $\sin 64°$
(2) $\cos 16.9°$	(5) $\cos 53.4°$
(3) $\tan 75°$	(6) $\tan 29.6°$

(b) Find the acute angle Θ for each of the following:

(1) $\sin \Theta = 0.5948$	(4) $\sin \Theta = 0.4067$
(2) $\cos \Theta = 0.6293$	(5) $\cos \Theta = 0.1392$
(3) $\tan \Theta = 0.2272$	(6) $\tan \Theta = 0.6619$

(c) A 10-foot pole casts a shadow 7 feet long. What is the angle of elevation of the sun at this time?

(d) A ship sails north 34 miles, then turns 90° to the right and travels 15 miles. What direct course could the ship have sailed in order to arrive at the same position?

(e) Find the vertical and horizontal components of a 150-pound force acting at an angle of 53° with the horizontal.

(f) A man stands 40 feet from the base of a tower. From his position the angle of elevation to the top of the tower is 65°. How tall is the tower?

(g) From the top of a cliff 650 feet high, the angle of depression to the waterline of a ship at sea is 16°. How far out is the ship?

(h) A paratrooper drops from a height of 3,000 feet. His path of descent makes an angle of 72° with the ground. When he reaches the ground, how far is he from a point that was directly below him when he jumped? It is assumed that the ground is level.

(i) Radar reports a plane at slant range 98,000 yards. The angle of elevation is 7.8°. What is the altitude of the plane?

(j) An approach to a bridge is to have an angle of elevation of 70° and must rise to a height of 23 feet above the level ground. How far from the bridge should the approach be started?

If you have successfully passed the pretest, do not study the entire chapter. Turn to the second half on "application to a.c. problems."

233. Examine the three right triangles in figure 177. You know they are similar triangles because their corresponding angles are equal. You know also that because they are similar the ratios of their corresponding sides are equal. What is of interest to you here concerning the three

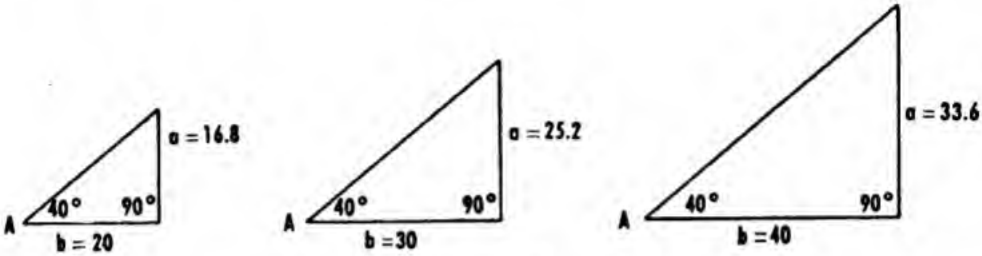

Figure 177.

triangles is the relation between the sides and the acute angles of the three triangles. Divide the altitude by the base $\left(\frac{a}{b}\right)$ in each equation—

$$\frac{a}{b} = \frac{16.8}{20} = 0.84, \frac{25.2}{30} = 0.84, \frac{33.6}{40} = 0.84.$$

Your result is the same for each triangle. Therefore, you can assume that in any right triangle (similar to the three shown in fig. 177) having an angle at A equal to 40°, the ratio of the altitude to the base is always 0.84. Your assumption is correct, for the value of the ratio $\frac{a}{b}$ of a right triangle depends only on the size of $\angle A$ and not on the lengths of the sides. The ratio $\frac{a}{b}$ in respect to the angle at A is called the TANGENT of $\angle A$ and is one of six functions of this angle.

234. In trigonometry it is customary to designate the angles with Greek letters. Throughout this chapter, the

acute angle formed by the hypotenuse and the base of the right triangle is referred to as angle theta (Θ). The other acute angle is designated phi (Φ). (See the triangle in fig. 178.) Using the three sides of the triangle in figure 178, you can express six ratios:

$$\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \text{ and } \frac{c}{a}.$$

Since all triangles do not have the sides identified by the letters a, b, and c, new terms are applied in trigonometry. (See fig. 179.) You must remember that they are always

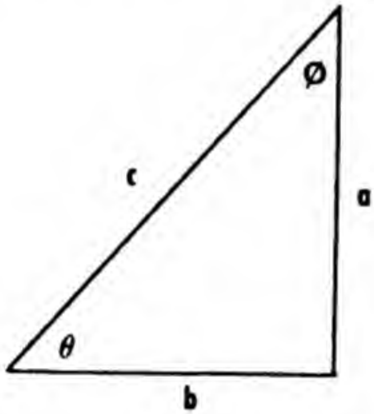

Figure 178.

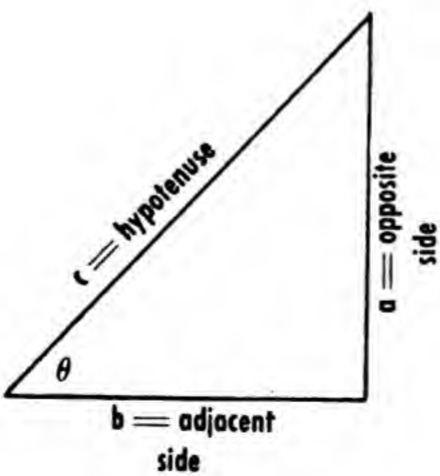

Figure 179.

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used in reference to one of the acute angles of the right triangle. Now, side a is called the side opposite angle Θ, and b is the side adjacent to Θ. Side c is always referred to as the hypotenuse.

235. The six ratios of sides of a right triangle as given in paragraph 234 can now be named:

$$\frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}} \text{ called the sine of } \angle \Theta \text{, written } \sin \Theta.$$

$$\frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}} \text{ called the cosine of } \angle \Theta \text{, written } \cos \Theta.$$

$$\frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} \text{ called the tangent of } \angle \Theta \text{, written } \tan \Theta.$$

$$\frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}} \text{ called the cotangent of } \angle \Theta \text{, written } \cot \Theta.$$

$$\frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}} \text{ called the secant of } \angle \Theta \text{, written } \sec \Theta.$$

$$\frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}} \text{ called the cosecant of } \angle \Theta \text{, written } \csc \Theta.$$

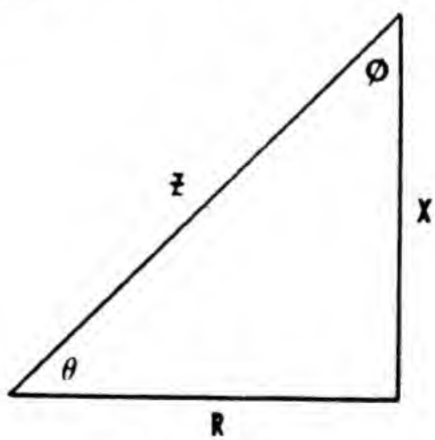


Figure 180.

236. The functions—sine, cosine, and tangent—cover most of the problems encountered in a. c. theory. You must memorize them so that you can tell at a glance any ratio of either acute angle, regardless of the position of the right triangle. The cosecant, secant, and cotangent are

reciprocals of the others. You will remember them easily, if you master the first three functions.

237. Try your hand at the following exercise, referring to figure 180:

	Answers:
(a) What is the sine of Φ?	$\frac{R}{Z}$
(b) What is the tangent of Θ?	$\frac{X}{R}$
(c) What is the cosine of Θ?	$\frac{R}{Z}$
(d) What is the cosecant of Φ?	$\frac{Z}{R}$
(e) What is the sine of Θ?	$\frac{X}{Z}$
(f) What is the cosine of Φ?	$\frac{X}{Z}$

STUDY GUIDE ON TRIGONOMETRIC TABLES

238. You learned from the study of the similar triangles in figure 177 that the ratio of the altitude to the base of a right triangle was fixed for any angle Θ. The sides of these particular triangles can be any length, but the tangent—that is, the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$—remains the same, since the numerical value for tan $40^\circ = 0.8400$. Mathematicians have calculated the values of the six ratios for angles of all sizes and have tabulated them. In appendix IV you will find a table giving the sine, cosine, tangent, and cotangent of angles from 0° to 90° at intervals of one-tenth of a degree. Let's turn back to this table and learn how to use it. You have a right triangle with $\angle\Theta = 29^\circ$. Find the sine of Θ. In the column headed "degrees" find 29. Opposite the 29 and in the column headed

"sin" you see the decimal 0.4848, thus sine $29^\circ = 0.4848$. This simply means that the ratio of the side opposite $\angle \Theta$ to the hypotenuse of the right triangle in question is 0.4848. Do the following exercises:

(a) Find the sine of each of these angles—14°, 75.4°, 61°.

(b) Find the cosine of each of these angles—24.7°, 89°, 43°.

(c) Find the tangent of each of these angles—62°, 51.8°, 39°.

Answers:

(a) 0.2419; 0.0677; 0.8746.
(b) 0.9085; 0.0175; 0.7314.
(c) 1.8807; 1.2708; 0.8098.

239. The following examples will show you how certain problems can be solved quickly by indirect measurement with the aid of a table of trigonometric functions.

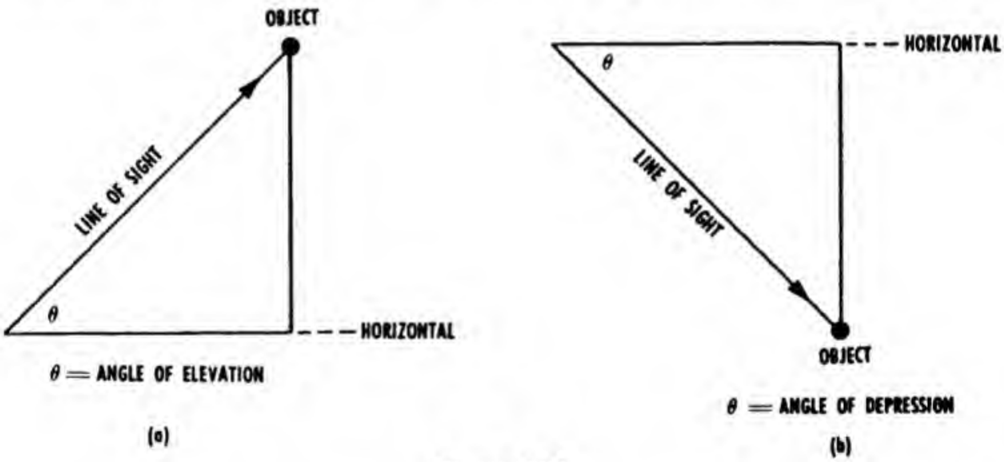


Figure 181.

Before attempting the solution of these problems, you should know the meaning of these two terms: (1) angle of elevation and (2) angle of depression. When an object is higher than the observer's eye, the angle between the horizontal and the line of sight to the object is called the ANGLE OF ELEVATION. (See fig. 181, a.) When an object is lower than the observer's eye, the angle between the line of sight to the object and the horizontal is called the ANGLE OF DEPRESSION. The solution of a problem is a great

deal easier if you sketch the problem and insert the given information. You need not draw to scale, but make a close approximation when constructing the angles.

Tangent

240. Example 1.—Before installing a television antenna on top of a pole, a technician must know the height of the pole so that he can order enough lead-in cable. Follow his sketch of the problem in the right triangle ABC in figure 182. First, he measures off a convenient distance such as 64 feet (AC) from the base of the pole. With the

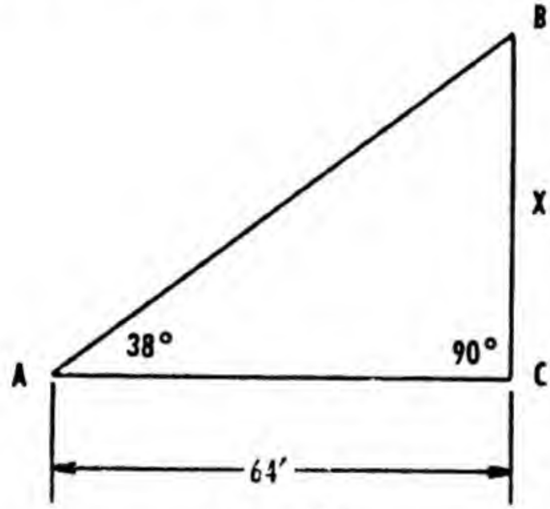


Figure 182.

proper measuring instrument he finds the angle of elevation at A equal to 38°. Then, letting X represent the height of the pole, and using the tangent, he sets up the equation:

Solution: $\tan \Theta = \dfrac{\text{opp side}}{\text{adj side}}.$

Substituting: $\tan 38° = \dfrac{X}{64}.$

From the table: $0.7813 = \dfrac{X}{64}.$

Transposing: $X = 64(0.7813).$

Simplifying: $X = 50$ feet.

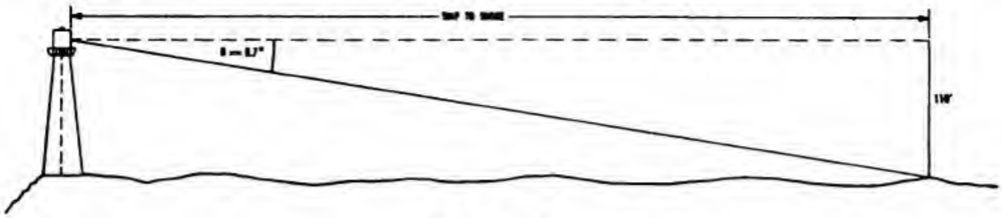


Figure 183.

To find the height of the pole with the aid of the trigonometric table requires only the measurement of one side and one angle. Note, however, that the angle at C must be a right angle.

241. Example 2.—Standing atop a 110-ft. lighthouse on the shore line, a sailor observes a ship at sea at a depression angle of 8.7° to the water line of the ship. How far at sea is the ship? (See fig. 183.)

Solution: $\tan \Theta = \frac{\text{opp side}}{\text{adj side}}.$

Substituting: $\tan 8.7° = \frac{110}{\text{adj side}}.$

From the table: $0.1530 = \frac{110}{\text{adj side}}.$

Transposing: $\text{adj side} = \frac{110}{0.1530}.$

Simplifying: adj side = 719 feet.

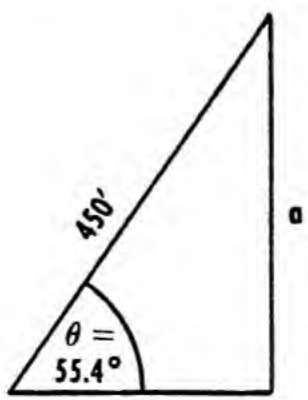


Figure 184.

The side adjacent to the angle represents the distance of the ship from shore, which is 719 feet.

Sine

242. Example 1.—The length of a kite string is 450 feet. The angle which the string makes with the ground is 55.4°. What is the altitude of the kite? Let the letter a represent the altitude. (See fig. 184.)

Solution: $$\sin \Theta = \frac{\text{opp side}}{\text{hyp}}.$$

Substituting: $$\sin 55.4^\circ = \frac{a}{450}.$$

From the table: $$0.8231 = \frac{a}{450}.$$

Transposing: $$0.8231(450) = a.$$

Simplifying: $$a = 307.4 \text{ feet.}$$

243. Example 2.—Sonar reports a submarine at range 1,675 yards at a depression angle of 9.8°. (See fig. 185.) How far under the surface is the submarine? Allow 4 yards from sea level to sound head.

Solution: $$\sin \Theta = \frac{\text{opp side}}{\text{hyp}}.$$

Substituting: $$\sin 9.8^\circ = \frac{X}{1{,}675}.$$

From the table: $$0.1702 = \frac{X}{1{,}675}.$$

Transposing: $$X = 0.1702(1{,}675).$$

Simplifying: $$X = 285.1 \text{ yards.}$$

The submarine is 289.1 yards under water.

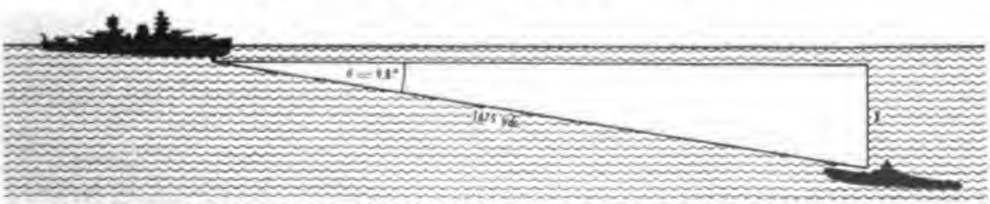


Figure 185.

244. Example 1.—A radio antenna is to be rigged from B to C in figure 186. ABC is a right triangle. The

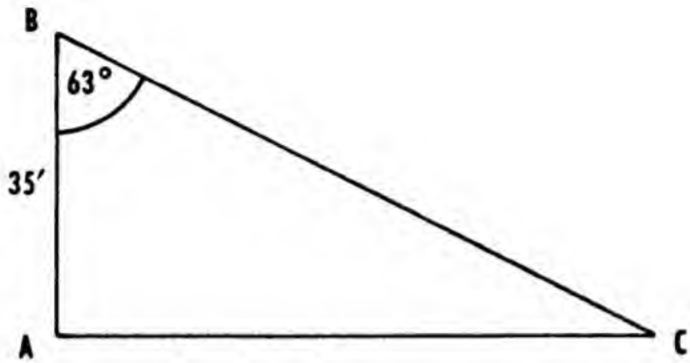


Figure 186.

distance AB is 35 feet, and angle B is 63°. How much antenna wire must be ordered allowing 3 feet extra for securing the ends?

Solution: $$\cos \Theta = \frac{\text{adj side}}{\text{hyp}}.$$

Substituting: $$\cos 63° = \frac{35}{BC}.$$

From the table: $$0.4540 = \frac{35}{BC}.$$

Transposing: $$BC = \frac{35}{0.4540}.$$

Simplifying: $$BC = 77.1 \text{ feet.}$$

Approximately 80 feet of antenna wire must be drawn from stores.

245. Example 2.—A 30-foot ladder rests against a building at an elevation angle of 68°. How far from the

base of the building does the ladder touch the ground? (See fig. 187.)

Solution: $\cos \Theta = \frac{\text{adj side.}}{\text{hyp}}$

Substituting: $\cos 68° = \frac{b}{30}.$

From the table: $0.3746 = \frac{b}{30}.$

Transposing: $b = 0.3746(30).$

Simplifying: $b = 11.2$ feet.

The base of the ladder is 11.2 feet from the bottom of the building.

246. If you have a clear understanding of the six examples given, you can work problems dealing with

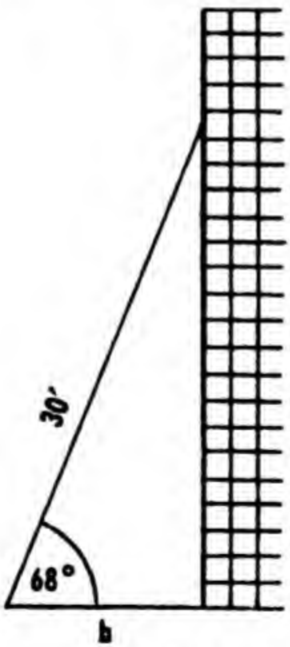


Figure 187.

trigonometric functions and combinations of them. For instance, you are now familiar enough with the trigonometric table to use it to find the value of a trigonometric function of an angle when the angle is given.

However often you will be given only the sides of a right triangle, and your problem will be to determine the angle. This operation is just the reverse of your other procedure.

247. Consider the following problem:

A ship travels north from point A for 25 miles to point C. The ship then travels east for 18 miles to point B. You

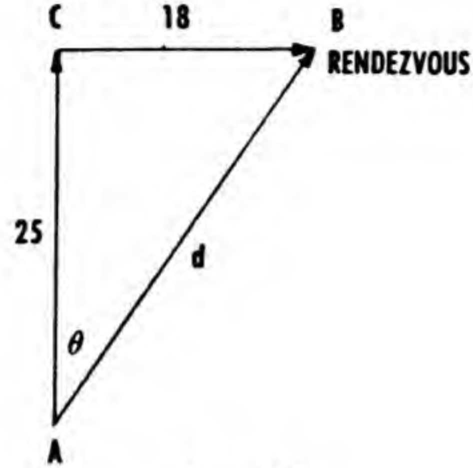


Figure 188.

are to start from the same point A and travel a direct route to point B to rendezvous with the ship. What is the proper course to steer? (See fig. 188.) First determine what function of the unknown angle is included in the given information. Here the bearing of the course AB from north is angle Θ, and the opposite and adjacent sides to $\angle \Theta$ are given, so that you use the tangent function

$\frac{\text{opposite side}}{\text{adjacent side}} = \frac{18}{25} = 0.7200$ (always carry out the division to four decimal places). Now turn to the trigonometric table to find the angle whose tangent is 0.7200. After a little practice in constructing the right triangles you will be able to draw your sketch somewhat to scale, and in that way you can roughly estimate the size of the angle and thus where to begin looking in the table. Trace down the columns headed "tan" until you find a decimal near what you have. In this case you'll find 0.7212

(close enough) and the angle is 35.8°. The course your ship must take to the rendezvous is 035.8 true. Just to complete the picture, how many miles on this route will you have to travel? You can solve this part of the problem by using the Pythagorean theorem or by using the sine or cosine, since you have the side opposite Θ and the side adjacent to Θ given. Below is the solution using the sine and letting the letter d represent the hypotenuse, which is the distance to be traveled.

Solution: $\sin \Theta = \frac{\text{opp side}}{\text{hyp}}.$

Substituting: $35.8° = \frac{18}{d}.$

From the table: $0.5850 = \frac{18}{d}.$

Transposing: $d = \frac{18}{0.5850}.$

Simplifying: $d = 30.8$ miles.

Your ship must travel 30.8 miles on course 035.8°.

248. One more problem and you should be all set.—Two forces are acting on an object at A. One force AB

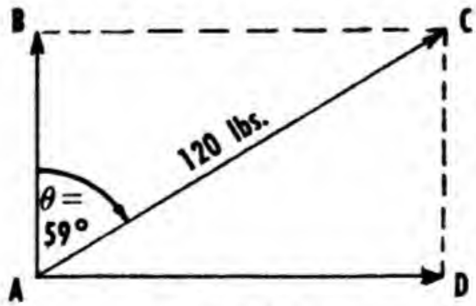


Figure 189.

pulls north, and the other force AD pulls east. (See fig. 189.) The resultant force AC of these two forces is 120 pounds, and its direction measured from north is 59°. What is the magnitude of the force pulling north and the magnitude of the force pulling east? First complete a

parallelogram to show the direction of the resultant force. In this parallelogram $BC = AD$.

Solution:

$$\sin 59° = \frac{\text{opp side}}{\text{hyp}} = \frac{BC}{120}. \qquad \cos 59° = \frac{\text{adj side}}{\text{hyp}} = \frac{AB}{120}.$$

From the table: $0.8572 = \frac{BC}{120}$. $\qquad 0.5150 = \frac{AB}{120}$.

Transposing: $BC = 0.8572(120)$. $\qquad AB = 0.5150(120)$.

Simplifying: $BC = 102.9$ lbs. $\qquad AB = 61.8$ lbs.

Therefore: $AD = 102.9$ lbs.

A force of 61.8 pounds is pulling north, and a force of 102.9 pounds is pulling east.

APPLICATION TO ALTERNATING-CURRENT PROBLEMS

249. With the aid of the Pythagorean theorem in chapter 7 you solved for the resultant of two out-of-phase voltages and for the impedance of series a.c. circuits. In the case of the voltages, the direction of the resultant was at some angle between the out-of-phase forces and thereby leading or lagging the current in the circuit. Likewise, the impedance was at an angle between the resistance and the reactance of the circuit. You already know the importance of this phase angle. You can solve problems in a manner similar to those covered in chapter 7 by knowing the values of one acute angle and one side of a right triangle. Also, with trigonometry you can solve problems dealing with the instantaneous values of current, voltage, and power. We will begin with simple problems on series a.c. circuits.

250. Example 1.—The impedance of a coil in an electrical circuit is 72 ohms at a leading angle of 50°. What are the values of the resistance and reactance of the circuit? (See fig. 190.) You should remember that when we speak of a leading or lagging angle in a series a. c. circuit, it is with respect to the current of the circuit,

which is the same through each component and is plotted at 0°. You will learn later that in parallel a. c. circuits the currents through each branch of the circuit may vary, but the amount of voltage is the same across each branch.

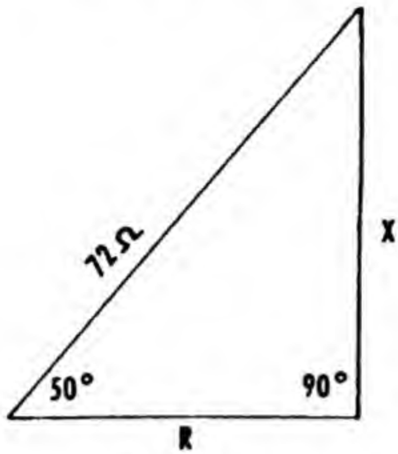


Figure 190.

Therefore, in parallel circuits the line voltage is plotted at 0° on a graph and is the reference for leading or lagging angles.

Solution: $\sin 50° = \frac{X}{72}$. $\qquad \cos 50° = \frac{R}{72}$.

$X = 0.7660(72)$. $\qquad R = 0.6428(72)$.

$X = 55.2$ ohms. $\qquad R = 46.3$ ohms.

Since the impedance angle is a positive 50°, the reactance in the circuit was plotted at +90°.

251. Example 2.—A circuit consists of a resistor and a coil in series with an a. c. generator. The generator voltage of 220 volts is leading the line current by 62°. Find the values of E_R and E_L. (See the diagram of the problem in fig. 191.)

Solution: $\sin 62° = \frac{E_L}{220}$. $\qquad \cos 62° = \frac{E_R}{220}$.

$E_L = 0.8829(220)$. $\qquad E_R = 0.4695(220)$.

$E_L = 194.2$ volts. $\qquad E_R = 103.3$ volts.

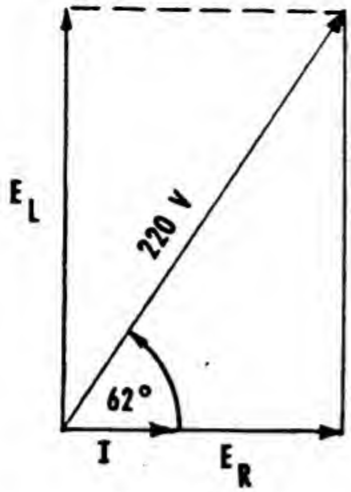


Figure 191.

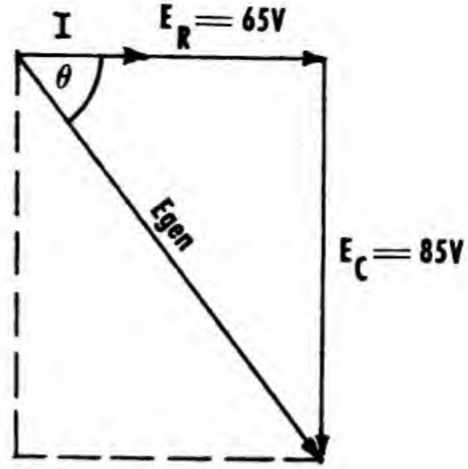


Figure 192.

252. Example 3.—A capacitor and a resistor are in series with an a. c. generator. With a voltmeter you measure a 65-volt drop across the resistor and an 85-volt drop across the capacitor. Find the generator voltage and the amount of lead or lag of the supply voltage with reference to the current in the circuit. (See fig. 192.)

Solution: $\tan \Theta = \frac{85}{65}$. $\sin 52.6° = \frac{85}{E_{gen}}$.

$\tan \Theta = 1.3077$. $0.7944 = \frac{85}{E_{gen}}$.

From the table: $\Theta = 52.6°$. $\frac{85}{0.7944} = E_{gen}$.

$$E_{gen} = 107v\underline{/-52.6°}.$$

In this problem you were given two sides of the right triangle. The first step in solving is to find Θ by means of the tangent. Having found this, you can use either the sine or the cosine to determine E_{gen}. As the components making up E_{gen} were plotted at 0° and −90°, the resultant voltage—which is the generator voltage—is at a negative angle between 0 and −90° and is lagging the current. As shown in the solution, the sine is used, and the completed answer is $E_{gen} = 107v\underline{/-52.6°}$.

253. The voltage and power waveforms you have seen thus far are called SINE WAVES. The following explanation shows how a sine wave is constructed.

254. In figure 193 a circle is divided into 12 equal parts, each containing 30°. A straight line for the desired time distance of one cycle (360°) is drawn to the right of the circle. This line also is divided into 12 equal parts, each part representing 30°. The radius lines drawn from

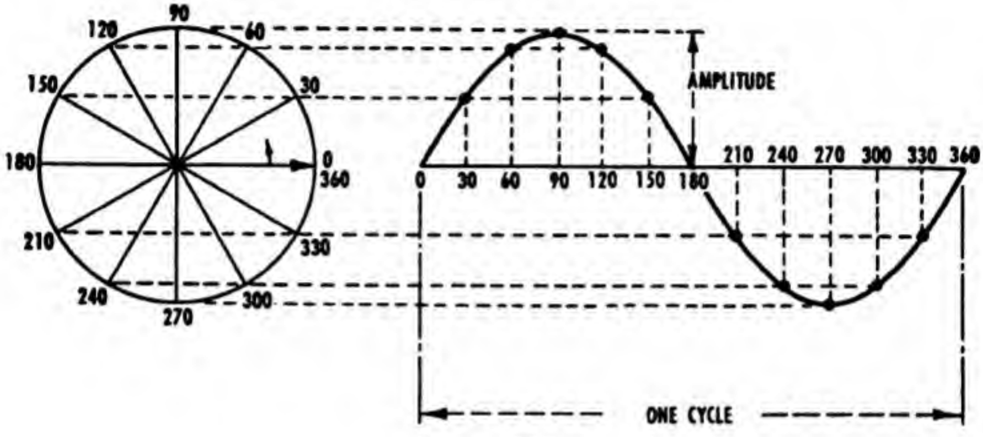
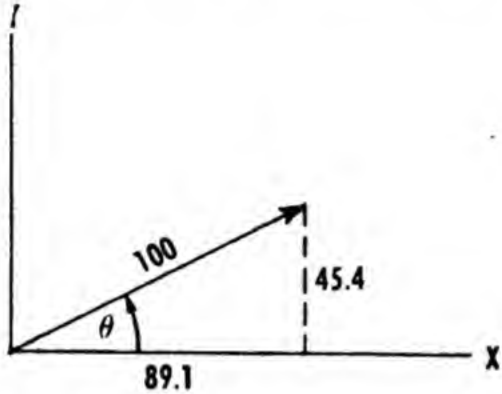


Figure 193.

the center of the circle to the outer edge are vectors. In chapter 7 you learned that a vector is a straight line representing a quantity that has both direction and magnitude. These radius lines will later represent values of current and voltage. The circle in figure 193 represents a vector rotating from 0° to 360°. The length of the vector (radius of circle) represents the maximum amplitude of the desired sine wave. The direction of the vector is the number of degrees it has rotated from the reference line at 0°, which is the stationary side of the angles generated. As shown, the rotating vector has been stopped at every 30° of its cycle. Horizontal lines are projected from each 30° point on the circle. Vertical lines are drawn from each 30° point on the horizontal time base. The points where the corresponding lines intersect are then connected by a smooth curve. This curve is known as a SINE CURVE or SINE WAVE.

255. Suppose we assign the vector a maximum value of 100 units and stop its rotation at 27°. Written in polar-vector form, this vector can be designated $100\underline{/27^\circ}$. On a line graph this vector can be stated in terms of its rectangular coordinates, or X and Y values. The solution by trigonometry is 89.1 units on the X-axis (horizontal) and 45.4 units on the Y-axis (vertical). (See fig. 194.) The vertical distance to the X-axis from the end of a



$\theta = 27°$

Figure 194.

vector is its INSTANTANEOUS AMPLITUDE. Thus, a vector whose maximum value is 100 units has an instantaneous amplitude of 45.4 units when it has rotated 27° of its cycle. The name "sine wave" is derived from the fact that the amplitude of the vertical component varies as the vector rotates, and since it is the sine of Θ its projection with respect to time gives us the sine-wave shape.

256. We can compare the action of the rotating vector to that of a simple a. c. generator. Suppose we place two magnets end-to-end with a small air space between them, the north pole of one facing the south pole of the other. Because unlike poles attract, there is a strong magnetic field between them. This field is shown by the flux lines drawn between the magnets (in fig. 195). When a loop of wire is moved through this magnetic field a voltage is induced in the loop. Consider a loop of wire rotating in

this magnetic field. Follow the stop-motion pictures of the rotating loop in figure 195. When the loop is in position 1 the conductors do not cut any lines of flux because they are moving parallel to the lines. Therefore, the induced voltage in the loop is zero. This position is comparable to that of the rotating vector at 0° (as in fig.

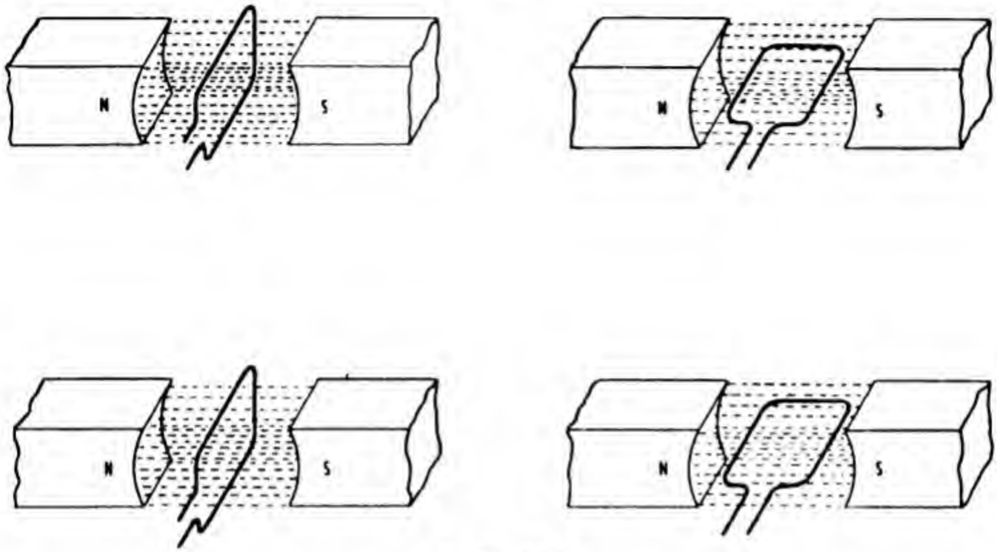


Figure 195.

193). As the loop rotates, it begins cutting the flux lines, thus inducing a voltage in the loop. When the loop reaches position 2 it has rotated one-quarter of a turn, or 90°. The conductors at this time cut a maximum number of lines of flux, and the induced voltage in the loop is a maximum positive. At position 3, after 180° of the cycle, again the conductors do not cut any flux lines, and the induced voltage is zero. At position 4 a maximum voltage is induced in the loop, but the conductors cut the flux lines in the opposite direction, so the induced voltage is negative. A complete rotation brings the loop back to position 1. If you were to plot the amount of voltage induced in the loop at every instant you would have an exact voltage curve, such as the sine curve in figure 193. You can see that this sine curve indicates the amplitude of the induced voltage at any given point in the rotation of the loop. The result of solving for the amount

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of voltage induced at any one instant with the sine function is called the INSTANTANEOUS VALUE OF VOLTAGE (written e_{inst}).

257. Now we can substitute the value E_{max} (maximum generator output voltage) for the rotating vector. The angle alpha (Greek letter written α) made by E_{max} with the reference line or stationary side of the angle varies in magnitude from 0° to 360°. The acute angle made by the rotating vector and the X-axis is designated Θ. Therefore $\angle \alpha$ equals $\angle \Theta$ for any angle between 0° and 90°. The

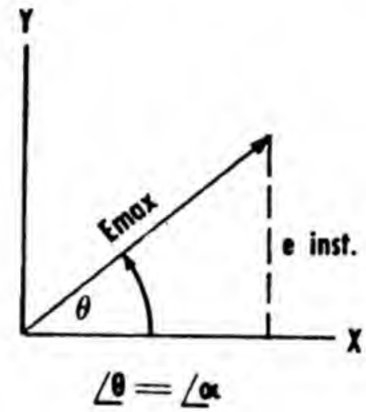


Figure 196.

instantaneous value of the voltage is the vertical component of the vector E_{max} and is represented by the vertical line drawn from the tip of vector E_{max} to the X-axis. This is shown in figure 196. Thus, the equation for the instantaneous value of the voltage stands—

$$e = E_{max} \sin \Theta$$

258. In our discussion so far we have limited ourselves to positive angles between 0° and 90° and to negative angles generated clockwise between 0° and -90°. But what about angles greater than 90°? First of all, examine the diagram in figure 197, a. As shown, a complete revolution of the vector from 0° to 360° is divided into four parts called QUADRANTS, each quadrant containing 90°. From 0° to 90° is the first quadrant, 90° to 180° the second quadrant, 180° to 270° the third quadrant, and 270°

to 360° the fourth quadrant. Notice now, when working with angles greater than 90° in the four quadrants, you come across $-X$ and $-Y$ values. On graphs of rotating vectors such as this, values of X measured to the right of the Y-axis are positive, and values of X to the left of the

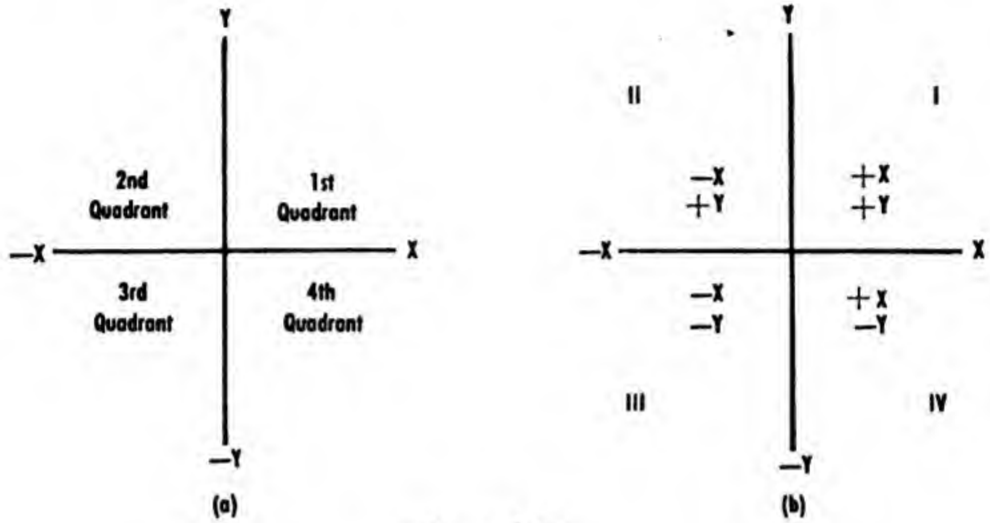


Figure 197.

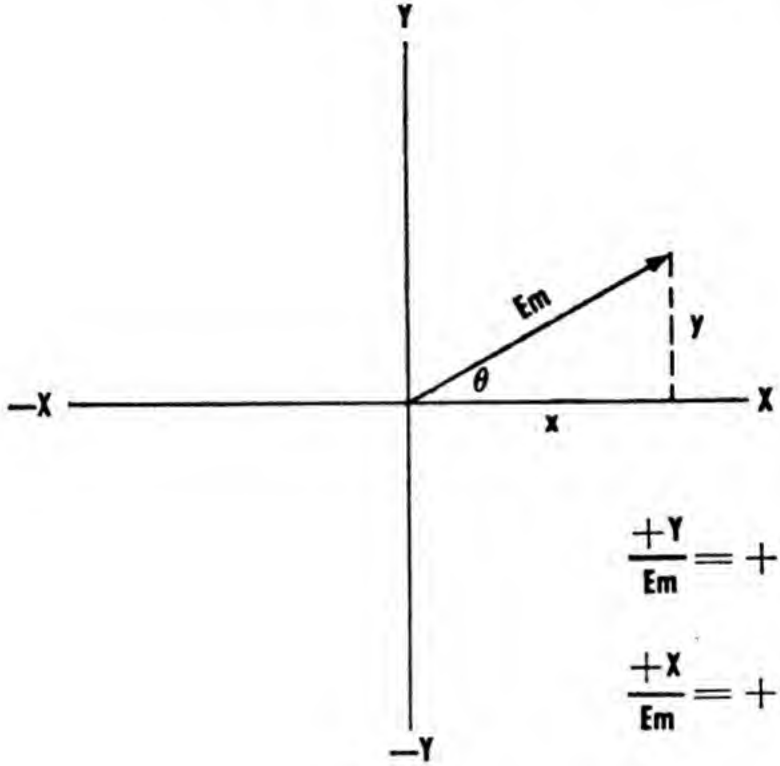
Y-axis are negative. Values of Y measured above the X-axis are positive, and values of Y below the X-axis are negative. Check this in figure 197, b. This introduces the fact that the signs of the trig functions in the four quadrants vary.

259. As we are dealing only with right triangles in this chapter, the angle opposite side e—the instantaneous value of the induced voltage on the Y-axis—must always be an acute angle. The trigonometric table in appendix IV shows the values of functions of angles from 0° to 90°. Therefore, the functions of angles greater than 90° must be reduced to the functions of an acute angle. The functions of any angle are defined in terms of the right triangle formed by dropping a perpendicular from the end of vector E_{max} to the X-axis, thereby forming a right triangle in any quadrant.

260. The radius vector (E_{max})—the hypotenuse of a right triangle—is always positive. As shown in figure 198,

the sign of any function of an angle in the first quadrant is positive since the X and Y values are positive.

261. Now consider an angle of 145° that terminates in the second quadrant. First drop the perpendicular from E_m to the X-axis, thus forming the right triangle. The acute angle Θ is 35°. Note that $\angle \alpha + \angle \Theta = 180°$.



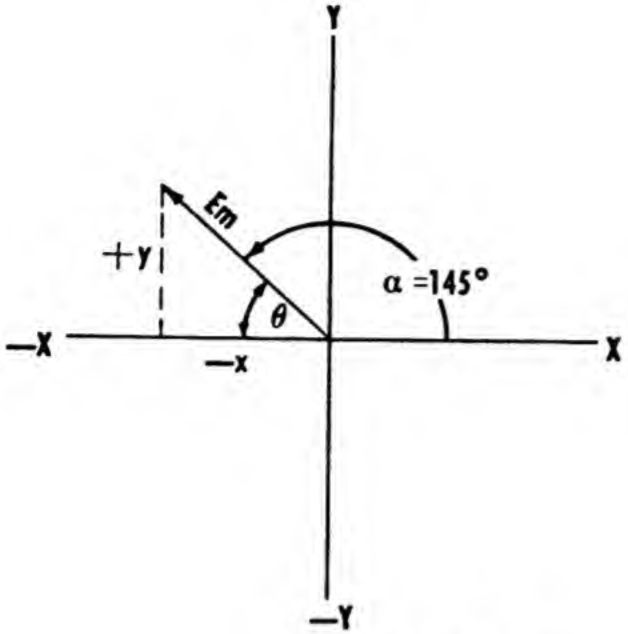
$$\frac{+Y}{Em} = + \sin \theta$$

$$\frac{+X}{Em} = + \cos \theta$$

$$\frac{+Y}{+X} = + \tan \theta$$

Figure 198.

Therefore, the angles are supplementary. A general rule in trigonometry is that a function of an angle has the same absolute value as the same function of its supplement. Thus, sin 145° = the absolute value of sin 35°. However, consider the signs of the functions of angle Θ. (See fig. 199.) When $\angle \alpha$ terminates in the second quadrant the instantaneous value of voltage always is positive since $\sin \Theta$ is positive in this quadrant.

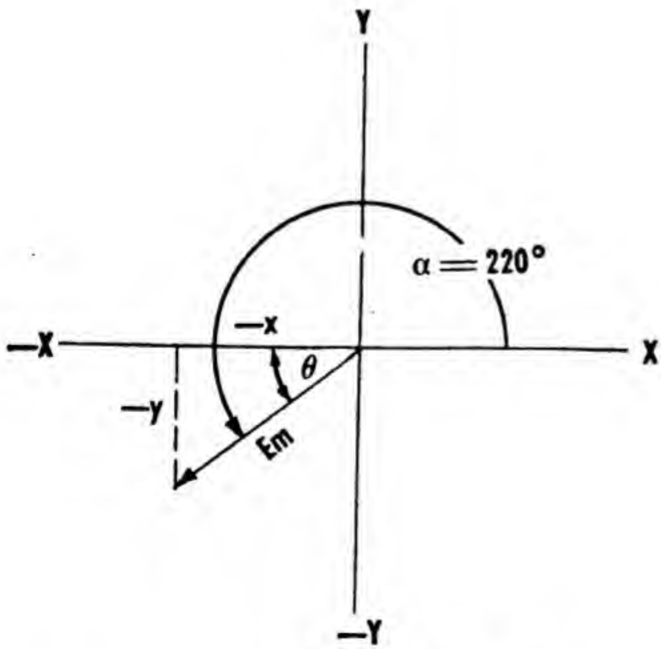


$$\frac{+y}{Em} = +\sin\theta$$

$$\frac{-x}{Em} = -\cos\theta$$

$$\frac{+y}{-x} = -\tan\theta$$

Figure 199.



$$\frac{-y}{Em} = -\sin\theta$$

$$\frac{-x}{Em} = -\cos\theta$$

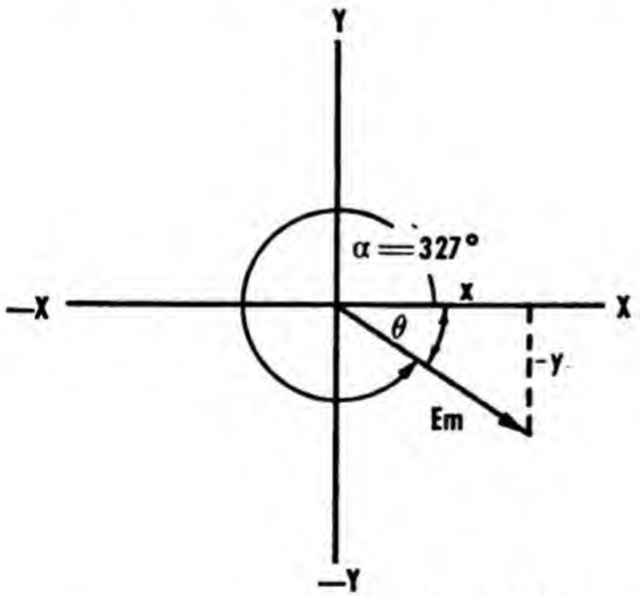
$$\frac{-y}{-x} = +\tan\theta$$

Figure 200.

262. The functions of angles in the third quadrant are again reduced to the functions of acute angles. In figure 200 is shown a generated angle with $\alpha = 220°$. To find the acute angle formed by E_m and the X-axis in the third quadrant, use the formula—

$$\Theta = \alpha - 180°.$$

Mathematicians have proved that the function of an angle in the third quadrant has the same absolute value as the same function of the acute angle between the radius vector and the X-axis. In this example $\Theta = 220° - 180° = 40°$. Now observe the signs of the functions as shown in figure 200, and you'll notice $\sin 220° = -\sin 40°$. As the value e in the third quadrant is a negative voltage you would prefix the minus sign to the sine of $\angle\, \Theta$.



$$\frac{-Y}{EM} = -\sin\theta$$

$$\frac{+X}{Em} = +\cos\theta$$

$$\frac{-Y}{+X} = -\tan\theta$$

Figure 201.

263. In the fourth quadrant the acute angle Θ is found by the formula $360° - \alpha = \Theta$. The graph in figure 201 shows a generated angle of $327°$. From the previous formula $\Theta = 33°$, you know that the functions of an angle in the fourth quadrant have the same absolute values as the same functions of an acute angle in the first quadrant. The signs of the functions must be prefixed in compliance

with the ratios shown in figure 201. What is the instantaneous value of the voltage in figure 200 if $E_{max} = 500$ volts?

Since:

$$\sin 327^\circ = -\sin 33^\circ$$
$$e = E_m\,(-\sin 33^\circ),$$
$$e = 500\,(-0.5446),$$
$$e = -272.3 \text{ volts.}$$

264. The function of an angle larger than 360° is found by dividing the angle by 360° and finding the required function of the remainder. Thus, to find the function of 755°, first divide by 360, which gives 2 and a remainder of 35°. Therefore, the angle 755° terminates in the first quadrant. Thus, sin 755° = sin 35°, because the sine is positive in the first quadrant.

265. Tying all the information given above into one general statement, you have—TO FIND ANY FUNCTION OF ANY ANGLE α, TAKE THE SAME FUNCTION OF THE ACUTE ANGLE Θ FORMED BY THE RADIUS VECTOR AND THE X-AXIS,

QUADRANT	SIN θ	COS θ	TAN θ
I	+	+	+
II	+	−	−
III	−	−	+
IV	−	+	−

Figure 202.

AND PREFIX THE PROPER ALGEBRAIC SIGN FOR THAT QUADRANT. The proper signs for the functions are shown in the chart in figure 202.

266. Perhaps a few examples will give you a clearer insight in solving for instantaneous values of voltage, current, and power in alternating-current circuits.

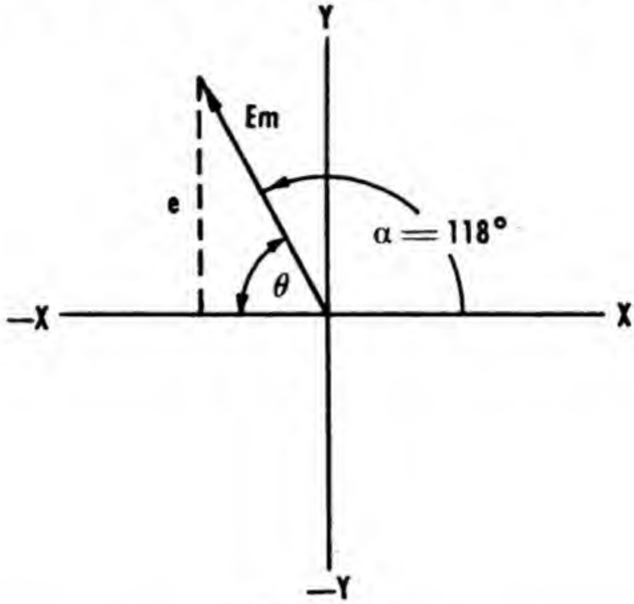


Figure 203.

267. Example 1.—What is the instantaneous value of an alternating voltage when it has completed 118° of its cycle? The maximum value is 400 volts. (See fig. 203.) Solution:

$$\Theta = 180 - \alpha = 180 - 118 = 62^\circ.$$
$$e = E_{\max} \sin \Theta.$$
$$e = 400\,(0.8829).$$
$$e = 353.2 \text{ volts.}$$

268. Example 2.—The maximum value of an a. c. voltage is 185 volts. What is the instantaneous value when E_m has reached 227° of its cycle? (See fig. 204.) E_m has been terminated in the third quadrant. $\Theta = 227 - 180 = 47^\circ$. Solution:

$$e = E_m \sin 47^\circ.$$
$$e = 185\,(-0.7314).$$
$$e = -135.3 \text{ volts.}$$

The minus sign is prefixed to the sine of 47° because Θ is in the third quadrant. To keep this straight in your mind, refer back to figure 197, b, and note that the vertical coordinate (Y value) of a vector in this quadrant is negative.

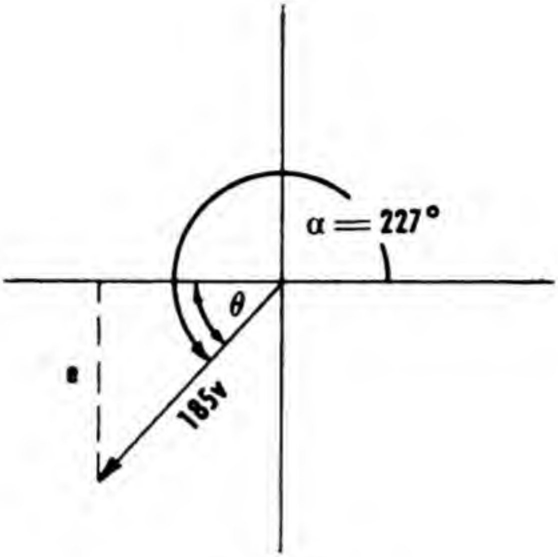


Figure 204.

269. Example 3.—The instantaneous value of an alternating voltage is 105 volts at 33°. What is its value at 230°? First you must find E_{max} from the information given. Having found E_{max}, go on from there to find the instantaneous value required. (See the diagram of the problem in fig. 205 and the solution that follows.)

Solution: $\sin \Theta = \frac{105}{E_m}$. $\Theta = 230° - 180° = 50°$.

$$E_m = \frac{105}{\sin 33°}. \qquad e = E_m \sin 50°.$$

$$E_m = 192.8 \text{ volts.} \qquad e = 192.8(-0.7660).$$

$$e = -147.7 \text{ volts.}$$

The instantaneous value of the alternating voltage after E_{max} has rotated 230° is –147.7 volts.

270. Example 4.—What is the instantaneous value of an alternating voltage 135° after its maximum positive

value of 325 volts. This problem may sound a little different to you, but see what you can do with it before looking at the solution that follows.

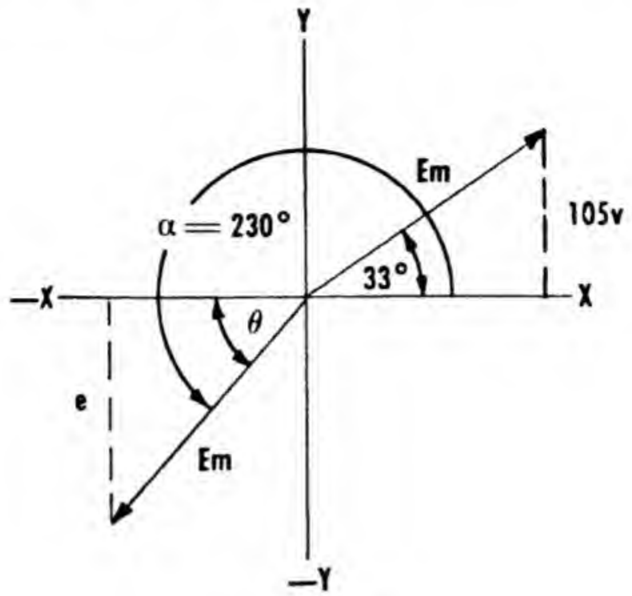


Figure 205.

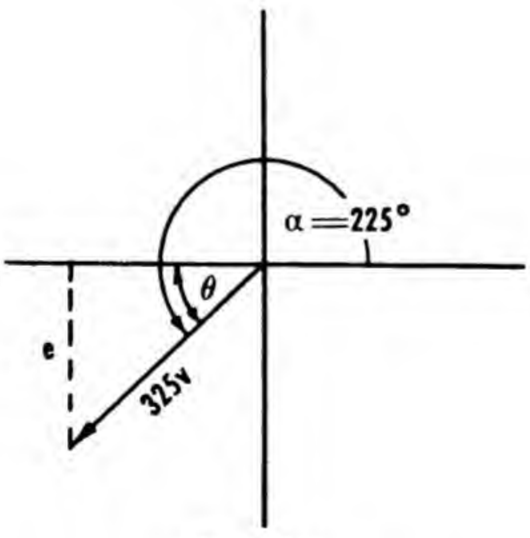


Figure 206.

271. Before drawing a sketch of the problem at hand, let's tackle half the problem. At what angle does a generator put out its maximum positive value? In other words, at what angle does the instantaneous value of

voltage equal E_{max}? From figures 193 and 195 you will find this occurs at 90°. Also a check of the trigonometric table in appendix IV indicates that the sine of $90° = 1.000$. It is at this time in the cycle that $e = E_{max}$. Therefore, $E_{max} = 325$ volts. The second half of the problem is to find the value e at a position 135° after this time or $135° + 90° = 225°$. Now follow figure 206 and the solution that follows.

Solution:

$$\Theta = 225° - 180° = 45° \text{ in the third quadrant.}$$
$$e = E_{max}(\sin 45°)$$
$$e = 325(-0.7071)$$
$$e = -229.81 \text{ volts}$$

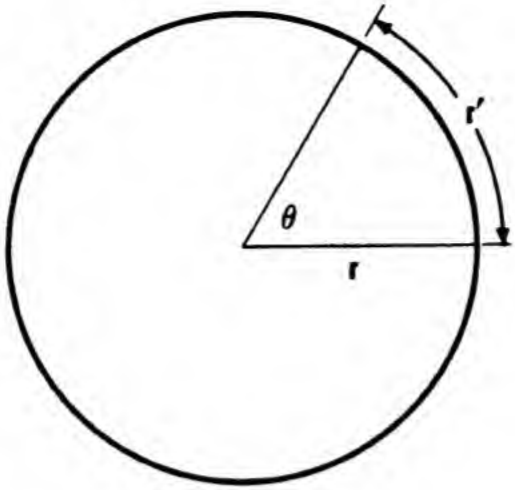
RADIANS

272. Angles, circles, and vectors are used in electricity and electronics in many familiar ways, such as in describing phase relations. For example, in the explanation of the voltage generated by a rotating loop in a magnetic field the sine-wave output was represented pictorially as a rotating vector that described a circle.

273. In the use of angles, circles, and vectors, the factor π appears often. The reason being the fact that the circumference of a circle is exactly 2π times the radius, or π times the diameter. This is the origin of the term "π"; it is the length of the circumference of a circle divided by the diameter.

274. Because π is so often used, a special name is given to the angle subtended by a length of the circumference equal to the radius. This angle, shown as Θ in figure 207, subtends a length of the circumference (r') equal to the radius (r). It is called a RADIAN. A radian is defined as an angle that, when placed with its vertex at the center of a circle, intercepts an arc on the circumference equal in length to the radius of the circle.

275. There is a simple and obvious relation between radians and degrees. One complete revolution of a radius



$r = r'$
$\theta = 1$ radian

Figure 207.

describes a circle of 360°. By the definition of a radian, one revolution also equals 2π radians. In other words:

$$1 \text{ revolution} = 360^\circ = 2\,\pi \text{ radians.}$$

Therefore—

$$1^\circ = \tfrac{2}{360}(\pi) \text{ radians,}$$

and

$$1^\circ = 0.01745 \text{ radians.}$$

Similarly: $$2 \text{ radians} = \frac{360^\circ}{\pi},$$

$$1 \text{ radian} = \frac{360^\circ}{2\pi},$$

and $$1 \text{ radian} = 57.3^\circ \text{ approx.}$$

Therefore, a radian is simply an angle equal to approximately 57.3°. To convert any given number of degrees to radians, multiply the number of degrees by 0.01745 or divide by 57.3.

276. Radians are used very often in electricity and electronics to describe an alternating current or voltage. In figure 208, for example, the single cycle of an a. c. voltage can be described either in degrees or in radians. Most books and texts find it more convenient to say the

voltage has completed 2π radians than to say 360°. Either statement means the same thing—that is, the voltage has completed one cycle.

277. In addition to their use in describing the phase of an a. c. wave, radians are used often to describe the frequency of the wave. The frequency is the rate at which

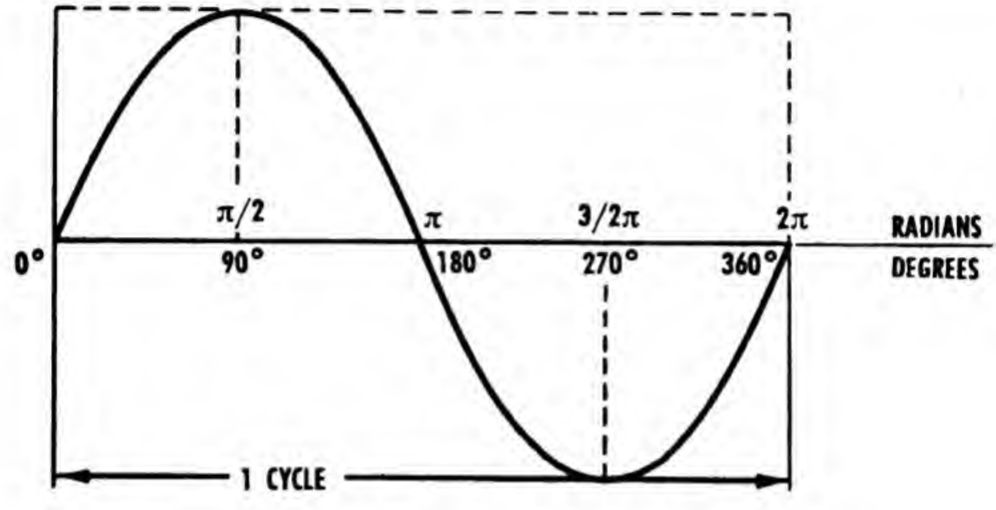


Figure 208.

the wave completes one cycle (360° or 2π radians). If a rotating vector is used to describe the wave, it is the angular velocity of the vector—that is, the rate at which r (in fig. 207) rotates. The frequency, f, can be described either as revolutions (cycles) per second or—since one revolution is 2π radians—as radians per second. In other words, if the frequency is f cycles per second, it is also $2\pi f$ radians per second. The term "$2\pi f$" is used so often to describe the frequency of a wave that it is given a special symbol. The symbol used to denote frequency in radians per second is the Greek letter omega (ω). Therefore—

$$\omega = \text{frequency in radians per second,}$$

and

$$\omega = 2\pi f,$$

where f is the frequency in cycles per second.

CHAPTER 9

VECTORS

In arithmetic you found that you could express many physical quantities by designating a certain number of units. For example, the area of a room is given as so many square feet, the distance between two poles as so many feet, the speed of a moving object as a number of linear units per unit of time, such as miles per hour or feet per second. Quantities that are fully described by a number (as in the examples given above), are called SCALAR quantities, and the numbers that are used to represent them are called SCALARS. A scalar quantity is one that has magnitude ONLY and that does NOT involve any idea of direction.

There are many quantities which need a more definite expression than is possible by designating magnitude only. For example, a force acting upon an object is not completely described until the direction, as well as the magnitude of the force, is given; also, to describe a moving object fully, the direction as well as the magnitude must be given. In electronics the whole system of circuit analysis is built around the idea of expressing the direction and magnitude of voltage and current.

PRETEST 9

278. (a) Vectors are used to show a quantity that has both .... and ....

(b) Vector quantities are represented by a directed ....

(c) A vector that is the sum or difference of two or more vectors is called the ....

(d) Two independent alternating voltages are acting on a circuit. One is 10 volts in magnitude at an angle of 30° with respect

to the reference; the other is 30 volts in magnitude at an angle of 120° with respect to the reference. What is the magnitude and direction of the resultant voltage?

(e) Find the resultant of the three voltages; 5 volts at an angle of 45° with respect to the reference, 10 volts at an angle of 120° with respect to the reference, and 20 volts at an angle of 345° with respect to the reference.

(f) Find the resultant of the three voltages in problem (e) algebraically.

(g) It is permissible to add vectors directly when they are in ________.

(h) It is permissible to subtract vectors directly when they are ________.

(i) A vector with a magnitude of 10 volts makes an angle of 53° with the horizontal. What are the rectangular components? Solve the problem graphically and algebraically.

(j) One of two vectors is 5 units in length and operates at an angle of 53°; the other is 8 units in length and operates at an angle of 147°. Find the resultant vector graphically.

(k) Resolve the two vectors of problem (j) into their rectangular components. After expressing the rectangular components as complex numbers, write the sum of these numbers.

STUDY GUIDE ON VECTORS

279. To show a quantity that has both magnitude and direction, we use VECTOR QUANTITIES. A vector quantity

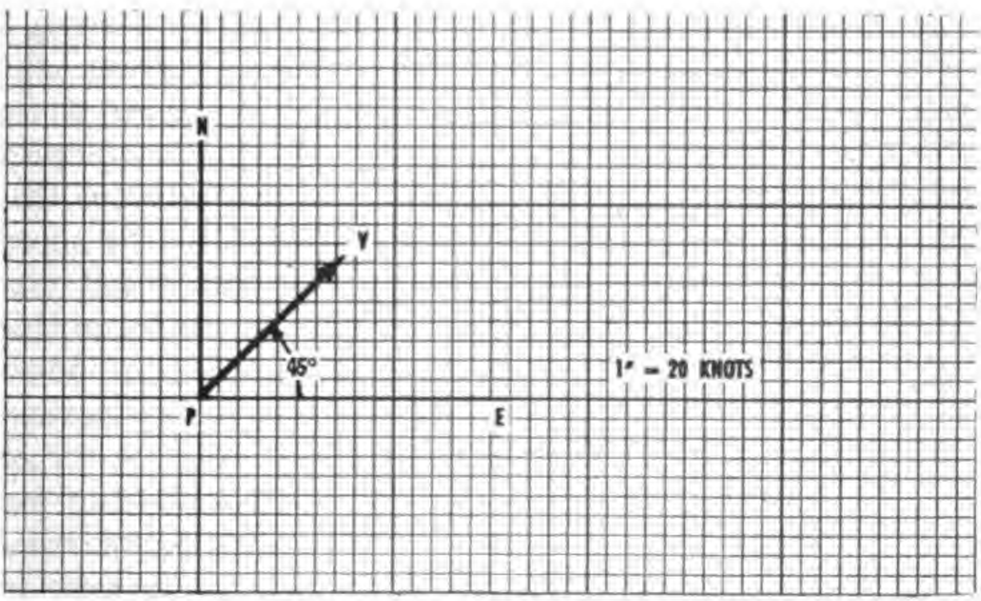


Figure 209.

is represented by a directed straight line, which is called a VECTOR. A vector then is a line that pictures both the magnitude and the direction of a force. Any vector may be moved as long as its direction and length remain the same.

280. Figure 209 shows a vector. A ship starting at point *P* steams in a northeasterly direction at a speed of 20 knots. The speed of the ship is represented by the length of the line, which is drawn to a convenient scale. The direction of the ship is represented by the direction of the line. Thus, the line *PV* is a vector, for it describes both the magnitude and the direction of the ship's velocity.

281. In a vector, then, there are three things to watch—

(a) A starting point, or a point where the force is applied.

(b) A line drawn to scale showing the magnitude (size) of the force.

(c) An arrow (or angle) showing the direction of a force.

ADDITION OF VECTORS

282. Two or more vectors may be added, but since a vector represents both magnitude and direction, the method differs from the procedure used for scalar quantities. The graphical method is used first, but the complex number method is also explained in the last part of this chapter. In figure 209 the vector *PV* represents the speed of a ship in a northeasterly direction. This speed was caused by the ship's propellers. Now, if the wind and tide coming from the north were acting on the ship, these forces would tend to push the ship in a southerly direction with a velocity of 10 knots. Thus, there are two forces acting on the ship represented by the two vectors in figure 210. To find the resultant motion of the ship, you must add the two vectors. Proceed as follows, referring to figure 211.

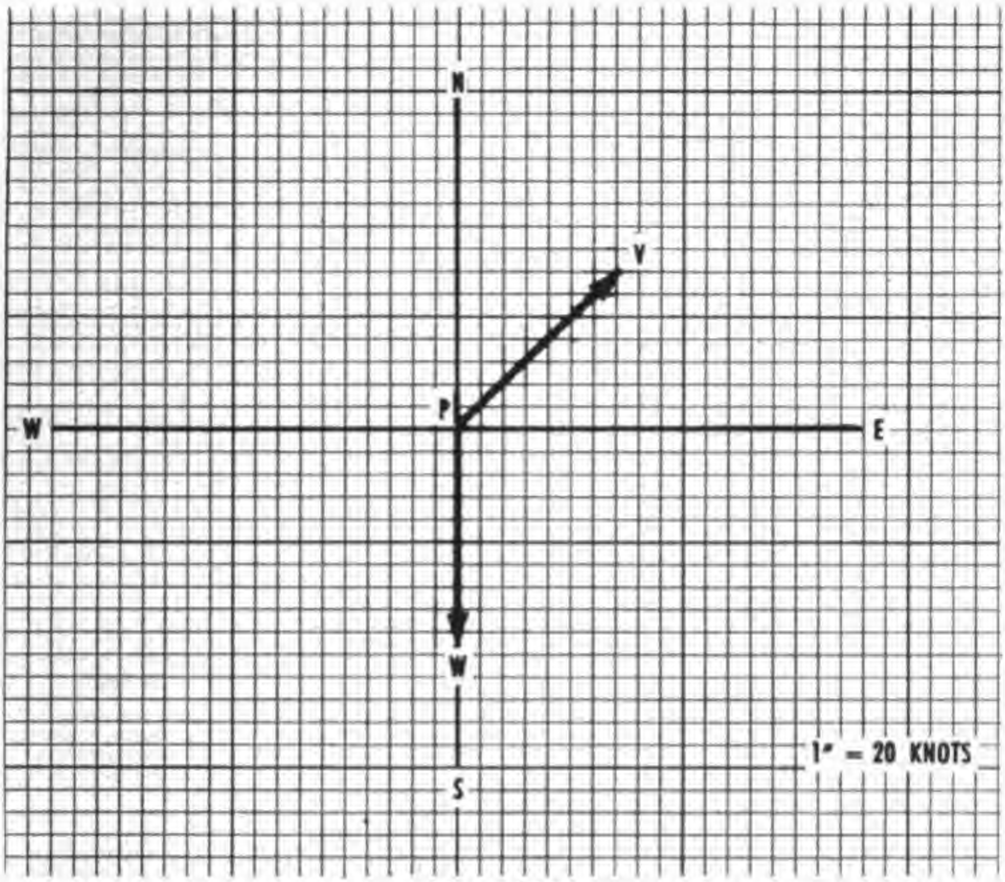


Figure 210.

(a) Using the two given vectors as the adjacent sides of a parallelogram, construct the other two sides, as shown by the dotted lines. The addition of the two vectors is given by the diagonal *PO*, as shown in figure 211.

(b) A vector that is the sum or difference of two or more vectors is called the RESULTANT. In this figure, vector *PO* is the resultant of the two vectors *PV* and *PW*. Vector *PO* shows both the magnitude and the direction of the effect when two forces combine to act as a single force. A resultant vector is the single vector that is equivalent to the two or more vectors that are being combined or added.

283. In figure 212 (using the same vectors that were used in the parallelogram example) is shown an alternate method for solving graphically for the resultant of two or more vectors. Again you start with vector *PV*, which is the force tending to move the ship from point *P* to point

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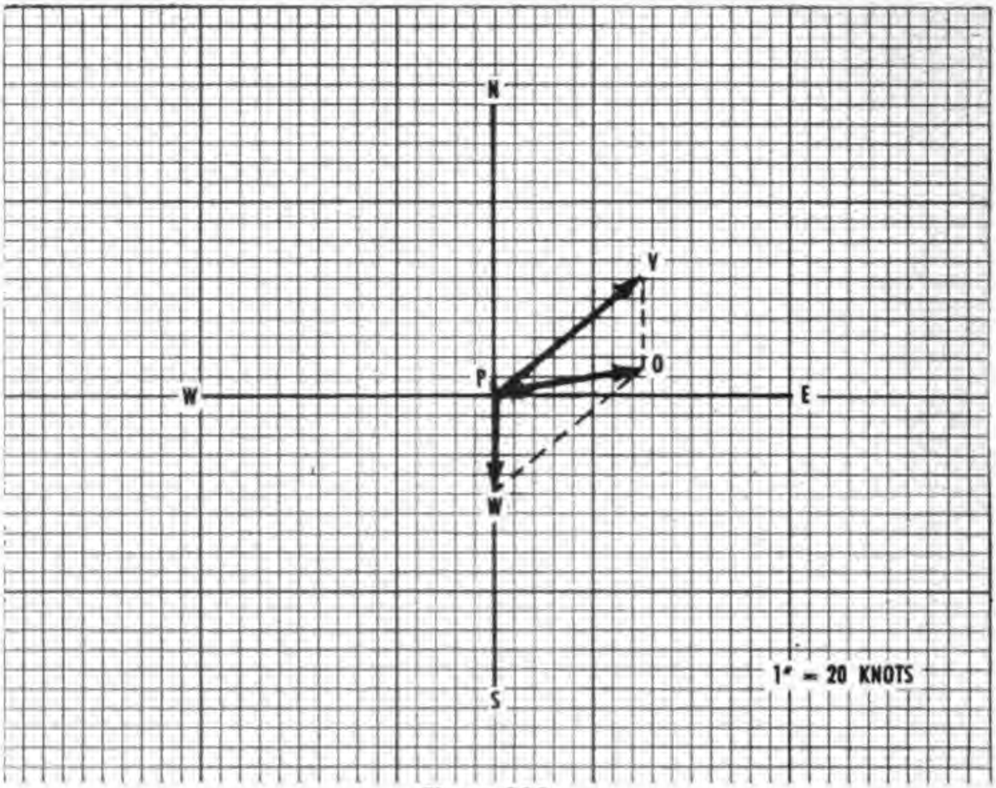

Figure 211.

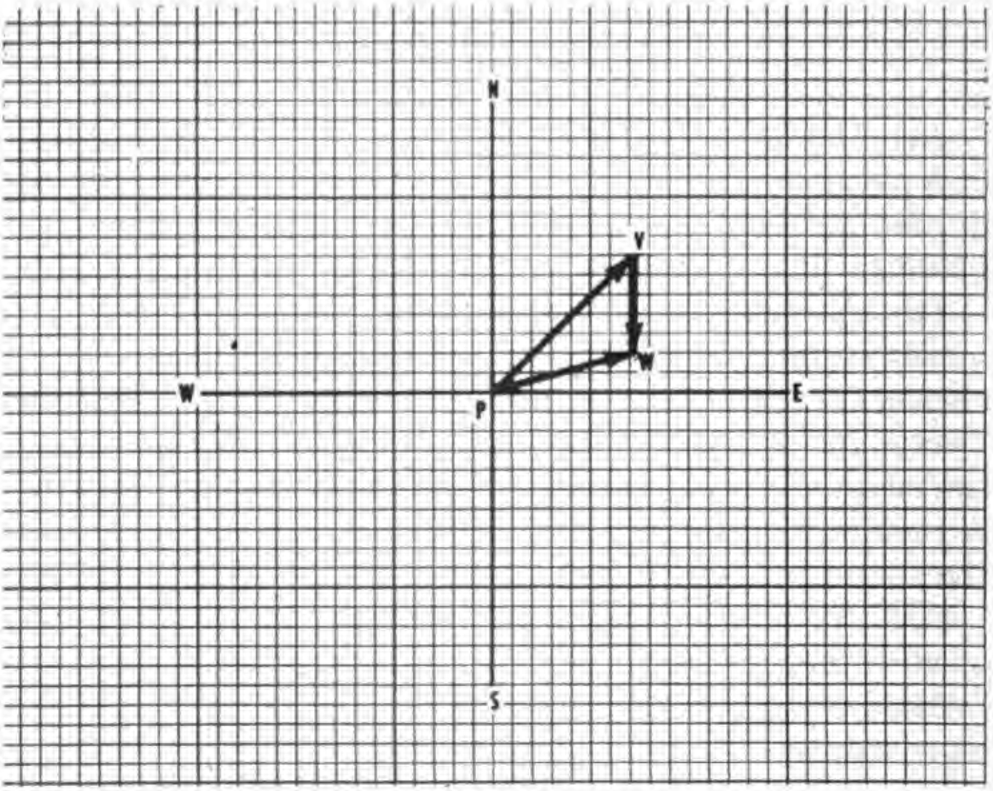

Figure 212.

V. Now, the vector VW, which is the other force acting on the ship, can be drawn from the end of the first vector or point V. The vector is drawn in its true direction. By connecting the beginning of the first vector and the end of the second, you obtain the resultant vector PW. By means of a protractor and ruler, you can prove that the resultant of figure 212 is equal both in magnitude and in direction to the resultant of figure 211. From the two examples given, you should be able to draw the following conclusion: Two forces acting upon a point or upon an object may be replaced by a single force called the resultant. The resultant force produces the same effect on the object as the joint action of the two forces. The simplest

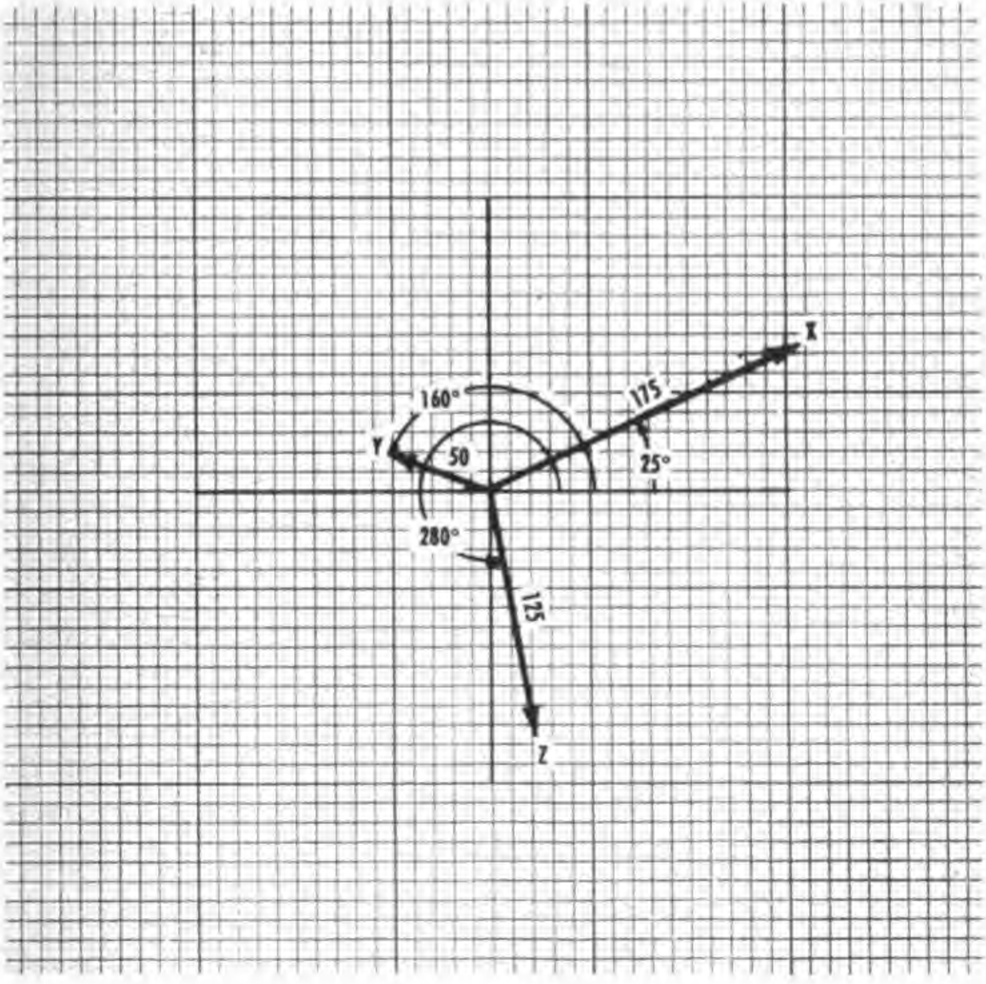

Figure 213.

method for the addition of vectors is by means of graphs. As shown in preceding paragraphs, there are two forms of the graphical solution for the resultant or sum of vectors. Refer to the following problem as an illustration of both forms. In each form the magnitude and direction of the forces involved are identical.

Parallelogram Method

284. Three forces—X, Y, and Z—are acting on point P, as shown in figure 213. Force X exerts 175 pounds at an angle of 25°, Y exerts 50 pounds at an angle of 160°, and Z exerts 125 pounds at an angle of 280°. What is the resultant force on point P? Solution: The resultant of vec-

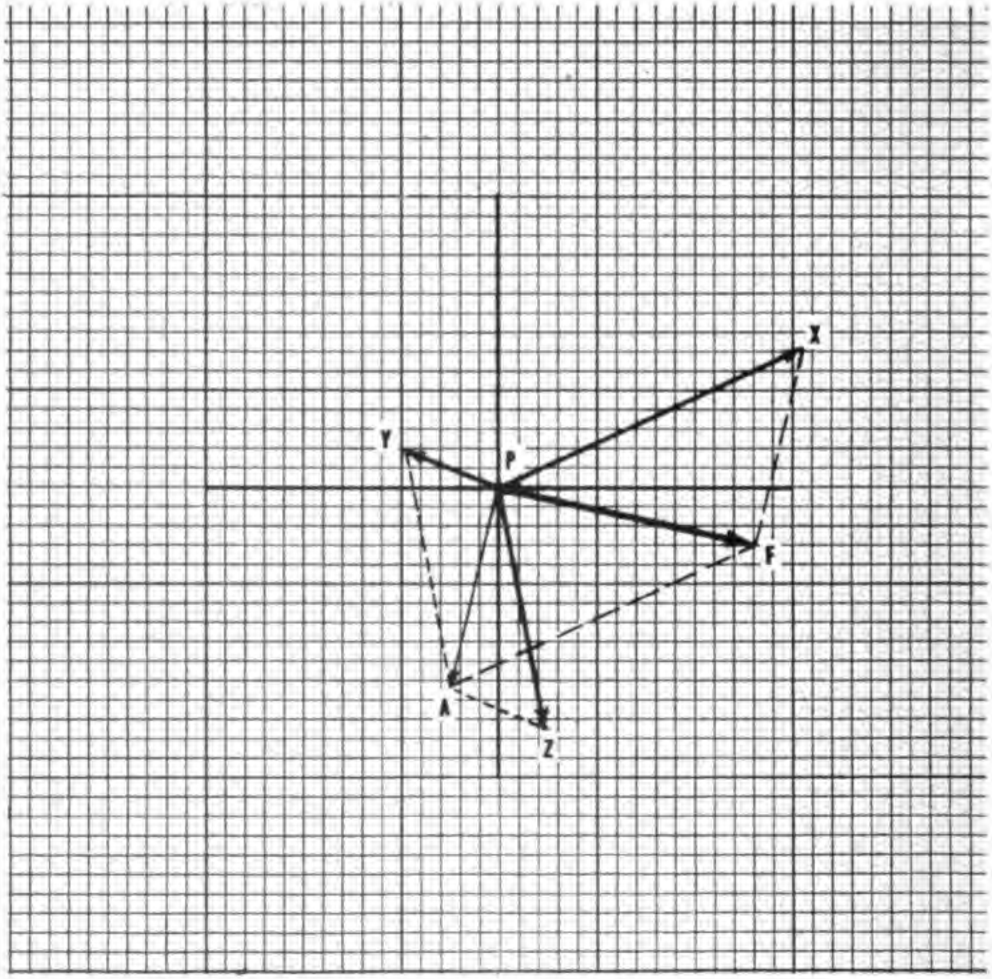

Figure 214.

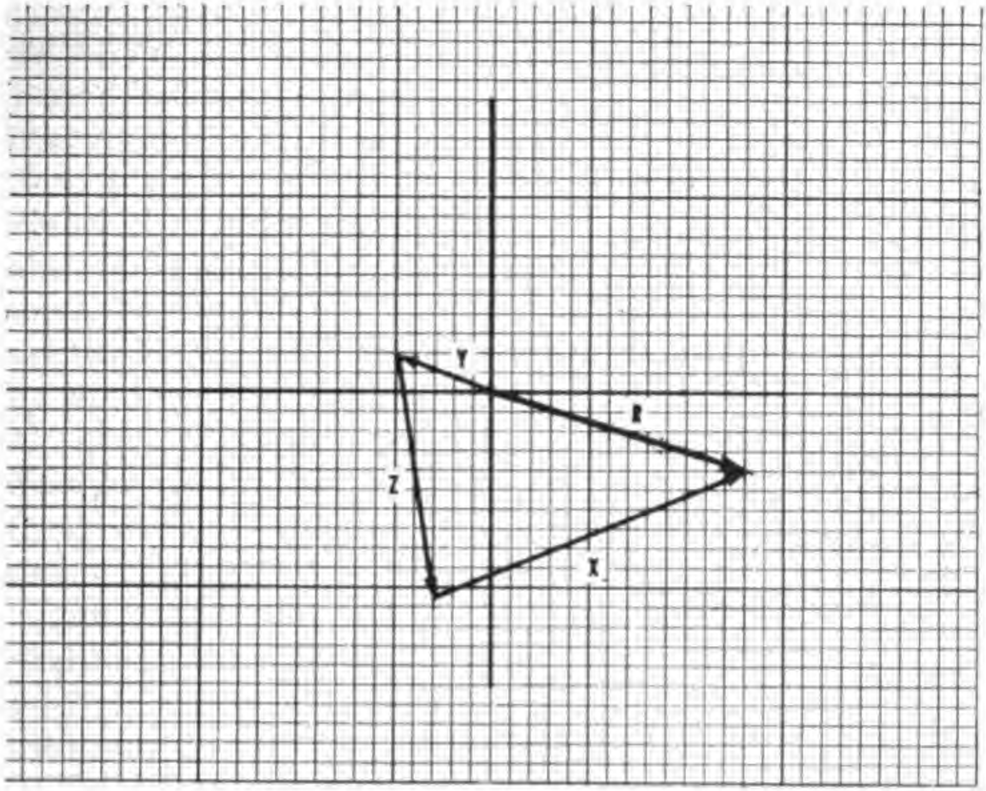

Figure 215.

tors X, Y, and Z can be found graphically by the parallelogram law for the addition of forces, as follows: Draw the vectors to scale. Find the resultant of any two vectors, such as Y and Z by constructing a parallelogram with PZ and PY as adjacent sides. (See fig. 214.) The resultant of PZ and PY is the diagonal PA of the parallelogram $PYAZ$. Effectively now, there are but two forces—PA and PX—acting upon point P. The resultant of these two forces is found by constructing another parallelogram with vectors PA and PX as adjacent sides. The final resultant force acting on point P, then, is the diagonal PF of parallelogram $PXFA$. Using a protractor and scale, you find the measurement of PF to be 138 pounds at an angle of 347°.

Polygon Method

285. Another way to solve the problem of figure 213 is as follows: As shown in figure 215, start by drawing any vector, say Y, to scale in the given direction. From the

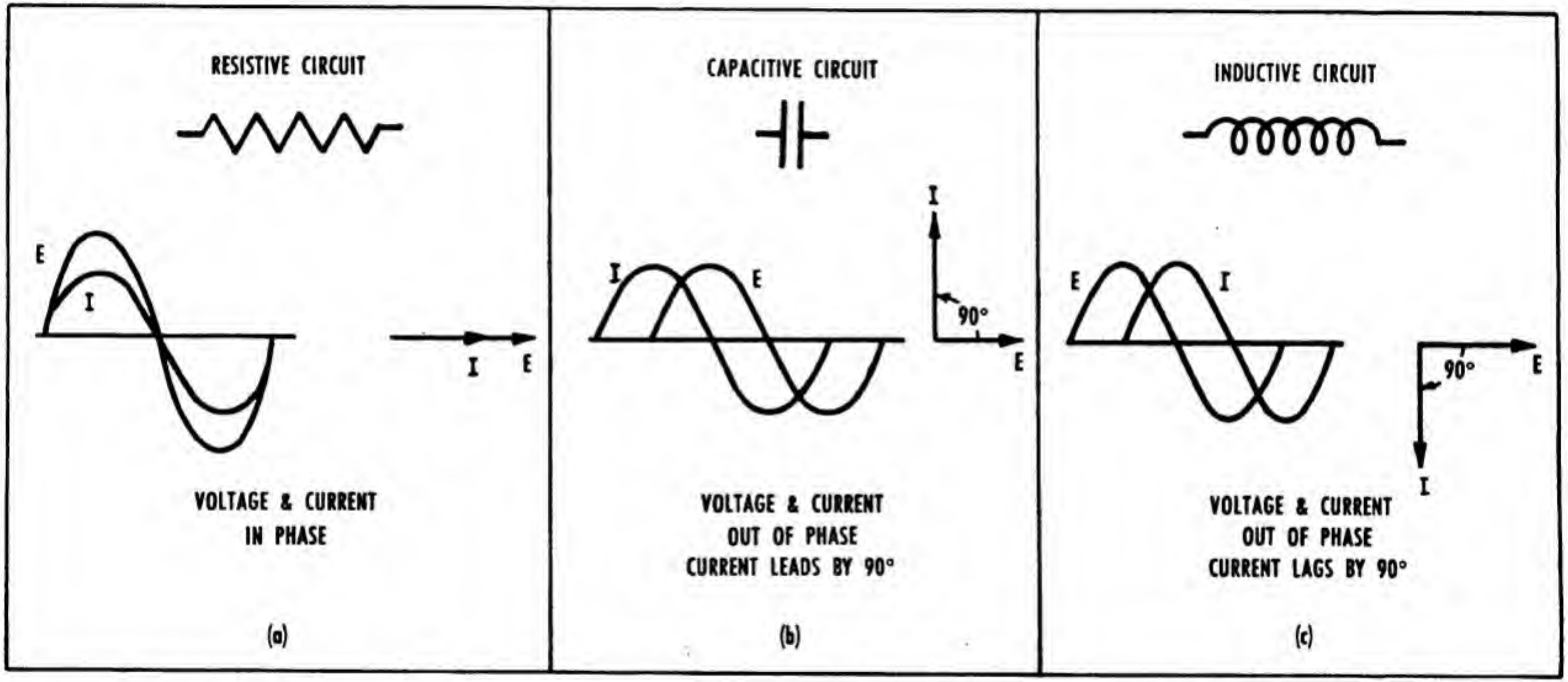
RESISTIVE CIRCUIT
E
I
I
E
VOLTAGE & CURRENT
IN PHASE
(a)
CAPACITIVE CIRCUIT
I
E
I
90°
E
VOLTAGE & CURRENT
OUT OF PHASE
CURRENT LEADS BY 90°
(b)
INDUCTIVE CIRCUIT
E
I
90°
E
I
VOLTAGE & CURRENT
OUT OF PHASE
CURRENT LAGS BY 90°
(c)

Figure 216.

end of vector Y, draw a second vector, say Z, carefully laying off its magnitude and direction. From the end of vector Z draw the last vector X with the proper magnitude and direction. Now, by connecting the end of vector X with the beginning of vector Y, you have the resultant vector R. Vector R, then, is the resultant of vectors X, Y, and Z. With a scale and protractor you find that the vector R is 138 pounds at an angle of 347°, the same answer as was obtained by the parallelogram solution. In solving by the short form, you may draw the vectors in any order as long as they are to scale and their direction is not changed. The final result is the same. By using either method, you may obtain the final resultant of any number of vectors.

286. In the introduction it was stated that the use of vectors is important in electronic circuit analysis. The need for vectors arises from the fact that in a practical alternating-current circuit the various voltages and currents are seldom in phase. If out-of-phase voltages or currents are to be added, the simplest method is by vectors.

287. Figure 216 shows the relationship of alternating voltage and current in the three components found in electronic circuits. To illustrate the use of vectors, consider the following examples:

288. In a series circuit made up of resistance and capacity (fig. 217, a), an alternating current of A amperes flows through both the resistance and capacity. Across the resistance the A amperes develop 50 volts in phase with the current; across the capacity the same current develops 100 volts, which lags the current by 90°. What is the total voltage across both the resistance and capacity? To find the answer to the above problem, the two voltages must be added together vectorially. In a series circuit the same current flows through both components. Its phases can therefore be used as a reference to determine the relation between the two voltages. Draw the current vector as the zero reference line, as in figure 217, b. No scale need be used on the current because it is used as a reference

only. Now, since the voltage across any resistance is in phase with the current, the voltage across the resistance can be shown as vector E_r, which is drawn in the same direction as I. The voltage across the resistance, E_r, is 50 volts, so it is drawn to a suitable scale, such as 10 volts = ¼″. The voltage across the capacitor lags the current through it by 90°. It then is represented by the vector E_c drawn at right angles below E_r and to a length corresponding to 100 volts, using the same scale. The voltage

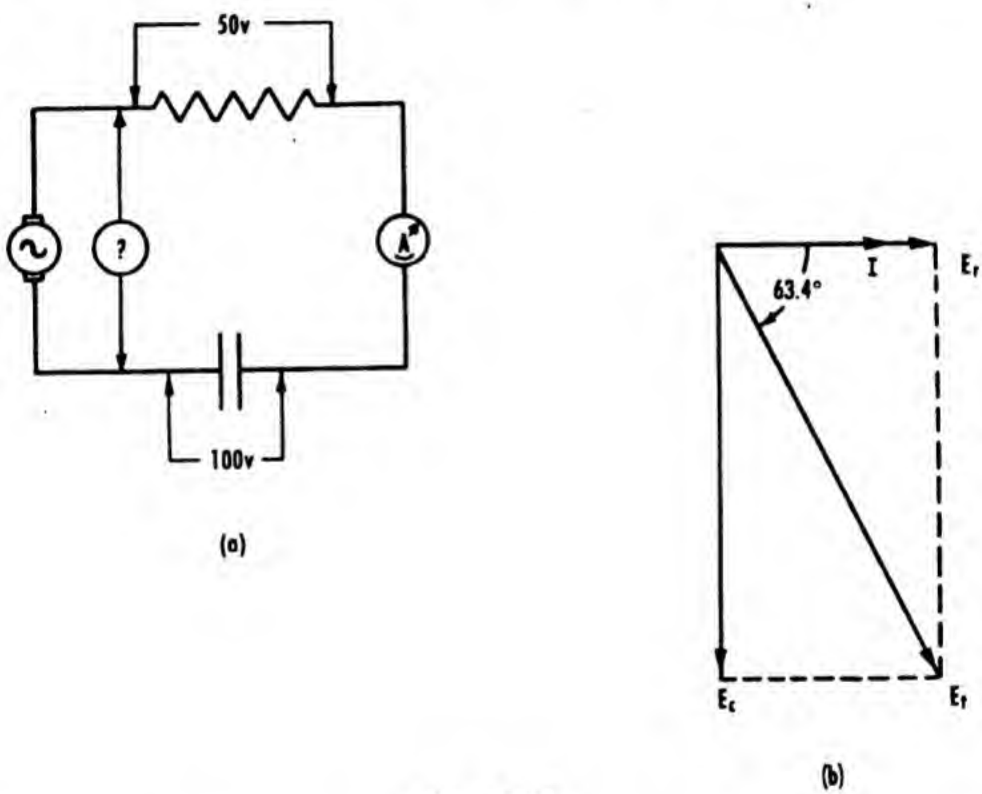

Figure 217.

across both components ($E_c + E_r$) can be found by applying the parallelogram law. The diagonal E_t represents the sum of the two voltages E_r and E_c. Using a protractor and scale, you find that the total voltage, E_t, is 111.8 volts, lagging the reference current by 63.4°. In a circuit, a voltmeter reading of E_t checks with this computed voltage.

289. In a series circuit made up of resistance and inductance, a current of A amperes develops 75 volts across

the resistance and 25 volts across the inductance. The voltage across the inductance leads the voltage by 90°. (See fig. 218, a.) Find the total (sum) voltage across both components, and its phase angle. The current, being the same

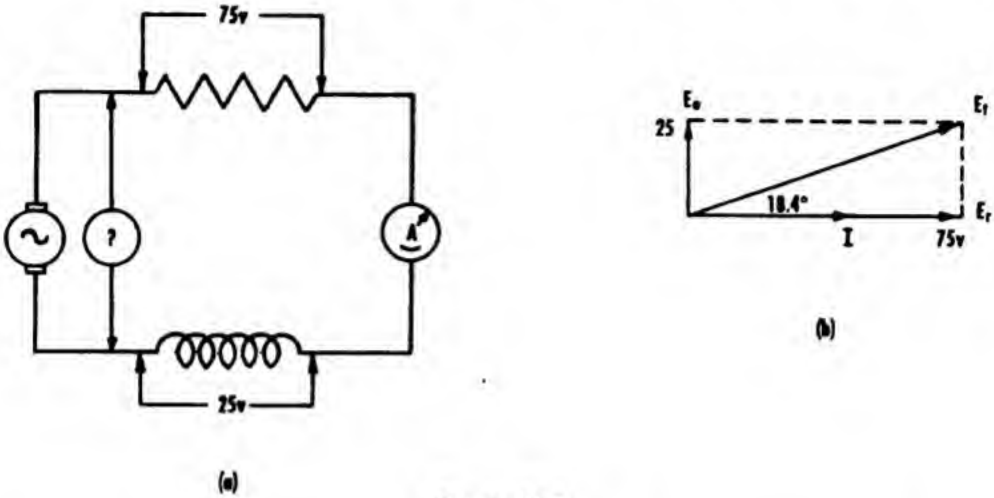

Figure 218.

through both components, can be used as a reference. (See fig. 218, b.) The 75 volts across the resistance is in phase with the current, A, so it is represented by vector E_r, drawn to a suitable scale. The voltage across the inductance leads the current by an angle of 90°, so it is represented by the vector E_l. The total (sum) voltage can be found by the parallelogram law, as shown by the dotted lines. The diagonal E_t represents the total voltage. By scale and protractor you can find that E_t is equal to 79 volts, leading the current by 18.4°.

290. Next consider a problem involving all three components. (See fig. 219, a.) Find the resultant voltage if a current of A amperes develops 100 volts across the resistance, 50 volts across the inductance leading the current by 90°, and 25 volts across the capacity lagging the current by 90°. Using the current as a reference, you can indicate the three vectors representing the three voltages (drawn to the same scale), as shown in figure 219, b. To find the resultant (sum), apply the parallelogram law again. (See fig. 219, c and d.)

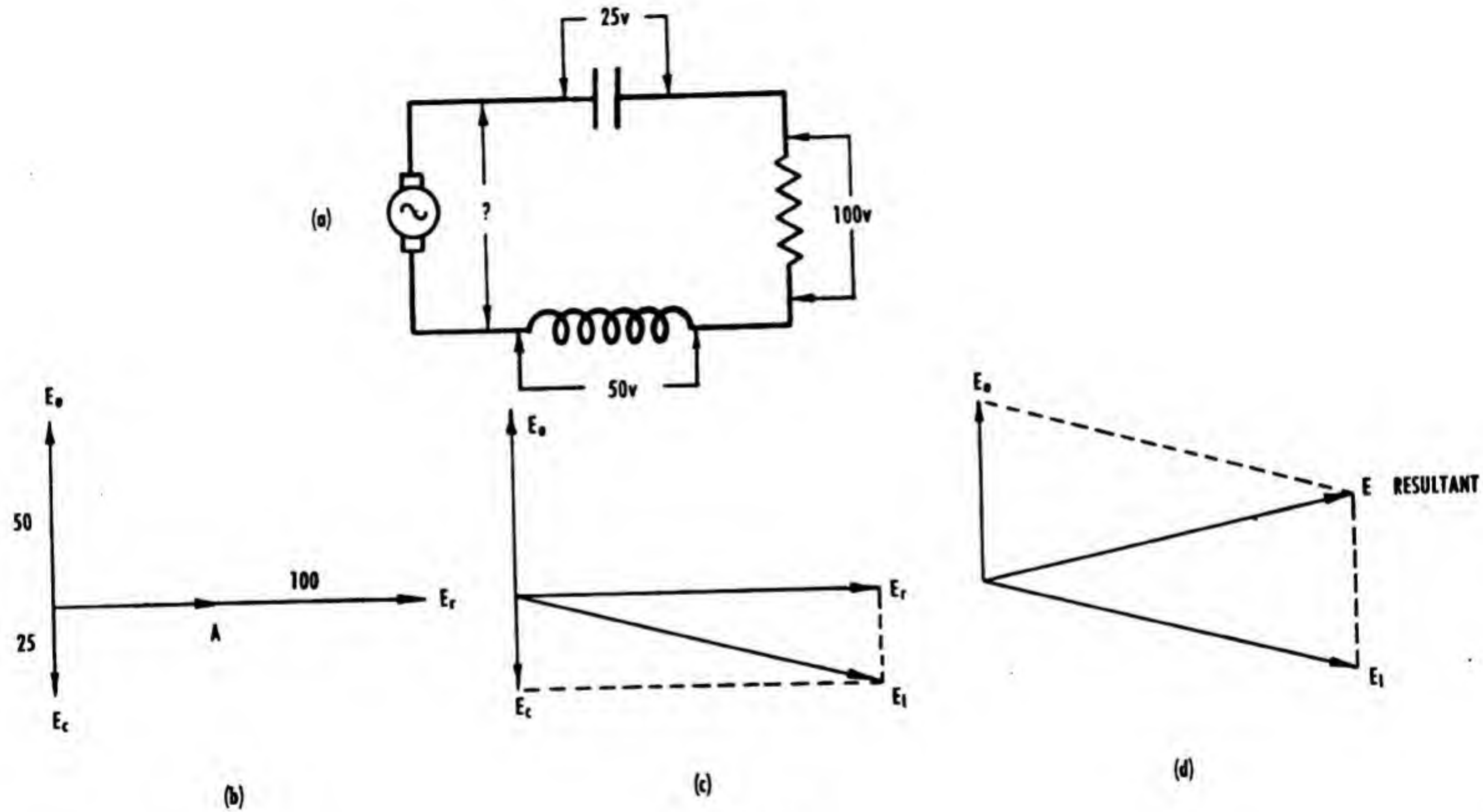
(a)
25v
?
100v
50v
(b)
E_a
50
25
E_c
A
100
E_r
(c)
E_a
E_r
E_c
E_l
(d)
E_a
E RESULTANT
E_l

Figure 219.

291. You will note that in figure 219, b, the voltage across the inductance (E_l) is acting in the opposite direction from the capacitor voltage E_c. In electrical terms you can say that E_l and E_c are 180° out of phase. An easier method of finding the resultant voltage (fig. 219, b) is to subtract E_c from E_l, since, as they are opposing one another, the larger overcomes the smaller. Taking the result of this subtraction and adding it vectorially to E_r by means of the parallelogram law, you obtain the resultant voltage. The final results of the two methods are identical. It should be remembered that the only time it is permissible to add or subtract the numerical value of vectors, is when they are in phase (for addition) and 180° out of phase (for subtraction).

SUBTRACTION OF VECTORS

292. The parallelogram method of subtraction is the reverse operation of addition — which was previously

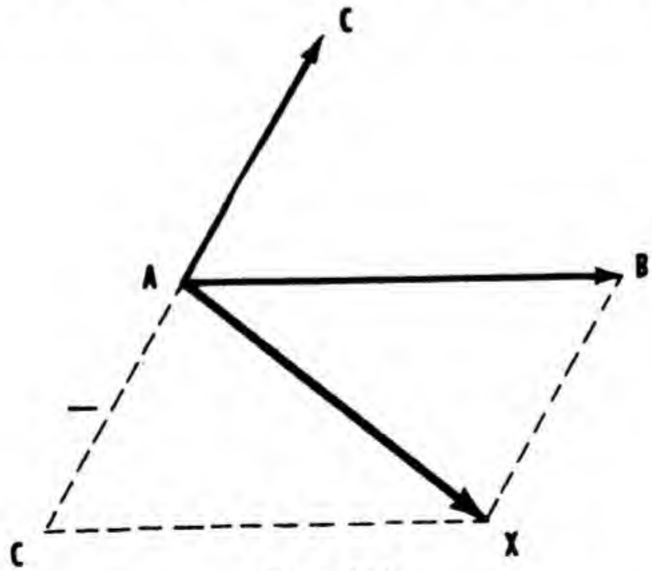

Figure 220.

described—with a slight modification. Consider the following problem: Subtract vector AC from AB (fig. 220). First, vector AC is reversed in direction, giving $-AC$. Then vectors $-AC$ and AB are added by the parallelogram method, giving the resultant AX which in this

problem is the difference. A simple check to verify the result is to add AX to AC. The sum or resultant will be identical with AB.

COMPONENTS OF A VECTOR

293. In the examples given previously, two or more vectors have been combined to form a single force. This process may be reversed. That is, a vector may be resolved into two components. When two components act at right angles to each other they are called RECTANGULAR COMPONENTS. This operation is used as a preliminary step when you add vectors by complex numbers. Any vector may be resolved into one component in line with the X-axis and another component in line with the Y-axis, which together may be written as a complex number. To add several vectors, which have been reduced to X- and Y-axis components, you add all X-axis components to find the X-axis component of the resultants, and add all the Y-axis components to find the Y-axis component of the resultant.

294. To find the rectangular components of a vector, consider the following example (refer to fig. 221). A coil with a combined resistance and inductance has a voltage drop across its terminals of 5 volts with a leading phase angle of 30° with reference to the current. What are the rectangular components? By placing the beginning of the vector at the origin of the X- and Y-axis, you can find the two rectangular components graphically and algebraically.

(a) Graphically the rectangular components of the vector OP can be found by dropping a perpendicular upon the X-axis from the endpoint P. This gives vector OA—which is the horizontal component of OP—and vector AP—which is the vertical component of OP. Notice that when the rectangle $OAPB$ is completely drawn, vector OB is also the vertical component of vector OP since it has the same length and direction as vector AP. Likewise, vector BP could be regarded as the horizontal component of vec-

tor *OP*. Numerical values of *AP* and *OA* may be obtained by measuring their lengths with the scale used to draw *OP*. The letter *j* is sometimes written in front of the numerical value of the vertical component to distinguish

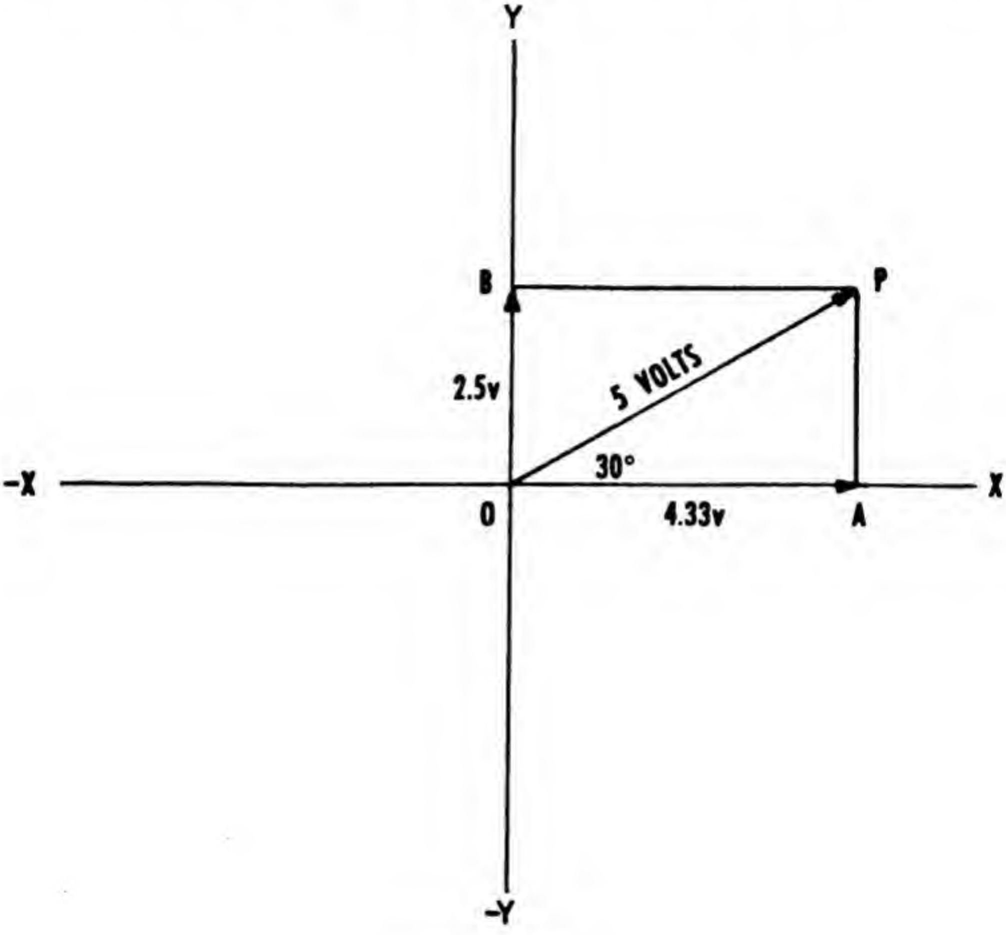

Figure 221.

it from the horizontal component. In basic arithmetic, you saw that a number multiplied by -1 rotates so that it is directed opposite from its original direction. This is a rotation of 180°. Multiplying by $\sqrt{-1}$ rotates the number 90°. In electronics, $\sqrt{-1}$ is represented by the letter *j*. Thus if *j* precedes a number, it is understood to be rotated a positive 90° from the reference or zero°. Likewise, a number preceded by $-j$ is to be rotated $-90°$ from the reference line.

(b) The horizontal and vertical components of the vector OP can also be found by solving a simple problem in trigonometry, as outlined in chapter 8.

$$\frac{\text{side adjacent}}{\text{hypotenuse}} = \text{cosine of the angle,}$$

$$\frac{OA}{OP} = \cos 30°,$$

$$OA = 5 \cos 30° = 5(0.866) = 4.33.$$

(Use the table of trigonometric functions in appendix D to find cos 30°.)

$$\frac{\text{side opposite}}{\text{hypotenuse}} = \text{sine of the angle,}$$

$$\frac{AP}{OP} = \sin 30°$$

$$AP = 5 \sin 30° = 5(\tfrac{1}{2}) = 2.5.$$

Checking with the Pythagorean theorem (chapter 7)—

$$5^2 = 4.33^2 + 2.5^2,$$
$$25 = 18.75 + 6.25,$$
$$25 = 25.$$

The description of vector OP by complex numbers is $\overrightarrow{OP} = 4.33 + j2.5$. Refer to chapter 10, *Essentials of Mathematics for Naval Reserve Electronics,* NavPers 10093, for a review of complex number notation.

295. The two foregoing methods of resolving a vector into its rectangular components may be summarized as follows—

(a) HORIZONTAL COMPONENT—Project the vector upon a horizontal line or multiply the cosine of the angle that the vector makes with the horizontal by the magnitude of the vector.

(b) VERTICAL COMPONENT—Project the vector upon a vertical line or multiply the sine of the angle that the vector makes with the horizontal by the magnitude of the vector.

296. If the resultant vector of two forces acting at right angles is to be found, the parallelogram method given in paragraph 282 may be used. The resultant may also be found by trigonometric means by solving for the hypotenuse of a right triangle.

VECTOR ALGEBRA

297. Any vector can be resolved into its rectangular components, and rectangular components can be expressed as a complex number. See chapter 10, *Essentials of Mathematics for Naval Reserve Electronics.* By making use of complex numbers and the operator *j*, voltage current and impedance vectors can be handled algebraically. This enables you to deal with circuit equations in general terms; it also helps you to simplify solutions. For example, a vector 5 units in length that operates at an angle of 53.1° may be expressed in terms of its rectangular com-

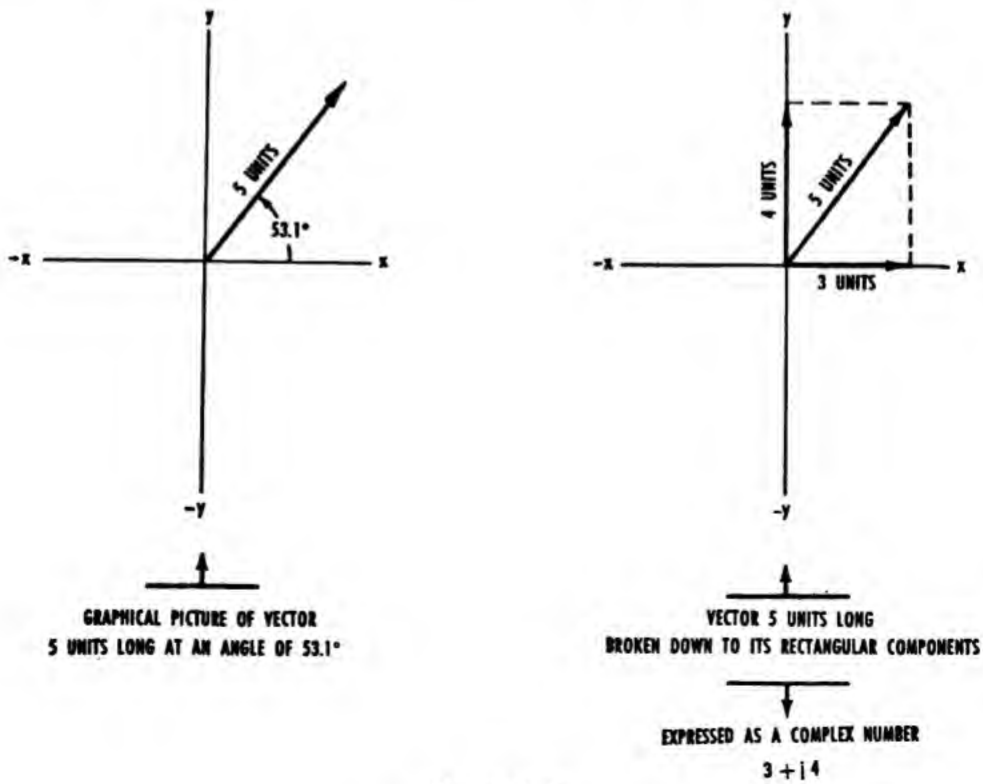

Figure 222.

ponents as the complex number $3 + j4$ (fig. 222). A COMPLEX NUMBER, then, is a means of expressing vectors that are in rectangular form. As stated in chapter 10, *Essentials of Mathematics for Naval Reserve Electronics*, complex numbers may be added or subtracted by treating them as ordinary binomials.

Example: ADD $5 + j4$ and $2.3 - j7.2$

SOLUTION

$$\begin{array}{r} 5.0 + j4 \\ 2.3 - j7.2 \\ \hline 7.3 - j3.2 \end{array}$$

Example: SUBTRACT $7.4 + j9.2$ from $29.2 - j17.6$

SOLUTION

$$\begin{array}{r} 29.2 - j17.6 \\ 7.4 + j9.2 \\ \hline 21.8 - j26.8 \end{array}$$

CHAPTER 10

CALCULUS—INTEGRATION AND DIFFERENTIATION

The mathematical methods that you have covered so far have been extremely useful tools in the solution of problems in the science of electronics. It would be impossible to analyze circuits and circuit components that are seen in our daily work without the use of mathematics. The first step that you learned in mathematics was the counting processes—addition, subtraction, multiplication, and division. Operations were performed with real and complex numbers. Real numbers comprise rational and irrational numbers. Rational numbers include integers, common fractions, and decimals which can be expressed as common fractions. The second step was the learning of algebra and the use of symbols denoting unknown quantities. The third step was the study of geometry, including the important Pythagorean theorem, which you used in calculating relations between voltages and between impedances. The fourth step was the learning of simple trigonometry.

Now, another step forward in the field of mathematics brings you to calculus. As an electronics technician you will read many advanced technical articles that contain references to either differential or integral calculus. In the explanation of phenomena whose rate of change depends on time, it is convenient to use calculus. Design computations involving the action of simple electronic circuits such as integrators and differentiators can be made most easily by use of differential or integral calculus. For these reasons a few facts about calculus will increase your knowledge of technical literature in general and the action of electronic circuits in particular. How-

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ever, it is not necessary for you to learn how to work problems by means of calculus.

The word CALCULUS means "a process of reasoning by the use of symbols." You are already familiar with this process of reasoning through the use of symbols—for example, $2\frac{1}{2}x$, $h^2 = a^2$, area $= bh$, and $\sin x = \dfrac{1}{\operatorname{cosec} x}$ are all symbols that are used to convey certain information to us. Each of these symbols belong to a branch of mathematics you have studied, and each is a calculus. The calculus with which you are now to become acquainted is integral and differential calculus. INTEGRAL CALCULUS is a process of calculating sums or adding up a large number of small things. DIFFERENTIAL CALCULUS, as the name implies, is a process of calculating by using differences or by breaking up a curve into a large number of small elements. As you will see, the former is the opposite of the latter—that is, in integral calculus small parts are added to get a curve or an area, whereas in differential calculus a curve is broken into small pieces to find the slope of the curve at any point.

FUNCTIONS

298. Before attempting to apply the methods of calculus, let us examine the ideas of functions and limits. These two ideas are the foundations on which modern mathematics is built. Since calculus is a part of modern mathematics, the importance of examining these ideas is at once apparent. First, look at a simple algebraic function. In algebra you became familiar with expressions like $x^2 + 2x - 3$. You know that this expression has no numerical value until a particular value is assigned to x. The value of this expression, then, is a FUNCTION of x, which is written $f(x)$ and read f of x. This means that the value of the expression depends on the value given to x. The equation is then written—

$$x^2 + 2x - 3 = f(x).$$

The $f(x)$ is merely a symbol that says that the value of the expression depends on the value that is given to x. For example, assign the value $x = 2$. By substitution,

$$2^2 + 2(2) - 3 = 5.$$

Thus, when x is 2 the function of x equals 5. This is written $f(2) = 5$, which means that $f(x)$ equals 5 when x is 2. In like manner, if $x = 3$, then

$$x^2 + 2x - 3 \text{ becomes } 3^2 + 2(3) - 3, \text{ or } 12.$$

Therefore, $f(3)$ is 12, or $f(x)$ is 12, when x is 3. By assigning any value to x and substituting this value in the expression, you can find the corresponding value of the function.

299. Find the values for $f(1)$, $f(-2)$, $f(-3)$, $f(4)$, and $f(5)$ in the following problems, which illustrate the method of computing values of functions:

(*a*) $2x^2 + 3x - 5 = f(x)$.

(*b*) $y^2 + 2y - 3 = f(y)$.

(*c*) $3z^2 + 4z - 7 = f(z)$.

Answers:

(*a*) $f(1) = 0; f(-2) = -3; f(-3) = 4; f(4) = 39; f(5) = 60$.

(*b*) $f(1) = 0; f(-2) = -3; f(-3) = 0; f(4) = 21; f(5) = 32$.

(*c*) $f(1) = 0; f(-2) = -3; f(-3) = 8; f(4) = 57; f(5) = 88$.

VARIABLES

300. In calculus you will see the expression $y = f(x)$ many times. Since you are interested in what happens to y as x changes, you can let x take on several different values and see what y becomes. Therefore, x in the formula is known as the INDEPENDENT VARIABLE, because you can assign any value you wish to it. The y is known as the DEPENDENT VARIABLE because its value depends on whatever value is given to x. What the equation $y = f(x)$ actually says is that the value of y depends on what value

you assigned to x. You can say the same thing by saying y is a function of x.

301. Some common functions that you are probably familiar with are shown in figure 223. Figure 223, a, shows the graph of the function $y = 3x$, which is a straight line. In figure 223, b, you see the graph of the function

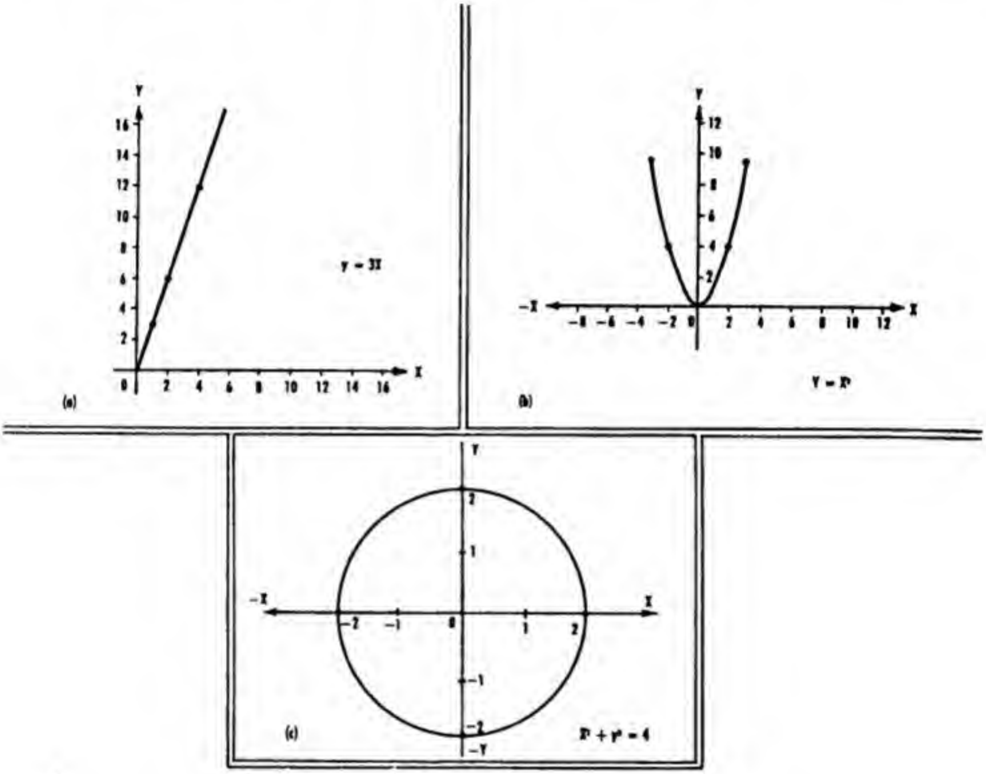

Figure 223.

$y = x^2$, which is a PARABOLA. Figure 223, c, pictures the function $x^2 + y^2 = 4$, which is a circle with a radius of 2. You can verify these facts quickly by choosing any value for the independent variable x and calculating what the dependent variable y is.

LIMITS

302. The idea of limits is as simple as that of functions, and it is time well spent to learn something about limits. Let us look at a SIMPLE LIMIT. Take the integer 1 and

divide it by 2. The result is $\frac{1}{2}$. If you divide by 2 again and again, you get $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and so on; each succeeding number is obtained by multiplying the preceding number by $\frac{1}{2}$. However, no matter how many times you divide the number, you can never reach zero. The best you can ever hope to do is to get very close to it. As the denominator becomes larger and larger, the fraction becomes smaller and smaller.

303. Although you cannot say that the fraction ever equals zero, you can say that as the denominator n becomes very large, the fraction $\frac{1}{n}$ APPROACHES zero as a limit. In other words, zero is the limit of $\frac{1}{n}$ as n becomes very large. An example of the use of this concept of a limit is in finding the length of the circumference of a circle by using an inscribed regular polygon. Suppose you inscribe a square in a circle, as shown in figure 224. You notice immediately that the sum of the sides of the square is much shorter than the circumference of the circle. Now inscribe an eight-sided regular polygon in the circle, as shown in figure 225. The difference in length between the sum of the sides of the polygon and the circumference of the circle is smaller. Notice that as the number of sides of the polygon is increased, this difference becomes smaller and smaller. When the inscribed regular polygon has a great many sides, the perimeter of this figure nearly equals the circumference of the circle. Actually the perimeter of the polygon and the circumference of the circle can never be equal, but their difference can be made as small as you please by using a polygon with enough sides. You can say, therefore, that the circumference of the circle is the LIMIT that the perimeter of the polygon approaches as the number of sides of the polygon increases.

304. A small change or DIFFERENTIAL is used in differential and integral calculus. Let us see just what a differential is. If there is a small change in a quantity x, then the

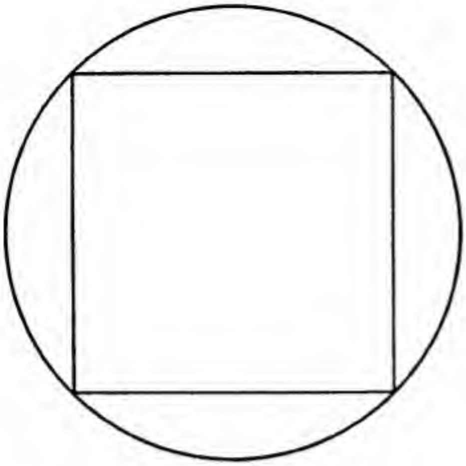

Figure 224.

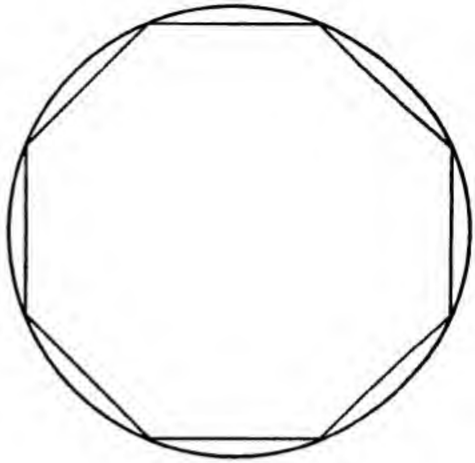

Figure 225.

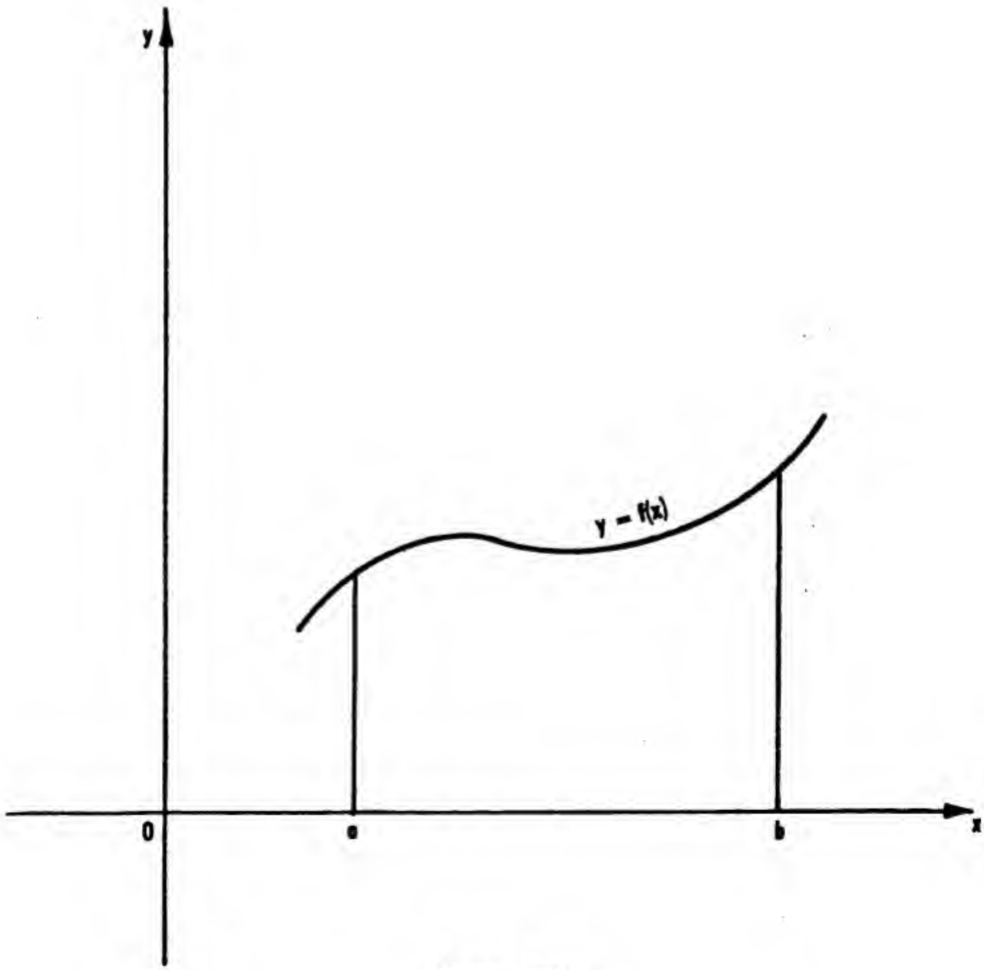

Figure 226.

quantity x has changed by the amount DELTA x, which is written Δx. If the change (Δx) is infinitely small, it is indicated by the symbol dx, called DIFFERENTIAL of x. Therefore, dx means an infinitely small change in x. Now that you know the language of functions, limits, and small changes, you can learn how they are used.

THE INTEGRAL

305. The basic concept in calculus is that of the integral. It implies a process of adding small quantities to get the total sum. Figure 226 is a curve represented by a function of x, that is, $y = f(x)$. Suppose you wish to find the area under the curve from the point $x = a$ to the point $x = b$. You can get an approximate value for the area by dividing it into a number of small rectangles, as in figure 227. The width of each rectangle is a small change in x, or Δx. The height of each rectangle is y. Therefore, the area of each rectangle is y times Δx, or $y(\Delta x)$. By assigning the proper values to y and Δx, you get the area of each rectangle. You can then add these small rectangles and get an APPROXIMATE value for the area under the curve. However, by examining figure 227, you will note that you cannot get the exact value because the top of each rectangle does not coincide with the curve. If you make the width of each rectangle narrower, as in figure 228, this difference becomes smaller. It is at this point that you use the principle of limits.

306. In figure 227 the area under the curve is divided into 8 rectangles. In figure 228 the same area is divided into 16 rectangles. If you double the number of rectangles, the difference between the area of all the rectangles and the area under the curve is approximately cut in half. If you divide the area into a larger and larger number of rectangles, this difference becomes very small. However, from the previous discussion on limits you know that regardless of how many times the difference in area is divided by two it can never be reduced to zero. You can

say then that the sum of the areas of the rectangles approaches the area under the curve as narrower rectangles are used. In other words, the area under the curve is the

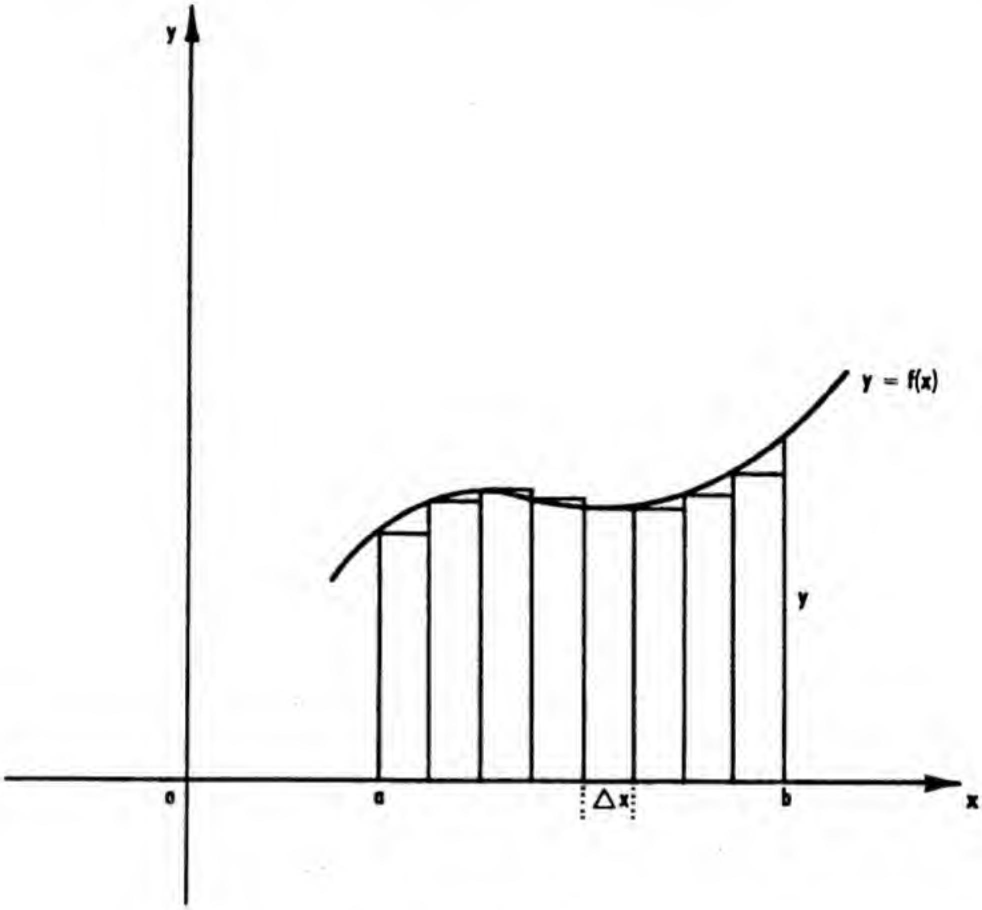

Figure 227.

limit of the sum of the areas of the rectangles as the rectangles become smaller and smaller. This idea can be expressed with the aid of the following symbols—

$$A = \int_a^b y\,dx.$$

The integral sign $\int$ is merely a distorted letter S, which indicates sum. The term $y\,dx$ is simply the area of the rectangle of height y and of width dx. The a and b on

the integral sign indicate that you are taking rectangles from the point $x = a$ to the point $x = b$. The letters b and

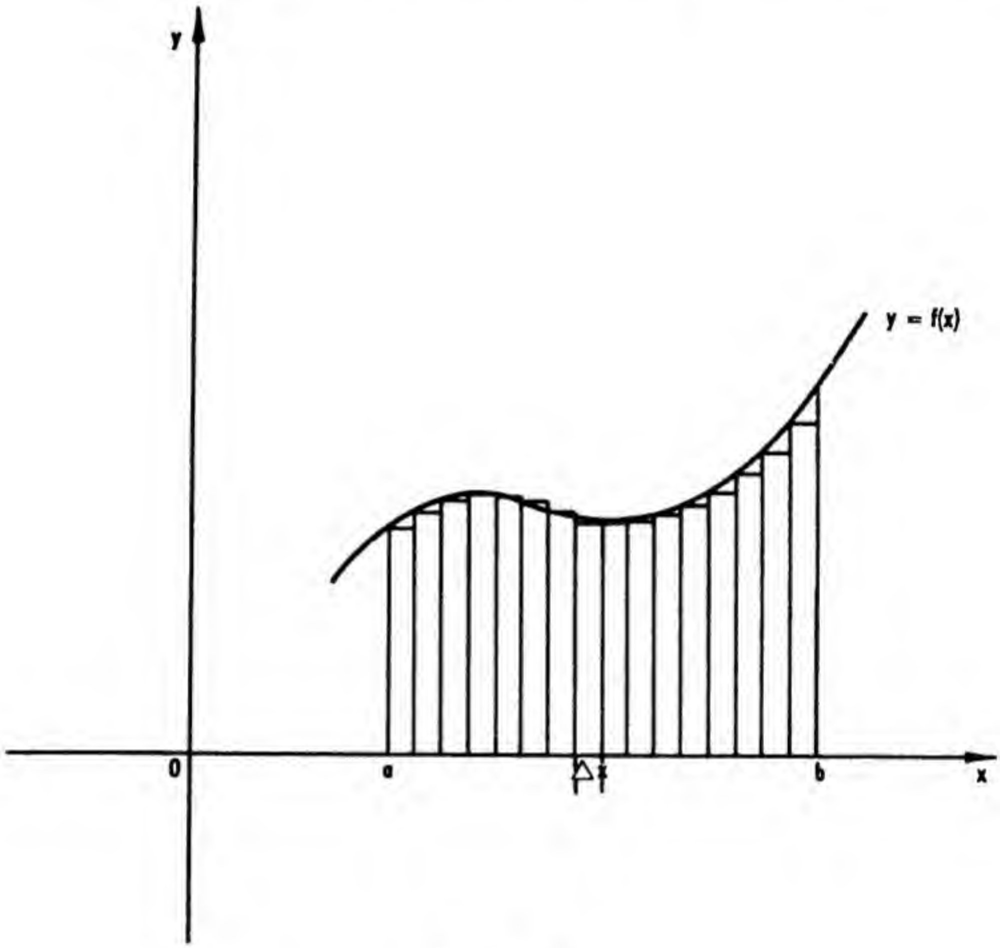

Figure 228.

a are called the UPPER LIMIT and LOWER LIMIT, respectively. Thus, the term $\int y\, dx$ simply means the sum of a large number of rectangles, each of which has an area $y\, dx$.

CALCULATING AREAS BY INTEGRATION

307. We shall now use the method of adding small quantities (integration) to find the area under a curve. Figure 229 is a graph of a straight line. The equation of the line $y = 3x$, which is simply verified by taking a couple

of values for x and calculating y. The area under the line from $x = 0$ to $x = 6$ is the same as the area of a triangle with a base equal to 6 units and a height of 18 units. As you know, the area of a triangle is one-half the height

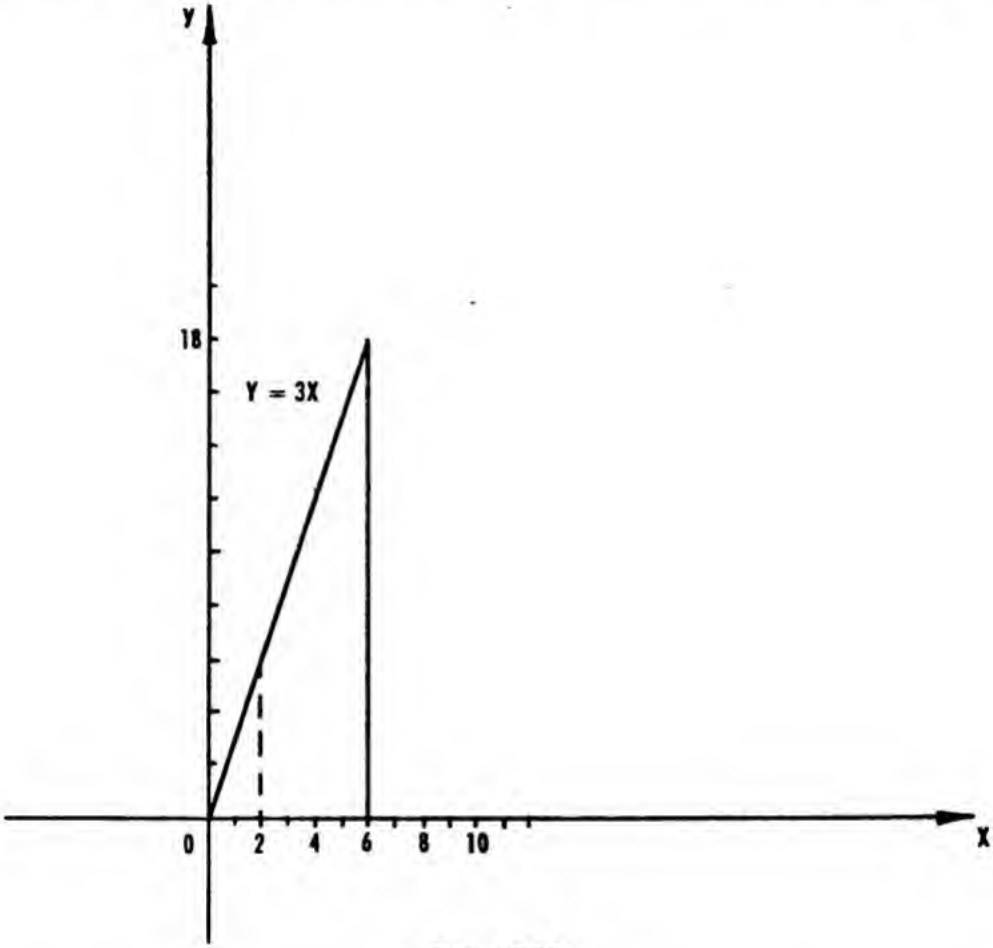

Figure 229.

times the base, which equals ½ (6) (18), or 54 square units. You can obtain this area also by integration. The formula for area is—

$$A = \int y\,dx.$$

308. Since you wish to obtain the area from $x = 0$ to $x = 6$, you can put the upper and lower units on the integral, thus—

$$A = \int_0^6 y\,dx.$$

The formula for y is—

$$y = 3x$$

Therefore—

$$A = \int_0^6 3x\,dx = 3\int_0^6 x\,dx.$$

You can look in a table of integrals and find that the integral of $x\,dx$ equals $\frac{1}{2}x^2$. (You will have to accept this without explanation for the time being.) Thus—

$$A = 3[\tfrac{1}{2}x^2]_0^6$$

Next find the value for the upper limit, which is $3(\frac{1}{2})(6^2)$ or 54, and subtract from it the value for the lower limit, which is $3(\frac{1}{2})(0)$ or 0. Therefore, the area is $54 - 0$, or 54 square units. If you wish to find the area under the curve from $x = 2$ to $x = 6$, the only difference is that you use 2 instead of zero as the lower limit. Thus—

$$\begin{aligned} A &= \int y\,dx \\ &= \int_2^6 3x\,dx \\ &= 3\int_2^6 x\,dx \\ &= 3[\tfrac{1}{2}x^2]_2^6 \\ &= 3(\tfrac{1}{2})(36) - 3(\tfrac{1}{2})(4) \\ &= 54 - 6 \\ &= 48 \text{ square units.} \end{aligned}$$

309. You probably are wondering (1) why integration is used instead of a formula for area and (2) where the integration formula $\int x dx = \frac{x^2}{2}$ comes from. In the example illustrated by figure 229, you found the area of a triangle. Although the formula for the area under a triangle is simple, the formula for areas under more com-

plicated curves, such as trigonometric curves, is not so simple. In most problems involving areas integration is the simplest method to use.

TABLES OF INTEGRALS

310. You can use integration only when you know the integral. Tables of integrals list the many integrals that have been solved by mathematicians. For example, a table will have in it—

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}, \text{ where n = any integer.}$$

This tells you that if $n = 0$, then—

$$\int dx = x.$$

If $n = 1$, then—

$$\int x \, dx = \frac{x^2}{2}.$$

If $n = 2$, then—

$$\int x^2 \, dx = \frac{x^3}{3}.$$

If $n = 3$, then—

$$\int x^3 \, dx = \frac{x^4}{4}.$$

By using these integrals, you can integrate the following expression:

$$\begin{aligned}
&\int (3x^3 - 3x^2 - 2x + 3)dx \\
&= 3 \int x^3 \, dx - 3 \int x^2 \, dx - 2 \int x \, dx + 3 \int dx \\
&= 3\left(\frac{x^4}{4}\right) - 3\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) + 3x \\
&= \frac{3x^4}{4} - x^3 - x^2 + 3x.
\end{aligned}$$

This is the integrated form of the original expression. If you wanted the area under a particular part of the curve $y = 3x^3 - 3x^2 - 2x + 3$, you would merely find the values

of the integrated form for the upper and lower limits and subtract. Tables of integrals also list integrals of trigonometric functions. For example—

$$\int \sin x\, dx = -\cos x,$$
$$\int \cos x\, dx = \sin x.$$

AREA UNDER A SINE WAVE

311. Let us use the last integral to find the area under a sine wave, which is a problem of practical importance to electronics technicians. Figure 230 is the graph of a sine wave, that is, $y = f(x) = \sin x$. Find the area under one-quarter of a cycle—from $x = 0°$ (0 radians) to $x = 90°$ $\left(\frac{\pi}{2} \text{ radians}\right)$.

The integral for an area is—

$$A = \int y\, dx.$$

The limits are 0 and $\frac{\pi}{2}$ radians. Therefore—

$$A = \int_0^{\frac{\pi}{2}} y\, dx.$$

The equation for the curve is $y = \sin x$. Therefore—

$$A = \int_0^{\frac{\pi}{2}} \sin x\, dx.$$

From the table you know that $\int \sin x\, dx = -\cos x$. Therefore—

$$A = -\left(\cos x\right)_0^{\frac{\pi}{2}}.$$

You also know that the $\cos 90° = 0$, and that the $\cos 0° = 1$. Therefore—

$$A = -(0 - 1) = 1 \text{ square unit.}$$

312. It is a well-known fact among mathematicians that, in practice, relatively few functions are integrable

by formulas. In view of this fact, several methods have been devised to compute the area under the curves of complex functions. One of these methods is to compute the area by measuring the length of the closed curve that

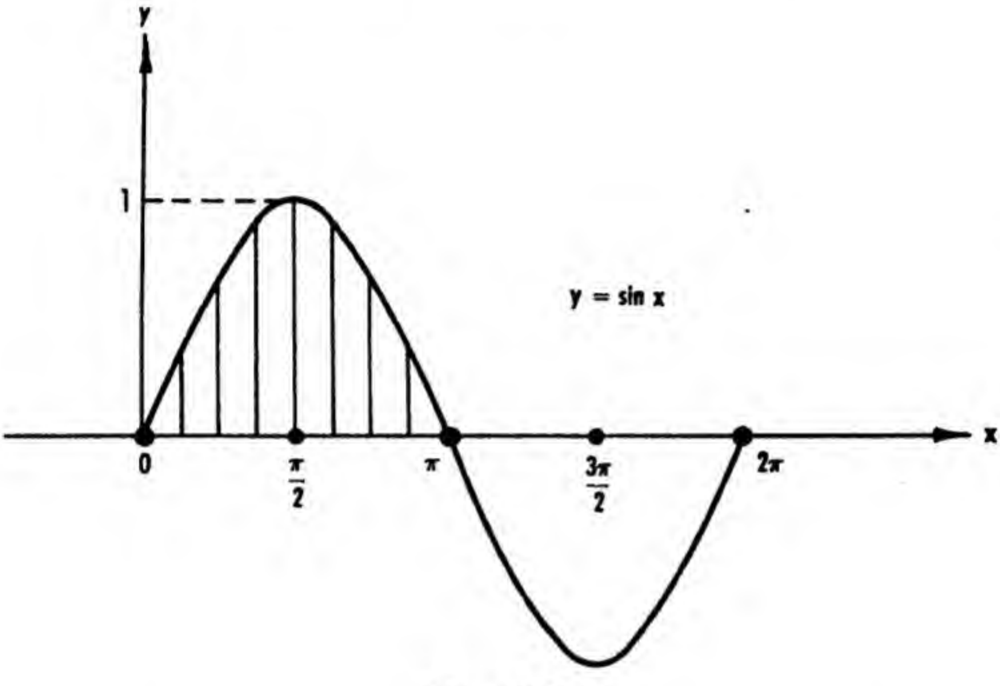

Figure 230.

encloses the area. This is done with an instrument known as a PLANIMETER. Another practical integration method is to use a thin metal or paper sheet of known weight per unit area. After drawing the curve accurately, you cut out and weigh the area within the curve that is to be measured, and since you know the weight per unit area of the paper or thin metal sheet, you can accurately compute the area under the curve. Still another integrating method is to use electronic circuits.

ELECTRONIC INTEGRATION

313. Figure 231 illustrates an electronic integration circuit. Integrating circuits are used a great deal in modern electronic equipments such as radar, loran, shoran,

sonar, and television. The integrator circuit gives an output voltage that is proportional to the area under an input voltage curve.

314. An ingenious device, called the electronic ANALOGUE COMPUTER, has been developed for the solution of

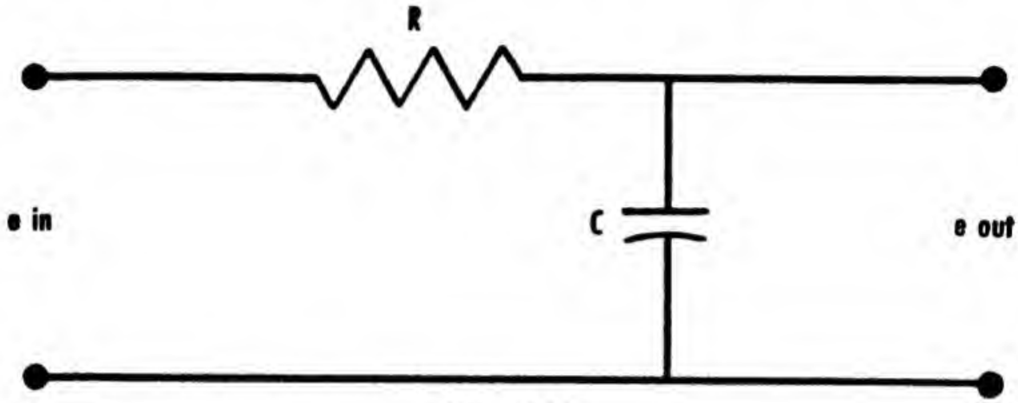

Figure 231.

higher-order equations. This device depends on the use of simple electronic integrating circuits to solve extremely complex equations. Integrating circuits are perhaps most commonly used in the sweep generator circuits of oscilloscopes. The integrating action of these circuits is obtained by using large values of resistor and capacitor. The large RC (resistor capacitor) constant enables the capacitor to charge linearly over a portion of its charging time and thus to provide a linear sweep.

DIFFERENTIAL CALCULUS

315. You now come to that branch of mathematics known as the DIFFERENTIAL CALCULUS. This branch of mathematics was discovered more than a hundred years after the discovery of the process of integration. The relation between these two branches of mathematics has already been pointed out, but it bears repeating. The integral calculus is a process for adding small quantities, whereas the differential calculus is a process using small differences. After learning some of the terms of the dif-

ferential calculus, you will examine the relation between these two branches more fully. You will see that they are exactly inverse procedures.

316. Differential calculus may be used to find the SLOPE of a curve. Some algebraic functions may be represented by curves. If you have a curve with x and y as coordinates of any point on the curve, and if the value of y depends on the value of x, you can say that $y = f(x)$. You will learn now that the slope of the curve $y = f(x)$ is an important and useful tool.

SLOPE OF A STRAIGHT LINE

317. When the curve is a straight line, as in figure 232, it is very easy to find the slope. The slope, which is the same anywhere on the curve, is simply the tangent of the

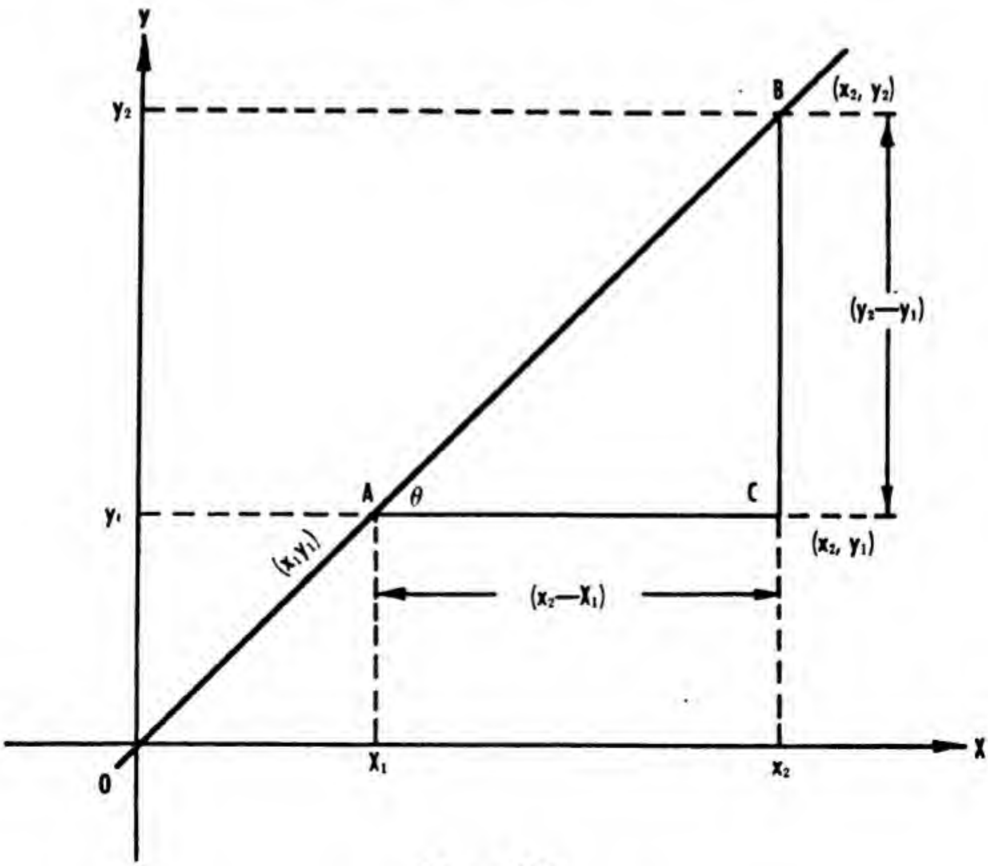

Figure 232.

angle Θ. You know the formula for the tangent of an angle of a right triangle—it is the length of the opposite side divided by the length of the adjacent side. In figure 232 the coordinates of point A are given as x_1 and y_1, and those of point B are given as x_2 and y_2. Since point C has the same y coordinate as point A, and the same x coordinate as point B, its coordinates are x_2 and y_1. The length of side BC, therefore, is $y_2 - y_1$, and the length of side AC is $x_2 - x_1$. The tangent of angle Θ is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

318. If the curve $y = f(x)$ in figure 232 is $y = 3/5x$, and if the right triangle ABC is drawn as shown, $y_2 - y_1$ is three units and $x_2 - x_1$ is five units. Therefore, $\tan \Theta = 3/5$. It doesn't matter where or how the right triangle is drawn, the ratio

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

always has the same value, $3/5$. Therefore, the slope of a straight line is constant.

SLOPE OF A CURVE

319. The curve in figure 233 is not a straight line. One way to find the slope of the curve at a point is to draw a tangent to the curve at that point and then calculate the slope of the tangent. When the curve is a straight line (fig. 232) the tangent and the curve coincide; but when the curve is not a straight line (fig. 233) the tangent and the curve do not coincide. However, the slope of the tangent to the curve at any point is the slope of the curve at that point. You will see that the value of the DERIVATIVE at any point is the slope of the curve at that point.

320. Figure 233 shows the curve $y = f(x)$. For the present you do not need to know the function representing the curve. Suppose that you wish to find the slope of the curve at point P_1. You know that the slope of the curve

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at P_1 is the slope of the tangent at P_1, but you will find it hard to draw an exact tangent at any one point. Therefore, start with the chord P_1P_2 between point P_1 and another point P_2 on the curve. If the coordinates of P_1 are x_1 and y_1,

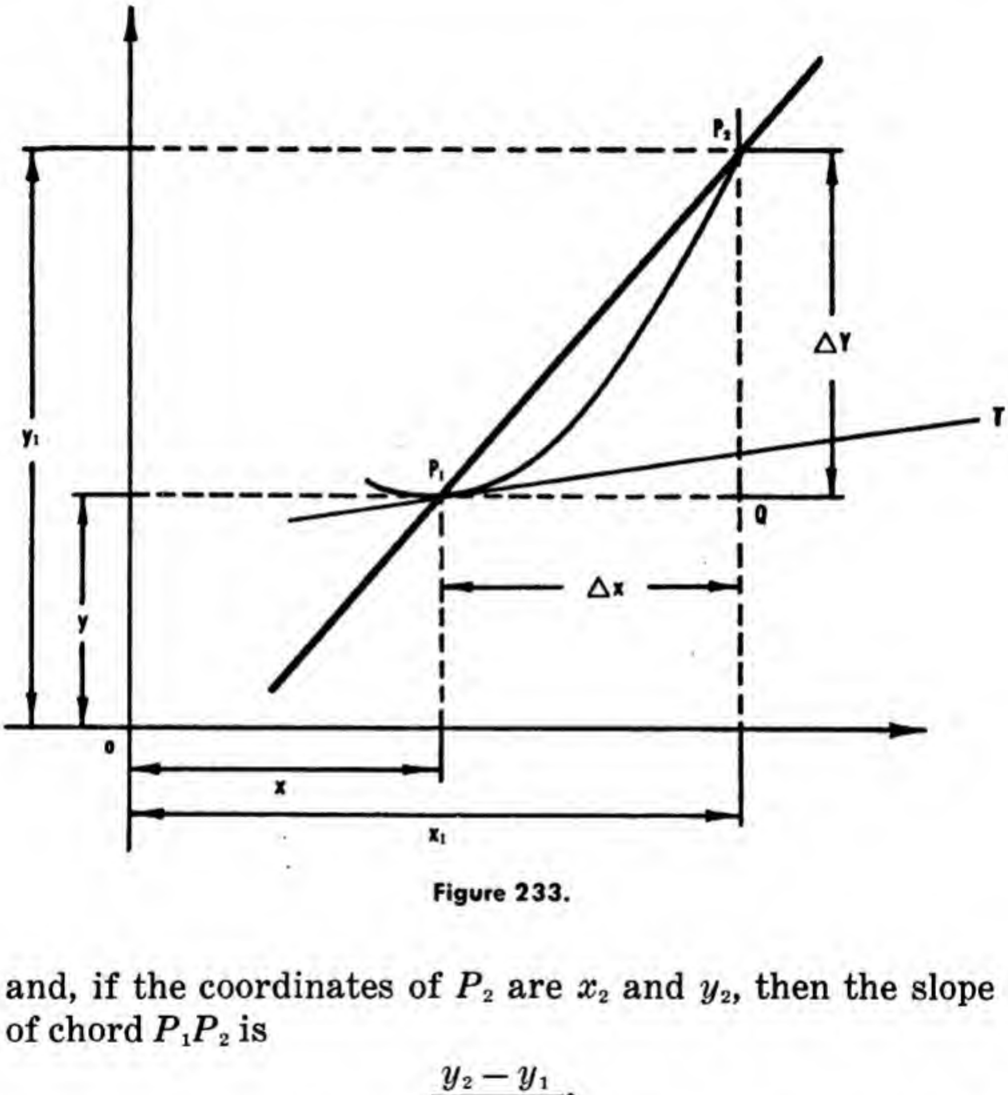


Figure 233.

and, if the coordinates of P_2 are x_2 and y_2, then the slope of chord P_1P_2 is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

Since $y_2 - y_1$ is the CHANGE in the y direction, you can call it delta y, or Δy. Similarly, since $x_2 - x_1$ is the CHANGE in the x direction you can call it Δx. The slope of chord P_1P_2, therefore, is $\frac{\Delta y}{\Delta x}$. Now call upon your old friend, the limit, to find the slope at point P_1 in terms of y and x.

321. What happens as you move point P_2 along the curve to point P_1? First, the sides Δy and Δx become smaller and smaller. Second, the position of chord P_1P_2 approaches the position of the tangent at point P_1. Therefore, you can say that the slope of the chord approaches the slope of the tangent at point P_1 as a limit when Δy and Δx become infinitesimals. An infinitesimal change in any variable or function is called a differential. When Δy and Δx become infinitesimals, the slope of chord P_1P_2 (which is $\frac{\Delta y}{\Delta x}$) becomes $\frac{dy}{dx}$ and equals the slope of the tangent at point P_1. The derivative $\frac{dy}{dx}$ is the ratio of the differentials dy and dx. The value of $\frac{dy}{dx}$, which equals the slope at point P_1, can be found by differentiating the function $y = f(x)$.

DIFFERENTIATION

322. The term $\frac{dy}{dx}$ is called the **DERIVATIVE OF y WITH RESPECT TO x** and is sometimes written $\frac{d}{dx}(y)$. It is an important term that you will meet often in calculus. It is simply the slope of the curve $y = f(x)$. The process of **DIFFERENTIATION** is a technique for finding the value of the derivative $\frac{dy}{dx}$. Since $y = f(x)$ when x is the independent variable and y the dependent variable, the derivative of y with respect to x, or $\frac{d}{dx}(y)$, is the derivative of the dependent variable with respect to the independent variable.

323. The importance of the slope of a curve lies in the fact that it is a measure of the rate of change of the dependent variable for changes in the independent vari-

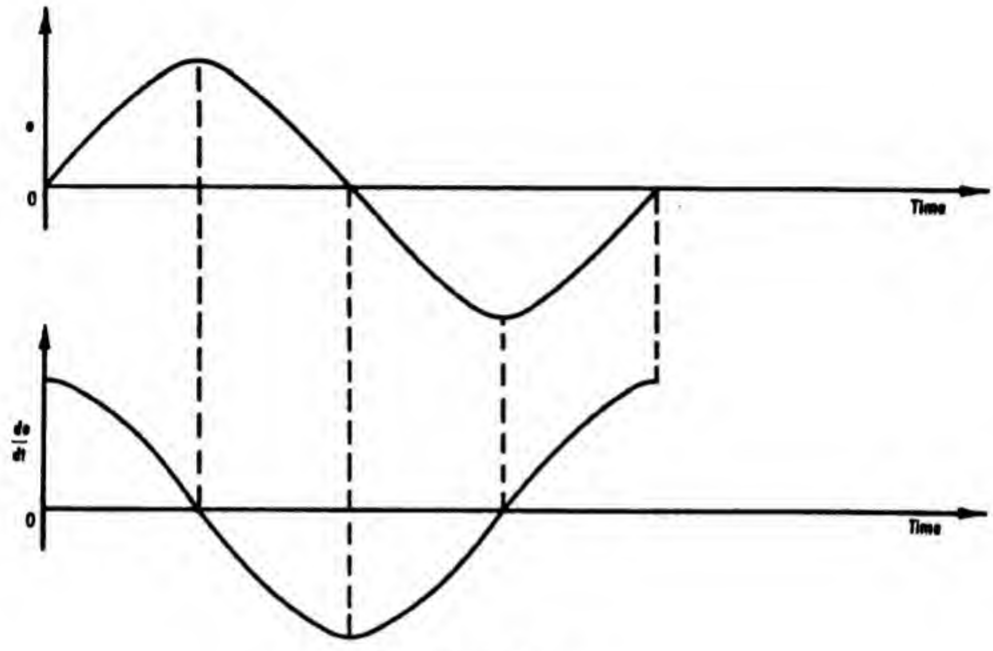


Figure 234.

able. Suppose you have a function that changes with time, such as the voltage sine wave in figure 234. Since voltage is the dependent variable and time the independent variable, you can say that $e = f(t)$, where e = voltage at any instant and t = time. The exact formula for the curve is—

$$e = E \sin (2\pi f) t,$$

where E = maximum voltage and f = frequency of the wave. The slope of the curve is $\frac{de}{dt}$. The value of the slope at the top and bottom of the wave is zero. Where the slope is zero the voltage is a maximum, and where the slope is a maximum the voltage is zero. The voltage is changing

most rapidly when the magnitude of the voltage is zero. The derivative $\frac{de}{dt}$ gives us the rate of change of voltage with time, so that $\frac{de}{dt}$ is zero at the maximum and minimum points on the sine wave, and $\frac{de}{dt}$ is a maximum when the voltage is zero.

324. The value of the slope or derivative is plotted immediately below the graph of the sine wave. Note that the derivative is also a wave but that it is 90° out of phase with the original wave. Since the graph of a cosine is a wave that is 90° out of phase with the graph of a sine, the second wave appears to be a cosine. If you refer to a table of derivatives you will find that the derivative of the sine is the cosine, or—

$$\frac{d}{dx}(\sin x) = \cos x$$

TABLES OF DERIVATIVES

325. The derivative of a function with respect to its independent variable is the slope of its curve or its rate of change in direction. Furthermore, the derivative of a sine is a cosine. The derivatives of many algebraic and trigonometric expressions have been worked out by mathematicians and are found in TABLES OF DERIVATIVES. In a table of derivatives you will find such expressions as—

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

These are trigonometric expressions. You will also find algebraic expressions, such as—

$$\frac{d}{dx}(x^n) = nx^{n-1}, \text{ where } n = \text{any integer.}$$

By letting $n = 1$, you obtain the derivative—

$$\frac{d}{dx}(x) = 1.$$

By letting $n = 2$, you obtain the derivative—

$$\frac{d}{dx}(x^2) = 2x.$$

By letting $n = 3$, you obtain the derivative—

$$\frac{d}{dx}(x^3) = 3x^2.$$

By letting n be any integer, you obtain the derivative of x^n with respect to x.

USE OF DERIVATIVES

326. One important use of derivatives in electronics is in the design of transformers and generators. The DERIVATIVE is a measure of the rate of change of a function. In a transformer, the secondary voltage depends on the rate of change of current in the primary. The curve representing the derivative of a sine wave is a cosine wave. The sine wave and cosine wave are exactly the same shape except for a phase displacement of 90°. This is an important fact that has many implications. Since the original sine wave and its rate of change (the cosine wave) are similar in appearance, you can send a sine wave through transformers and tuned tanks and obtain outputs that depend on the rate of change of the sine wave but they are exactly the same in shape. This is the reason why sine waves are used so often in electronics and electricity. It is the only wave shape that can pass through a chain of transformers with zero distortion.

327. Another use of derivatives is in the generation of a square wave. Figure 235 shows a saw-tooth wave. If you differentiate the function representing the saw-tooth wave you obtain its rate of change. The rate of change of the saw-tooth wave is plotted in the lower part of figure

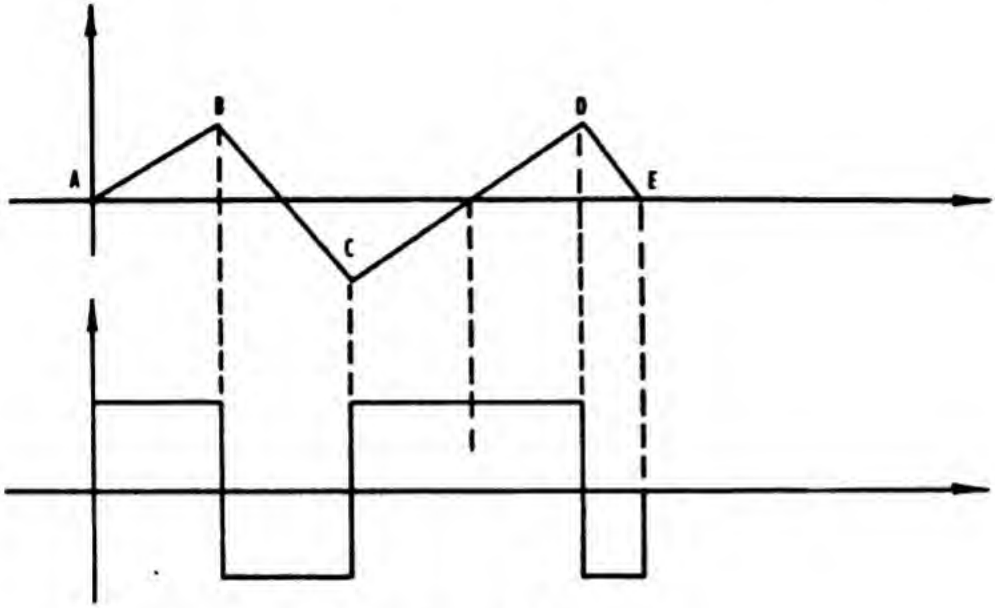


Figure 235.

235. From point A to point B and from point C to point D the rate of change of the saw-tooth wave is a constant positive value. Similarly, from B to C and from D to E the rate of change is a constant negative value. The plot of the rate of change is the square wave shown below the saw-tooth wave.

328. Other important uses of differentiation and derivatives are in calculations of motion. For example, the velocity of a ship is the derivative of the distance covered with respect to time. In other words, velocity is rate of change of position. Similarly, acceleration is the rate of change of velocity with time, or acceleration is the derivative of velocity with respect to time. Since velocity is the derivative of distance with respect to time, you can ex-

press velocity as $\frac{ds}{dt}$ where s = distance and t = time. Since acceleration is the derivative of velocity with respect to time, you can express acceleration as $\frac{d}{dt}$ (velocity), or $\frac{d}{dt}\left(\frac{ds}{dt}\right)$. The last expression is written as $\frac{d^2s}{dt^2}$ and is called a SECOND DERIVATIVE.

329. Another interesting use of differential calculus is in finding the maximum and minimum points of a

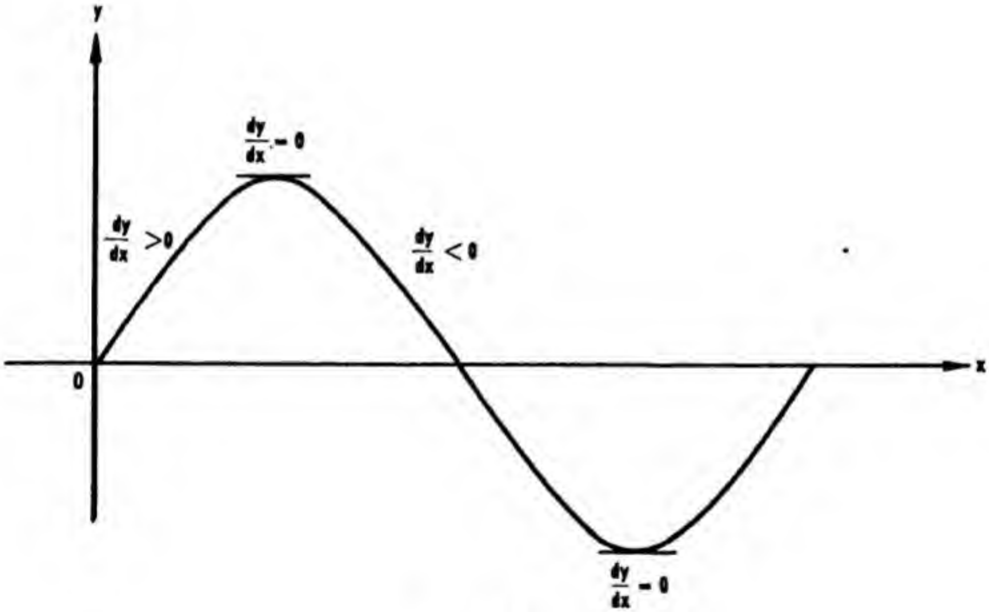


Figure 236.

function. In figure 236 you see that the slope of the curve is zero at the maximum and minimum points of the curve. If a function representing a curve has a maximum or minimum value, that value always occurs when the slope or derivative is zero. This principle is used in the design of such things as pipes and troughs, which should have maximum capacity with minimum area.

DIFFERENTIATOR CIRCUIT

330. A circuit that automatically differentiates an input wave is shown in figure 237. Since the derivative of the input is merely the rate of change of the input, the input must be changing in some way or else nothing is

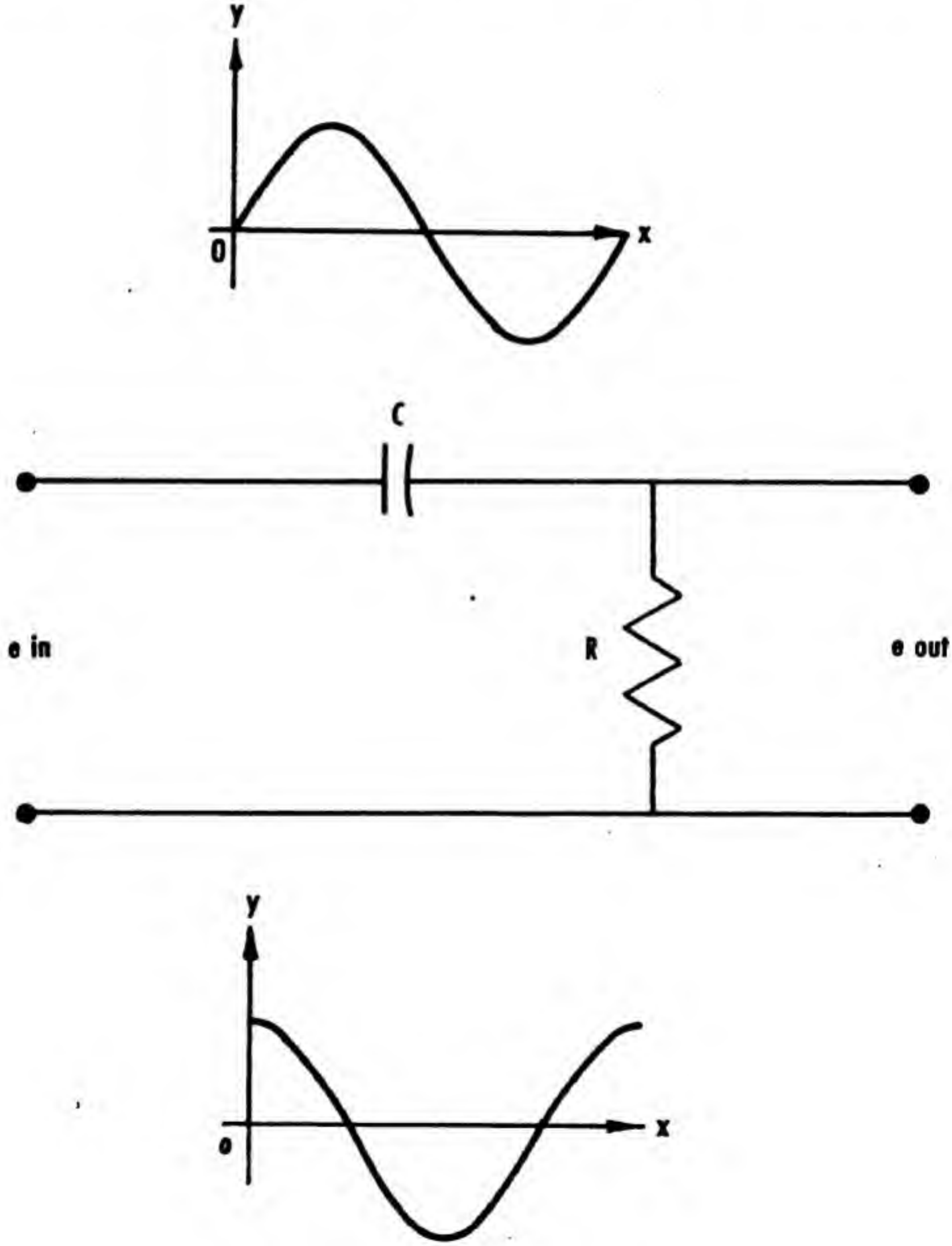


Figure 237.

obtained at the output. Since the magnitude of the output voltage must be exactly proportional to the rate of change of the input, the RC constant must be very small. This is the only thing that distinguishes a differentiator circuit from an ordinary RC coupling circuit. As long as the RC constant is small compared to the time required for one cycle of the input wave, the capacitor has a larger reactance than the resistor. When this occurs, the capacitor and resistor are like a voltage divider that divides the input voltage. Since the reactance of the capacitor depends on frequency the output voltage increases as the frequency increases, thus differentiating the input wave or extracting its rate of change.

COMPARISON OF INTEGRATION AND DIFFERENTIATION

331. Integration and differentiation are inverse processes like multiplication and division. In differential calculus you are concerned with finding the differential of a function, whereas in integral calculus you are given a differential and asked to find the function. Follow the process and see how it works—

$$dy = d(x^3) = 3x^2\,dx,$$

$$\int 3x^2\,dx = 3\int x^2\,dx = 3\left(\frac{x^3}{3}\right) = x^3.$$

Thus, you end with the same expression you started with, which shows that integration and differentiation are like multiplication and division—inverse processes. The similarity and dissimilarity of integration and differentiation are shown also by comparing the integrator circuit (fig. 231) and the differentiator circuit (fig. 237).

APPENDICES

APPENDIX A

REMEDIAL WORK

(For answers see Appendix C)

SECTION I

1. What fraction of a complete rotation is 1°?
2. A degree is divided into .... minutes. A minute is divided into .... seconds. Therefore 1° contains .... seconds.
3. An angle of 180° is called a .... angle.
4. If you can rotate a gun director 2.3 revolutions before reversing it, through how many degrees will it have turned?
5. What part of one complete revolution is a turn of 72°? 135°? 200°?
6. Measure the four angles in figure 238.

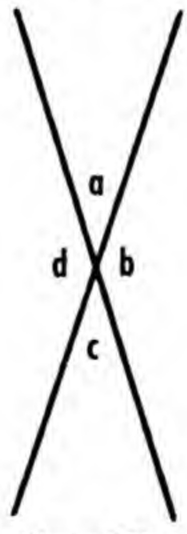


Figure 238.

7. Refer to the illustration in problem 6. If $\angle A$ is acute, can $\angle d$ be acute? Explain why this is true.
8. What kind of angle do the hands of a clock make at 1300? 1500? 1600? 1800?

9. When you read a voltmeter, you should be directly in front of the instrument to avoid errors caused by ....
10. Add:

(a)	26° 24′ 30″ 16° 08′ 15″	(b)	14° 15′ 35″ 16° 45′ 50″

Subtract:

(c)	37° 15′ 30″ 20° 14′ 40″	(d)	29° 27′ 00″ 40° 15′ 30″

11. Express the answers to (a) and (b) in problem 10 as decimals to the first decimal place.

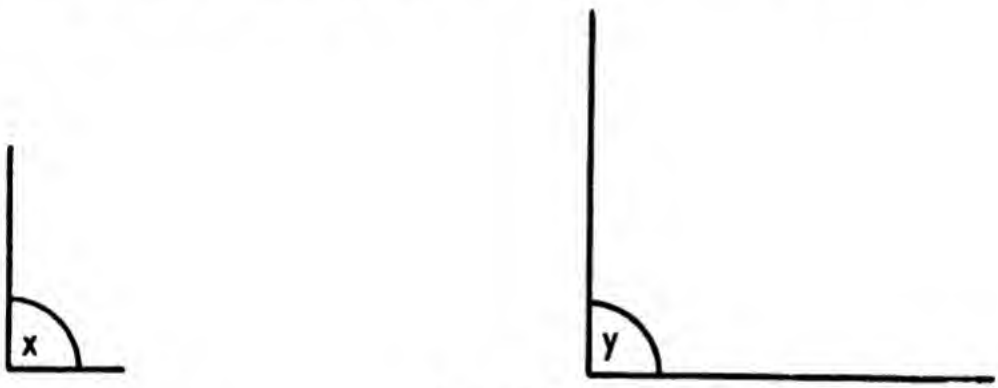


Figure 239.

12. Because a clockwise turning in navigation increases a ship's course, it may be mathematically interpreted as a .... turn. A counterclockwise turning decreases a ship's course and may be interpreted as a .... turn.

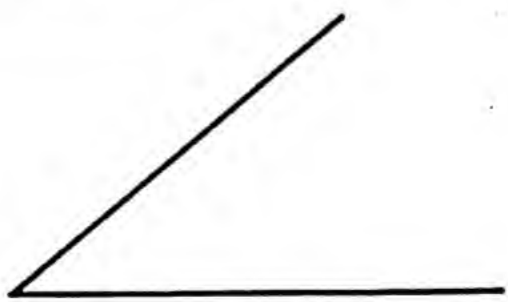


Figure 240.

13. In electronics problems involving alternating current, angles are generated in a .... direction.
14. A ship on course 045° turns 71° counterclockwise. What is the new course?
15. A ship on course 350° makes a 45° turn in a clockwise direction. What is the new bearing of the ship's course?

16. If a ship has a list of 12° what angle does a normally vertical bulkhead make with the horizon?
17. Draw a diagram showing each of the following bearings: 080°; 110°; 155°.
18. Your ship is on course 325° true. A lookout reports a buoy bearing 072° relative. What is the true bearing of the buoy?
19. Look at $\angle x$ and $\angle y$ in figure 239. Are they the same size?
20. A student measured the angle in figure 240 and stated it was 140°. What mistake did he make?

SECTION II

1. Draw a straight line 3 inches long. Mark the ends A and B. Using your compass and with A as center, draw a circle having a 1-inch radius. Using the point where this circle intersects AB (called point C) as a center, draw a second circle having a 2-inch radius. Using B as center and with a 1½-inch radius, draw a third circle, calling the point where it intersects AB, D. If you have drawn the circles correctly, $AC=1$ inch, $CD=½$ inch, and $DB=1½$ inches.
2. Draw a straight line 3 inches long and use your dividers to divide it into four equal parts. Check your results with a ruler; each segment should be ¾ of an inch long.
3. Use your dividers and the horizontal scale to find the lengths of RS, TZ, JM, and XY in figure 241.
4. Copy the angles in figure 242. Check your results with a protractor.
5. Draw a straight line 2⅛ inches long and find its midpoint by constructing its perpendicular bisector.
6. Lay a circular object on a paper and trace around it, forming a circle. Now, find the center of this circle by constructing the perpendicular bisectors of two chords.
7. Draw a straight line 4 inches long. Mark the ends A and B. Locate points C and D 1 inch from each end of AB. Construct perpendiculars to AB at C and D. Use your dividers and a straightedge to extend the perpendicular at D to a point E so that $DE = CD$. Extend the perpendicular at C about 3 inches to G. Then from point E construct a line perpendicular to CG; call the intersection of this perpendicular with CG point F. If you have done your work correctly $CFED$ will be a square.
8. Use your protractor to draw a 75° angle. Bisect this angle, using your compass, and then check your results with the protractor.

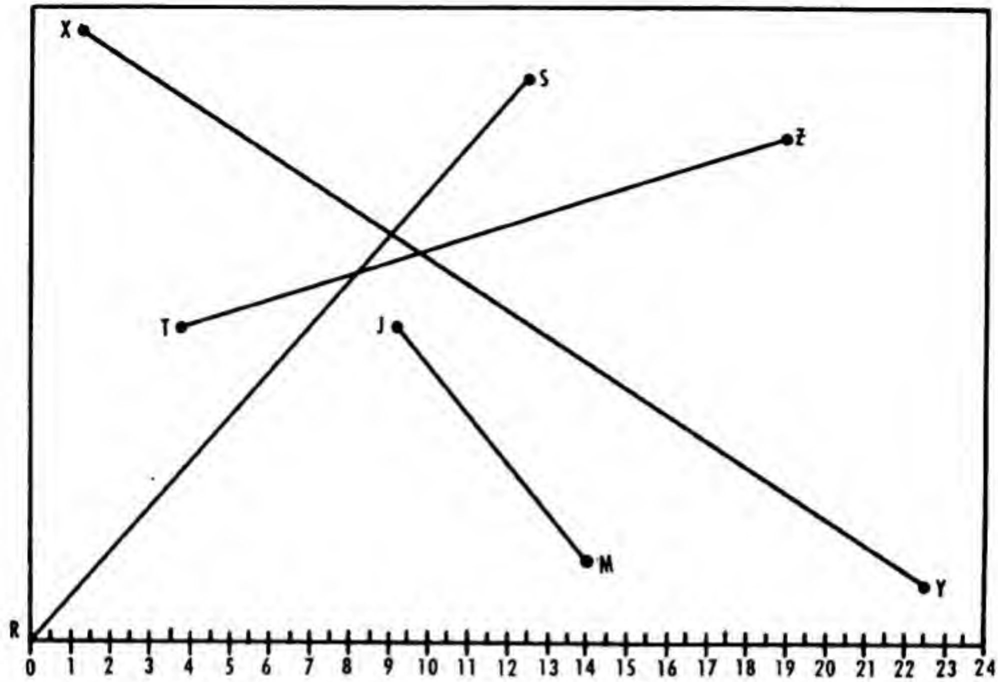


Figure 241.

9. Construct a square, using a compass and straightedge, with each side 1½ inches long. What is the perimeter of this square?
10. Draw a rectangle having a base of 3 inches and an altitude of 1½ inches. Draw the diagonals of the rectangle. What is the size of the smallest angle formed by a diagonal and a side of the rectangle? (Use your protractor.)
11. Construct a regular octagon.
12. Construct a regular hexagon, using a circle.
13. Construct an equilateral triangle, without using a circle.
14. Construct a regular pentagon:
    (a) By the method described in paragraph 36.
    (b) By drawing a circle and then using only a pair of dividers and a straightedge.

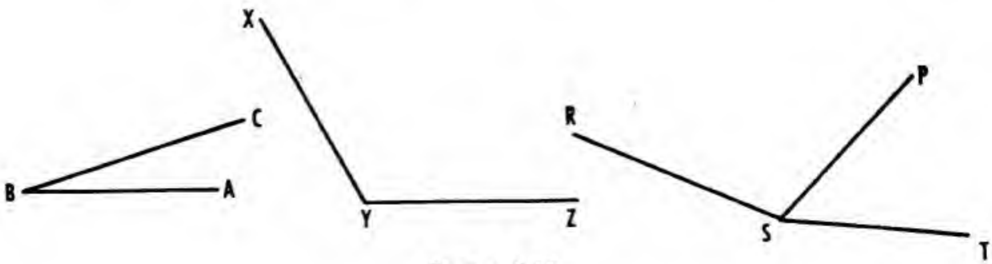


Figure 242.

15. Construct the following triangles, using compass, straightedge, and protractor:
(a) $\angle A = 30°, AB = 2$ inches, $\angle B = 45°$.
What are the lengths of the other two sides?
(b) $AB = 1$ inch, $\angle B = 82°, BC = 1\frac{1}{2}$ inches.
How large are the other two angles?
(c) $AB = 2$ inches, $BC = 2\frac{1}{2}$ inches, $CA = 1\frac{1}{2}$ inches.
How large are the three angles?

SECTION III

1. Figure 243 shows two parallel lines cut by a transversal. Why does $\angle x = \angle a$? Why does $\angle b = \angle x$? Does $\angle b = \angle c$? Why? Why does $\angle c = \angle x$?

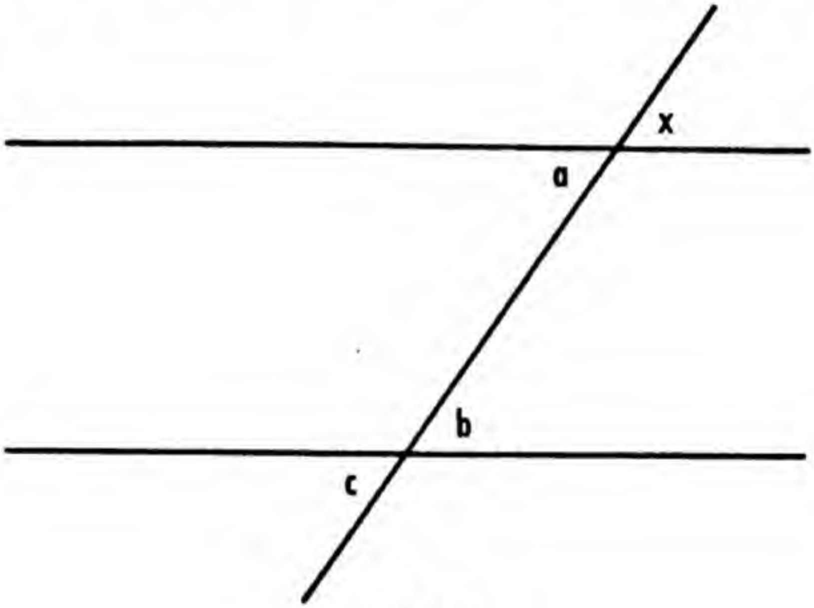


Figure 243.

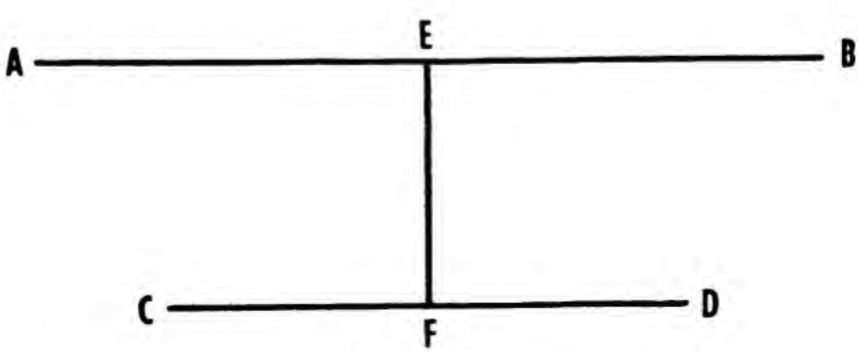


Figure 244.

2. Draw a line AB and construct its perpendicular bisector CD. Construct a perpendicular to CD at a point 1 inch from the point at which AB and CD intersect. In what way are all the angles of the figure alike?
3. Figure 244 shows a special type of receiving antenna used with television and FM radios. The bar EF is perpendicular to both AB and CD. Use your compass, dividers, and straightedge to copy this figure.
4. A radio antenna is made up in the shape of a rhombus. Draw a diagram of this antenna, making each side 1½" long and the smallest interior angle 80°.
5. Draw a parallelogram having one interior angle of 75°. What are the sizes of the other interior angles?
6. A radio chassis has a square-topped surface. Construct a diagram of this surface making each side 2¼" long and using the following method: Draw a straight line and mark off a segment AB 2¼" long. Using your compass, construct perpendiculars at each end of AB. Using your dividers, mark off lengths of 2¼" on both perpendiculars above the line AB. Connect the perpendiculars to form the fourth side of the square. Check the accuracy of the right angles formed by the square, using your protractor.

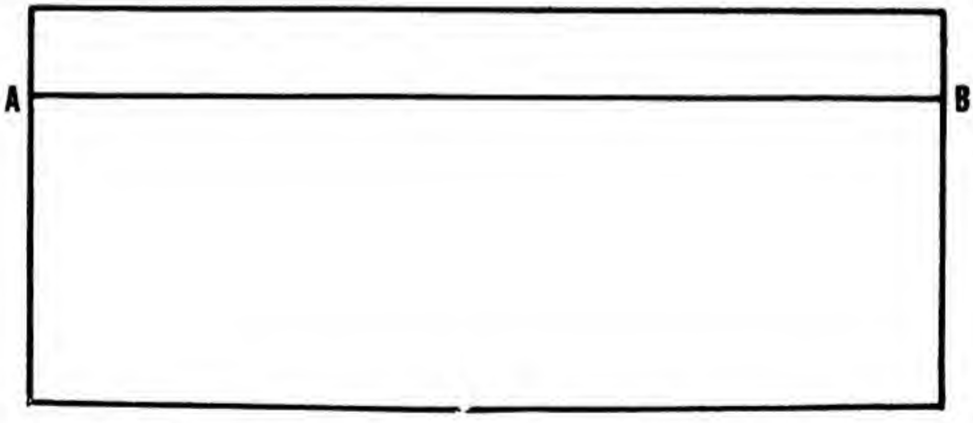


Figure 245.

7. Figure 245 represents a radio chassis. Five tubes are to be placed an equal distance apart on the line AB. The tubes at each end of the line are to be the same distance from the end of the chassis as they are from the adjacent tubes. Copy the diagram and divide the line AB into six equal parts, locating centers so that the five tubes may be correctly placed.
8. Construct an equilateral triangle having 1½" sides. Construct two more 1½" lines, connected to the vertexes of the triangle

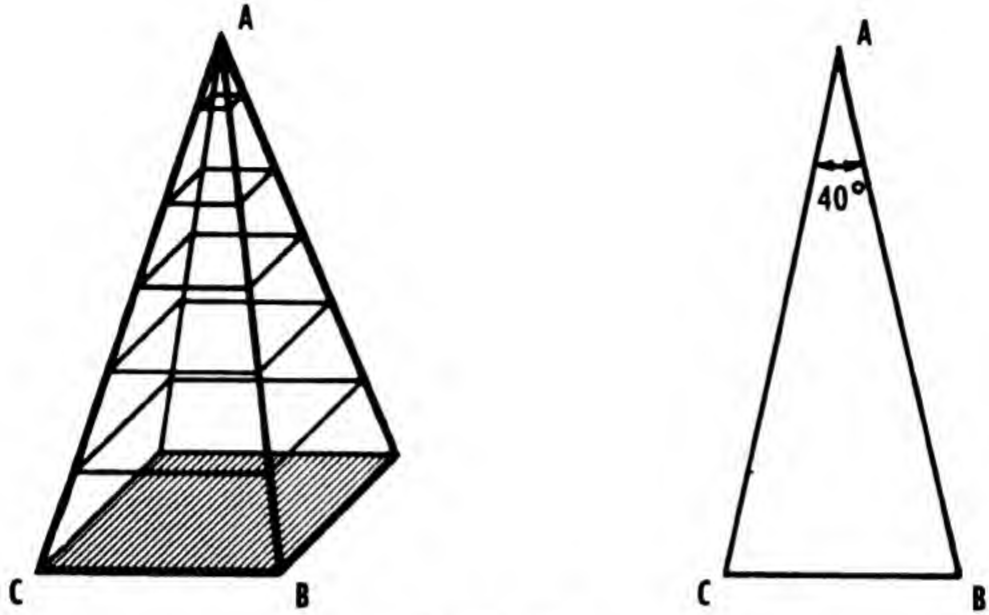


Figure 246.

so that they form a rhombus with 1½″ sides. What are the sizes of the interior angles of the rhombus?

9. Figure 246 shows a tower used to support a radio transmitting antenna. Each side of the tower is shaped like an isosceles triangle. The angle formed between the two equal sides is 40°. What are the sizes of the base angles ($\angle ACB$ and $\angle ABC$)?

10. What are the perimeters of the polygons in figure 247?

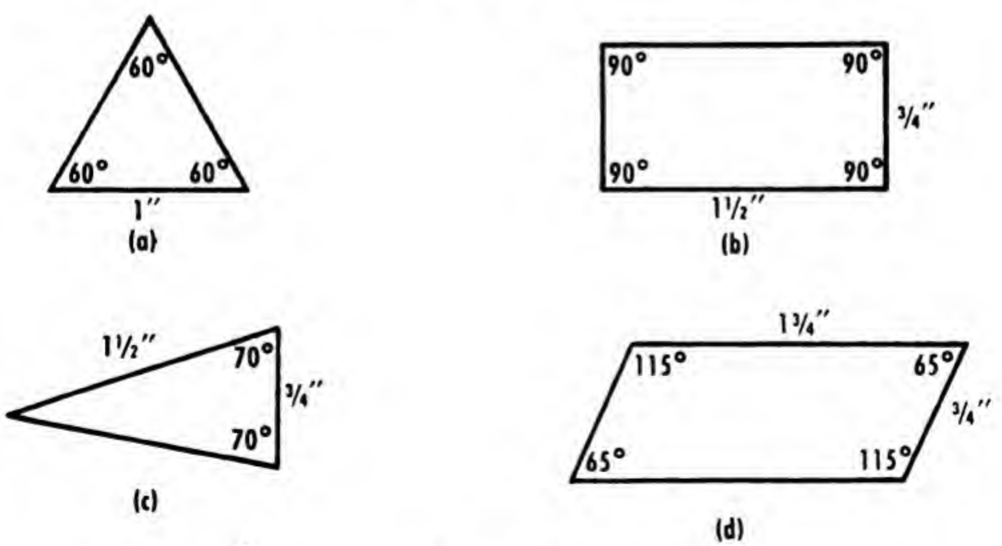


Figure 247.

921364—51——18

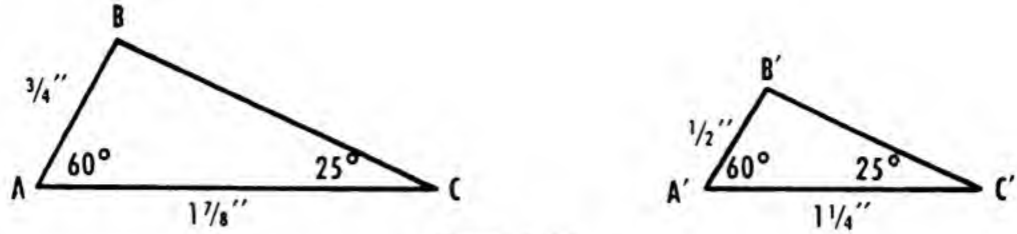


Figure 248.

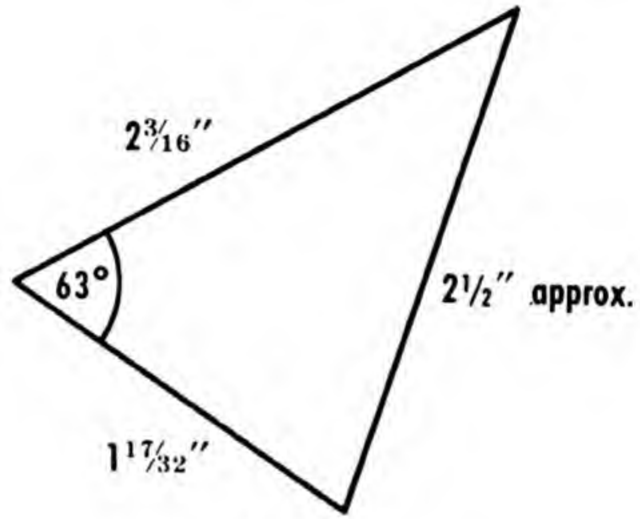


Figure 249.

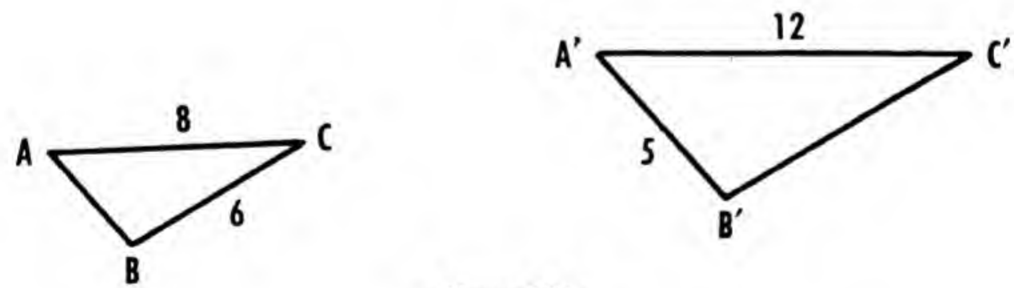


Figure 250.

Figure 251.

SECTION IV

1. Using only the information given, state two reasons why the triangles in figure 248 are similar.
2. Using the scale 1¾" : 1", construct a triangle similar to triangle ABC in figure 249. Indicate the lengths of the sides in the triangle that you construct.
3. The triangles in figure 250 are similar. Complete the proportions and solve for the unknown sides.

   (a) $\frac{AB}{5} = \frac{?}{?}$; $AB = ?$ (b) $\frac{B'C'}{?} = \frac{12}{?}$; $B'C' = ?$
4. Using a scale ¼" = 200′, draw lines representing the following distances—

   ¼ mile; ½ mile; 700 ft.; 1,000 ft.
5. Using a scale 1 inch : 20 miles, find the distances represented by—

   2½"; 6¾"; 1⅞"; 5"; 7½".
6. A radio transmitting room is 48 ft. by 33 ft. What are the dimensions of the room on a scale drawing if the scale used is ⅜" = 1 ft.?
7. The scale ¾" = 5 ft. corresponds to what representative fraction?
8. An original model of a ship is made to the scale 1/450. A lifeboat on the deck of the model measures ⅝". What is the true length of the lifeboat?
9. From memory, draw the symbols for the following electrical parts—

   (a) capacitor (c) inductor
   (b) resistor (d) antenna
   (e) fuse
10. Two radio tube sockets are shown in figure 251 as viewed from the bottom. Mark the numbers by each hole that correspond to the tube pins.
11. When you use an ohmmeter to test an electrical circuit, the power to the equipment must be ....
12. An ohmmeter connected across a capacitor gives a zero reading on the scale; this indicates the capacitor (good) (bad) (shorted) (open). Check two.
13. What physical appearance quickly distinguishes a transformer from a choke coil?
14. The larger the diameter of wire the (larger) (smaller) its resistance.

15. A transformer has a primary winding of 4 turns and a secondary winding of 20 turns. If 5 volts a.c. are applied to the primary, how many volts will be developed across the secondary?
16. On all layout and detail drawing, the distance between holes is always measured from ....

SECTION V

1. The vertical distance above or below the horizontal axis of any point on a graph is its ....
2. The horizontal distance to the right and left of the vertical axis of any point on a graph is called an ....
3. When a point on a graph is located by reference to the horizontal and vertical scales, the two distances are referred to as .... of the point.
4. Referring to figure 96, what was the average light-bulb cost in 1940? What was its efficiency?
5. Refer to figure 101. If you wished to operate the tube with 120 volts on the plate, and if you wanted 8 milliamperes of plate current, what value of grid voltage (E_c) would you put on the tube?
6. Refer to figure 101. If the tube given had 200 plate volts and the plate milliamperes was 10, what must be the grid volts (E_c)?
7. Refer to figure 103. With −7 volts of grid voltage, how much plate current will be flowing?
8. Refer to figures 111 and 112. What should be the positions of knob A and B of control "P" in order to have an output frequency of 230 kilocycles?
9. Refer to figure 114. If a resonant frequency of 4 megacycles is desired and a 100-micromicrofarad (μμf.) capacitor is available, what size inductance in microhenries is needed for working with the capacitor?

SECTION VI

1. A ship's compartment is 10.5′ long and 7.3′ wide. What is the area of the deck?
2. Find the area of each of the triangles in figure 252.
3. Find the area of each of the following parallelograms—

	Base	*Height*
(a)	12′	10′
(b)	110 yds.	90 yds.
(c)	30 cm.	19 cm.

4. Find the area of each of the following trapezoids—

	Upper base	*Lower base*	*Height*
(a)	10 in.	12 in.	6 in.
(b)	15 in.	10 in.	8 in.
(c)	4.5 in.	8.3 in.	9 in.

5. Find the area of the largest circle you can cut from a square piece of metal whose sides are 10 inches.
6. Find the area of a square 9 feet on a side.
7. A tool chest measures 3 ft. × 1 ft. × 1.5 ft. How much space does it occupy?

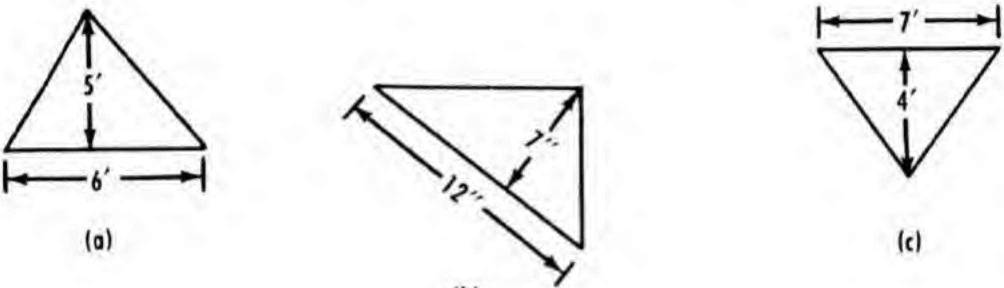


Figure 252.

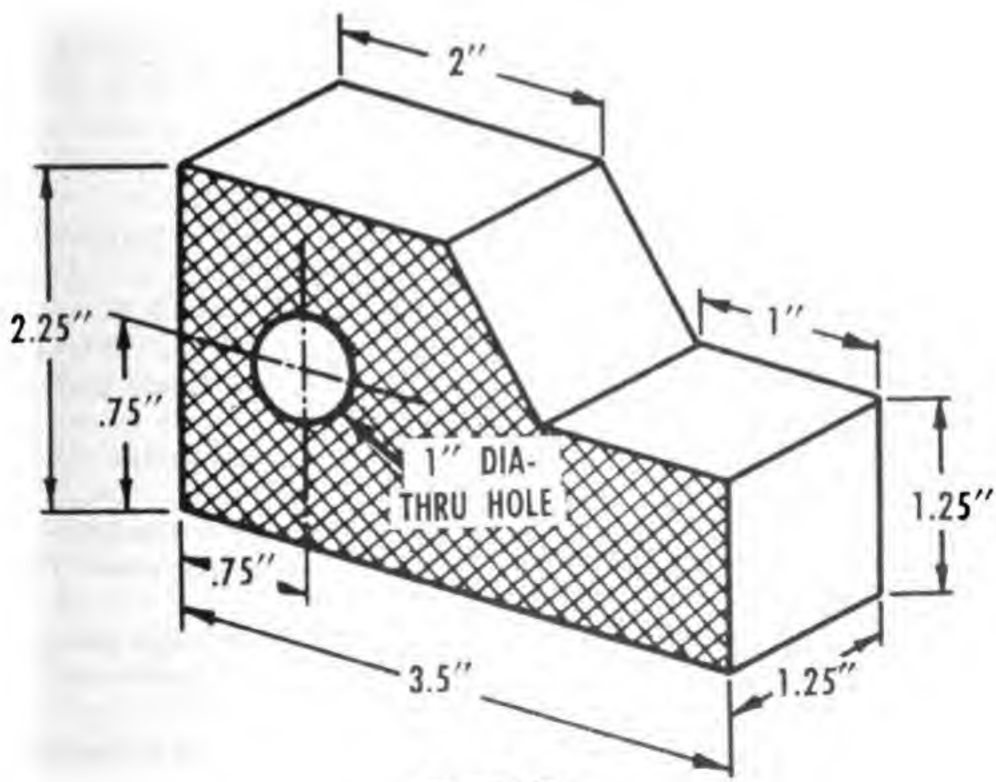


Figure 253.

8. What is the volume of a cable that has a radius of ½ in. and a length of 2,000 ft.?
9. A steel shaft has a diameter of 4 in. and is 8 ft. long. If steel weighs 490 lbs. per cubic foot, what is the weight of the shaft?
10. A concrete column in the form of a hexagonal prism, 8 ft. high, has a base with an area of 3.5 sq. ft. At 154 lbs. per cubic foot, what is the weight of the concrete in the column?
11. Find the volume of the object in figure 253. Note the hole.
12. Find the area of the hatched surface (drawn lines) in figure 253.
13. Find the total exposed area (including the interior area of the cylindrical bore) of the object shown in figure 253.

SECTION VII

1. Referring to the triangle in figure 254, find the values of the missing sides to three significant figures.

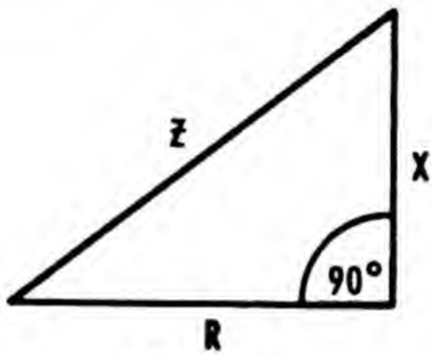


Figure 254.

(a) $R = 5, X = 15, Z = ?$ (d) $X = 13, Z = 31, R = ?$
(b) $X = 10, Z = 17, R = ?$ (e) $R = 3, X = 5, Z = ?$
(c) $R = 34, Z = 50, X = ?$ (f) $X = 4, Z = 14, R = ?$

2. On course 090° a ship steams at 16 knots. A strong current is running due south at 3 knots. What is the distance traveled after 1 hour?
3. In a series a. c. circuit, the voltage across a capacitor .... the current through it by ...... degrees.
4. A resistor of 40 ohms and a capacitor having a reactance of 65 ohms are connected in series across an a.c. generator. Find the impedance of the circuit.
5. In a series a.c. circuit the voltage across a pure capacitor and the voltage across a pure inductor are .... degrees out of phase.
6. On a graph, draw one cycle each of two voltage waveforms equal in magnitude, with E_1 leading E_2 by 90°.

7. An a.c. generator supplies 220 volts to a series circuit consisting of a resistor and an inductor. A voltmeter shows a drop of 74 volts across the resistor. What is the value of E_L?
8. A resistance of 50 ohms and a capacitor having a reactance of 35 ohms are connected in series across 110 volts a.c. Find the impedance of the circuit and the amount of current flow (I) through the circuit. (Total current, $I = \frac{E_{total}}{Z}$.)
9. The area of a square is 529 sq. in. What is the length of the diagonal?
10. An a.c. circuit has R, C, and L in series; $Z = 55$ ohms, $X_C = 80$ ohms, $X_L = 60$ ohms. Find R.

SECTION VIII

1. A 26-foot pole casts a shadow 20 feet in length. What is the angle of elevation of the sun at that time?
2. A road slopes downward at an angle of 17° with the horizontal. What distance below level ground will a man be after walking 300 yards down the road?
3. A ladder 28 feet long leans against a building with its base 6½ feet from the building. What size angle does the ladder make with the ground?
4. From the top of a cliff 320 feet high a man sights a ship at sea at a 16.4° depression angle. How far away is the ship?
5. A square lot has a diagonal 230 feet long. How long is one side of the lot?
6. In the following right-triangle problems, solve for the unknowns—
   (a) $Z = 140, \Theta = 38.4°, R = ?\ X = ?$
   (b) $R = 62, \Theta = 21°, X = ?\ Z = ?$
   (c) $Z = 312, R = 90, X = ?\ \Theta = ?$
7. Find the instantaneous value of voltage after E_{max} of 250 volts has rotated 220° of its cycle.
8. The instantaneous value of an alternating voltage is 69 volts at 162°. What is the maximum value?
9. The instantaneous value of an a.c. voltage is 110 volts at 48°. What is the instantaneous value at 207°?
10. An a. c. voltage has a maximum value of 610 volts. What is the instantaneous value after E_{max} has completed 900°?
11. Convert each of the following to radians—
   (a) 30°
   (b) 34.3°
   (c) 61.5°

12. Express the following angles in degrees—
    (a) 3.1 radians
    (b) 0.4 radian
    (c) 7.3 radians

SECTION IX

1. Find the resultant voltage of the following vectors—
    (a) 18 volts at 0° and 11.6 volts at 90°
    (b) 54 volts at 0° and 145 volts at 90°
    (c) 16 volts at 0° and 8.5 volts at 0°
    (d) 55 volts at 270° and 32 volts at 180°
    (e) 26 volts at 0°, 42 volts at 90°, 9.6 volts at 270°, 50 volts at 90°, 30 volts at 0°, and 139.4 volts at 270°.
2. Find the horizontal and vertical components, denoted by x and y, respectively, of the following vectors and express each as a complex number—
    (a) 30 at 36.9°
    (b) 14.3 at 69.1°
    (c) 117 at 166.2°
    (d) 40 at 270°
    (e) 182 at 285.1°
    (f) 52 at 25.8°
    (g) 20.4 at 116.5°
    (h) 19.4 at 340.6°
    (i) 101 at 180°
3. Work all the problems given in pretest 9.

APPENDIX B

ANSWERS TO PRETESTS

CHAPTER 1

USING ANGLES TO TELL DIRECTION

1. (a) Amount of turning;
   (b) 40°;
   (c) 240°; 300°; 144°;
   (d) ¾; ⅛; 1/12;
   (e) Obtuse;
   (f) 26° 55′ 41″;
   (g) 15.5°; 20.2°; 25.45°;
   (h) $x = 140°$; $y = 33°$; $z = 46°$;
   (i) 020°;
   (j) 255°;
   (k) 020°;
   (l) 035°;
   (m) True north;
   (n) Bow;
   (o) 240°.

CHAPTER 2

LINES AND ANGLES

17. (a) Radius. (b) See paragraph 21. (c) See paragraph 23.
    (d) Draw any two of the three straight lines connecting the three points in pairs; then locate the center of the circle at the intersection of the perpendicular bisectors of these two lines. Using the distance from this intersection to any one of the three points as a radius, draw the required circle.

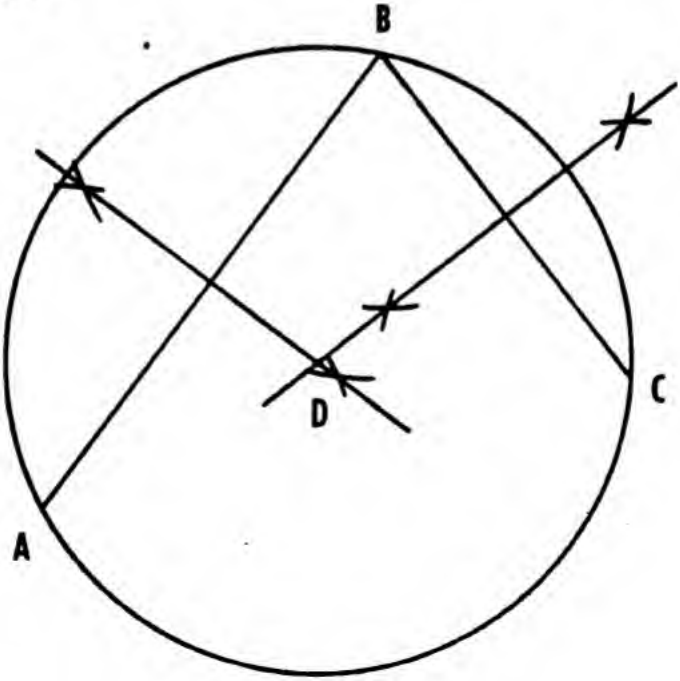


Figure 255.

(e) 8 inches; 12 yards; 4S units.

(f) Opposite sides of a rectangle are equal. The sum of the two altitudes equals the perimeter minus the sum of the two bases or $2a = p - 2b$. Therefore, $2a = 7 - 2(2) = 3$. Each altitude equals $\frac{3}{2}$ or $1\frac{1}{2}$ inches.

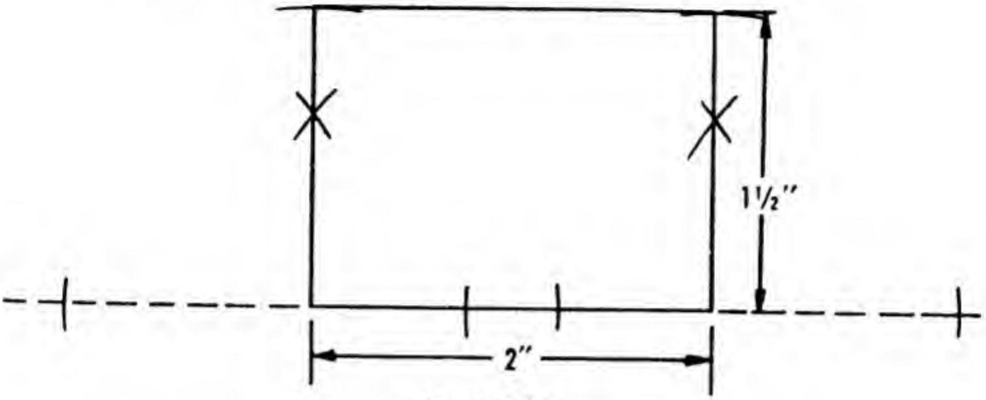


Figure 256.

(g) Regular.

(h)

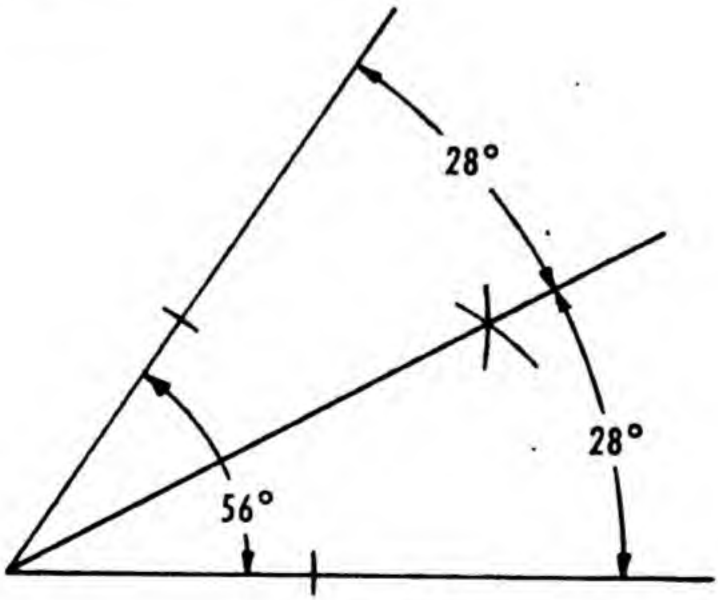


Figure 257.

(i) See paragraph 35.

(j) Tangent.

(k) See paragraph 34.

(l) See paragraph 39.

(m) See paragraph 38.

(n) See paragraph 26.

(o)

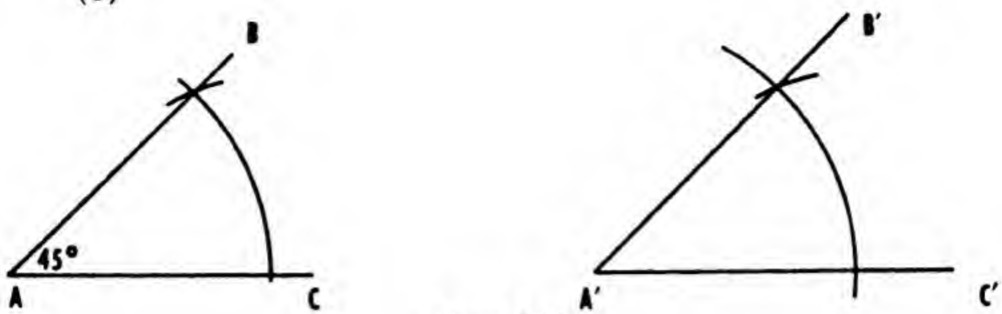


Figure 258.

CHAPTER 3

PRACTICAL CONSTRUCTIONS INVOLVING PARALLEL

40. (a) Plane.
    (b) See text, paragraph 45; chap. 1, paragraph 9; and chap. 3, paragraph 42 and paragraph 44.
    (c) See text, paragraphs 46 and 47.
    (d) See text, paragraphs 49, 50, and 51.
    (e) 70°; 50°.
    (f) One; One; Three.
    (g) 70°.
    (h) Yes.
    (i) 21½″.
    (j) 540°.
    (k) 60°; 0°.
    (l) 90°; 140°; 50°.
    (m) Larger.
    (n) Smaller.
    (o) 40° and 140°.

CHAPTER 4

BASIC IDEAS OF SCALE DRAWING

59. (a) Sides.
    (b) 6; 135; 1.6; 3.6; 48.
    (c) $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$.
    (d) 3½″ and 5″.
    (e) 78′9″.

(f)

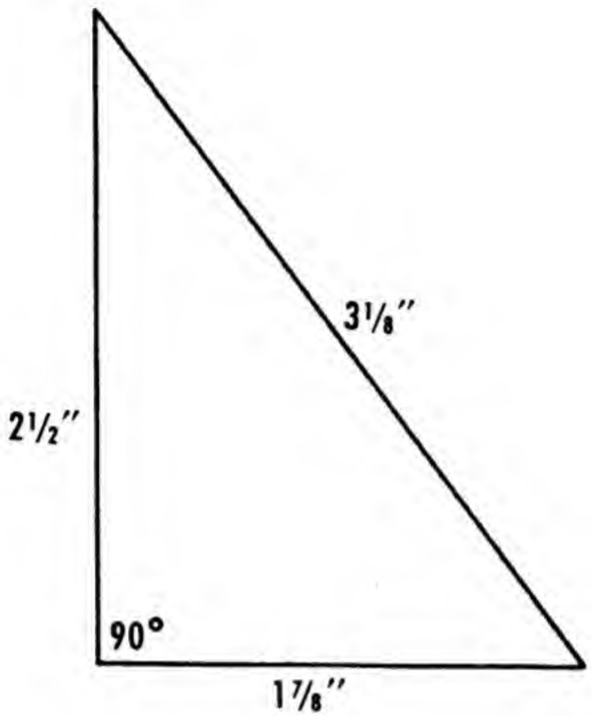


Figure 259.

(g) 34′2″ (approximately).

(h) (1) $\frac{1}{50}$; (2) $\frac{4}{3}$; (3) $\frac{7}{73}$; (4) $\frac{1}{40}$; (5) $\frac{3 \times 10^{-6}}{2}$; (6) 150.

(i) 150 mi.; 275 mi.; 331¼ mi.; 437½ mi.

(j) $\frac{1}{267200}$ or approx. $\frac{1}{250000}$

(k) 31.6 mi. (approximately).

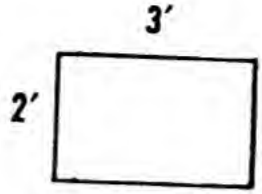
(l) 48′

(m)

SCALE LENGTH	⅛″	⅜″	½″	¾″	1″	2½″	4″	5″
ACTUAL LENGTH	1′	3′	4′	6′	8′	20′	32′	40′

Figure 260.

(n)



SCALE ¼″ = 1′

Figure 261.

(o)

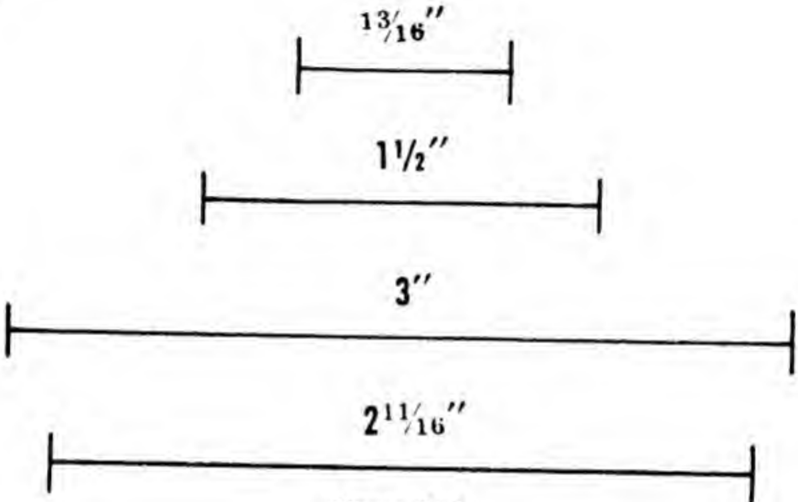


Figure 262.

(p)

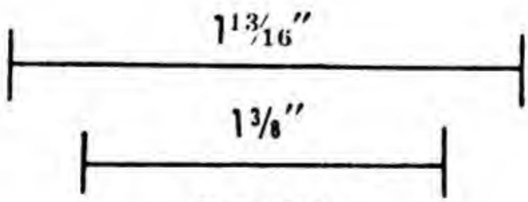


Figure 263.

CHAPTER 5

GRAPHS

126. (a) Ratio.
    (b) 28 cents.
    (c) Bar; circle; line.
    (d) Parts; whole.

(e)

Figure 264.

(f) 60 percent.
(g) 1.2K; 4.3K.
(h) 8 milliamperes.
(i) 3.9 to 1.8 milliamperes.
(j) Three.
(k) 3.65 megacycles.

CHAPTER 6

FORMULAS USED IN PRACTICAL GEOMETRY

159. (a) (1) parallelogram; (2) $A = bh$; (3) 120 sq. in.
(b) (1) triangle; (2) $A = \frac{1}{2}\ bh$; (3) 4.02 sq. in.
(c) (1) circle; (2) $C = \pi d$, $A = \pi r^2$; (3) 62.8 cm., 314 sq. cm.
(d) (1) trapezoid; (2) $A = \frac{1}{2}\ h\ (b + B)$; (3) 60 sq. in.
(e) (1) cube; (2) $V = S^3$; (3) 68.92 cu. in.
(f) (1) rectangular prism; (2) $V = lwh$; (3) 192 cu. in.
(g) (1) circular cylinder; (2) $V = \pi r^2 h$; (3) 254.47 cu. in.
(h) (1) square pyramid; (2) $V = \frac{Bh}{3}$; (3) 18 cu. in.
(i) (1) circular cone; (2) $V = \frac{\pi r^2 h}{3}$; (3) 56.55 cu. in.
(j) (1) sphere; (2) $A = 4\pi r^2$, $V = \frac{4\pi r^3}{3}$; (3) 78.54 sq. in., 65.45 cu. in.

CHAPTER 7

THE PYTHAGOREAN THEOREM

197. (a) (1) 29; (2) 38; (3) 163; (4) 220.5.
(b) $X = \sqrt{Z^2 - R^2}$.
(c) Minus; other.
(d) (1) 12.8; (2) 17.86; (3) 24; (4) 23.3; (5) 11.18; (6) 15.
(e) 30.41 knots.
(f) 28.2 feet.
(g) 28.3 feet.
(h) Yes.

CHAPTER 8

ESSENTIALS OF TRIGONOMETRY

232. (a) (1) 0.7349; (2) 0.9568; (3) 3.7321; (4) 0.8988; (5) 0.5962; (6) 0.5681.
(b) (1) 36.5°; (2) 51°; (3) 12.8°; (4) 24°; (5) 82°; (6) 33.5°.
(c) 55°.
(d) 023.8°.
(e) $X = 90.3$ lbs., $Y = 120$ lbs.
(f) 85.78 feet.
(g) 2,267 feet.
(h) 975 feet.
(i) 13,299 yards; approx. 6.6 nautical miles.
(j) 187.3 feet.

CHAPTER 9

VECTORS

278. (a) Magnitude; direction.
(b) Straight line.
(c) Resultant.
(d) 31.6 volts at 101.6°.
(e) 19.1 volts at 21.5°.
(f) 19.1 volts at 21.5° (same as in problem e).
(g) Phase.
(h) 180° out of phase.
(i) $Y = 7.99$ volts, $X = 6.02$ volts.
(j) 9.1 units at 114°.
(k) $X = 3$ units, $Y = 4$ units; $Y = 4.36$ units, $X = -6.7$ units; $-3.7 + j8.36$ units.

APPENDIX C

ANSWERS FOR REMEDIAL WORK

SECTION I

1. $\frac{1}{360}$
2. 60; 60; 3,600.
3. Straight.
4. 828°.
5. $\frac{1}{5}$; $\frac{3}{8}$; $\frac{5}{9}$.
6. $\angle a = 35°$; $\angle b = 145°$; $\angle c = 35°$; $\angle d = 145°$
7. No. The sum of $\angle a$ and $\angle d$ must equal a straight angle (180°), therefore if $\angle a$ is acute (less than 90°) then $\angle d$ must be obtuse (greater than 90°).
8. Acute; right; obtuse; straight.
9. Parallax.
10. (a) 42°32′45″; (b) 31°01′25″; (c) 17°00′50″; (d) As applied to navigation 349°11′30″.
11. 42.5°; 31.0°.
12. Positive; negative.
13. Counterclockwise.
14. 334°.
15. 035°.
16. 78°.
17.

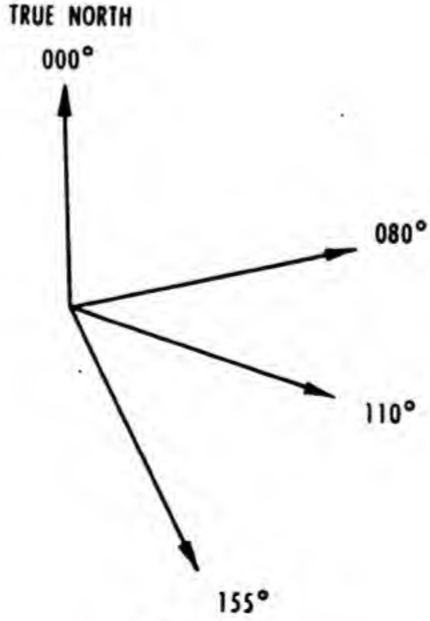


Figure 265.

18. 037°.

19. Yes.

20. He read the wrong scale on the protractor.

SECTION II

1.

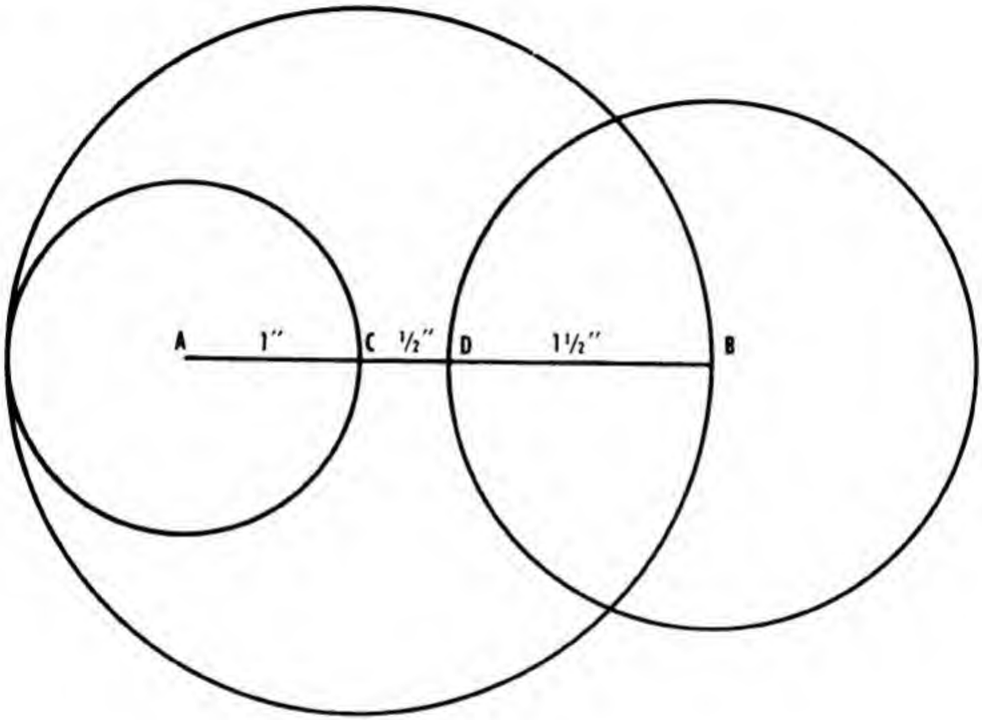


Figure 266.

2.

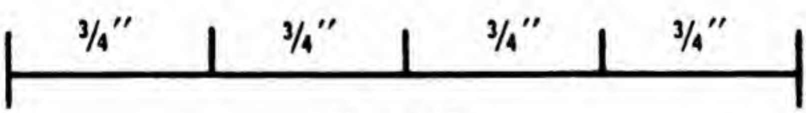


Figure 267.

3. $RS = 19$ units;
   $TZ = 16$ units;
   $JM = 7\frac{1}{2}$ units;
   $XY = 25$ units.

4. See paragraph 22.

921364—51——19

5.

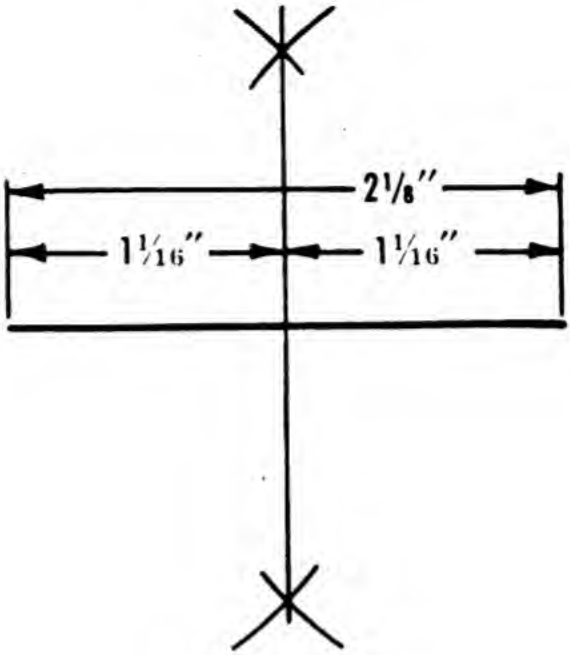


Figure 268.

6.

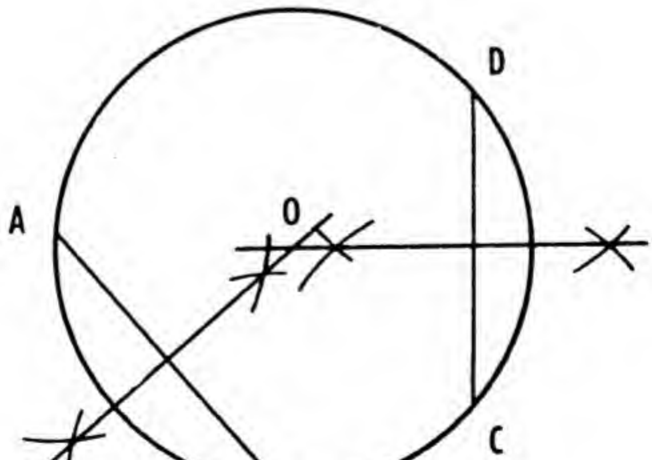


Figure 269.

7.

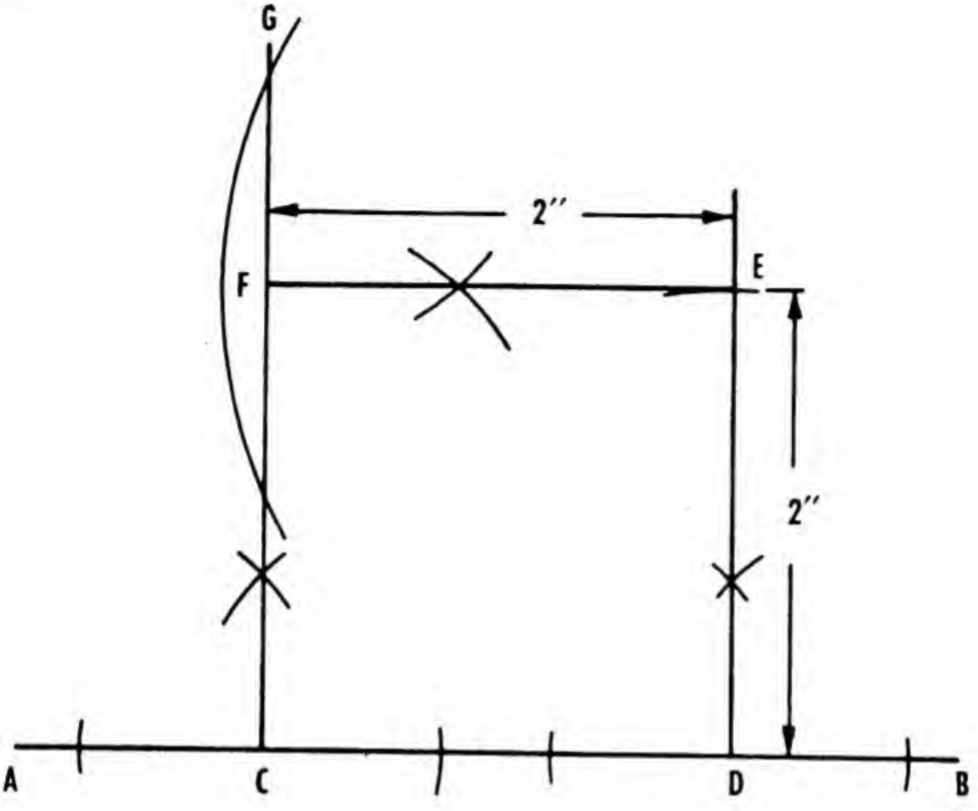


Figure 270.

8.

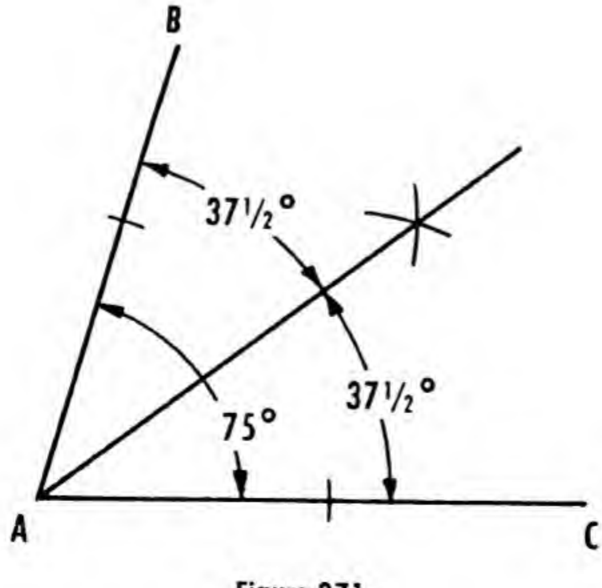


Figure 271.

9. See paragraph 28; perimeter = 6 inches.
10. See paragraph 30; the smallest angle is about $26\frac{1}{2}°$, formed by the base and a diagonal.
11. See paragraph 33.
12. See paragraph 34.
13. See paragraph 35 (b).
14. (b) See paragraph 37.
15. (a) $AC = 1\frac{1}{2}''$, $BC = 1\frac{1}{16}''$.
    (b) $\angle A = 61°$, $\angle C = 37°$.
    (c) $\angle A = 90°$;
    $\angle B = 37°$;
    $\angle C = 53°$.

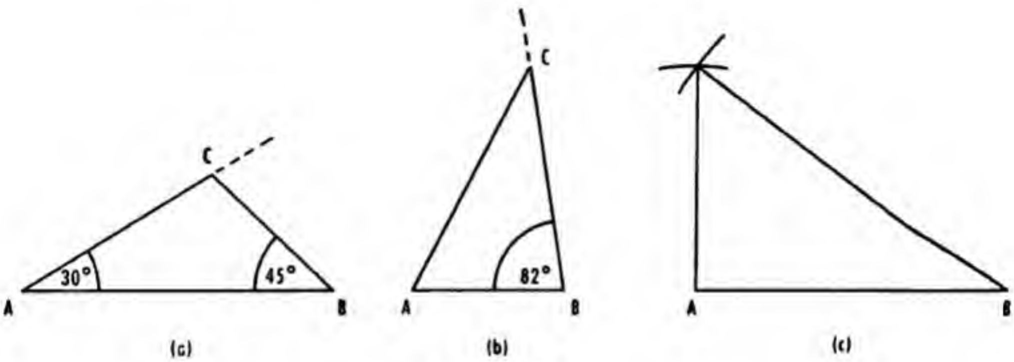


Figure 272.

SECTION III

1. $\angle a = \angle x$, vertical angles are equal. See text, paragraph 58. $\angle b = \angle x$, corresponding angles are equal. See text, paragraph 42. $\angle b = \angle c$, vertical angles are equal. $\angle c = \angle x$, both $\angle c$ and $\angle x$ are equal to $\angle b$.
2.

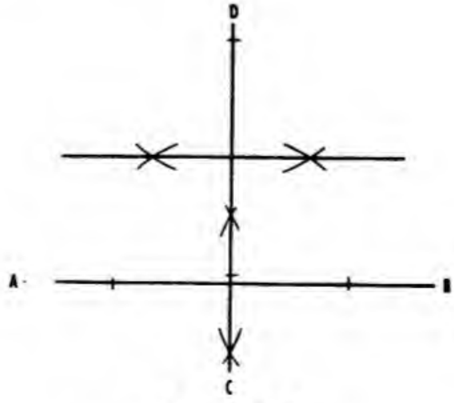


Figure 273.

3. Your figure should show all construction lines. Check the right angles of all perpendiculars by using a protractor.

4.

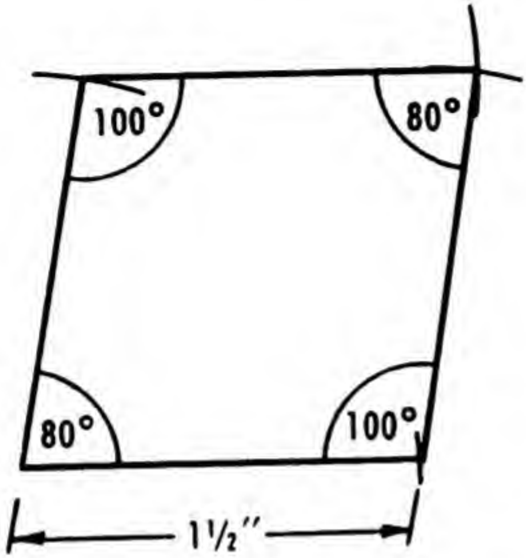


Figure 274.

5.

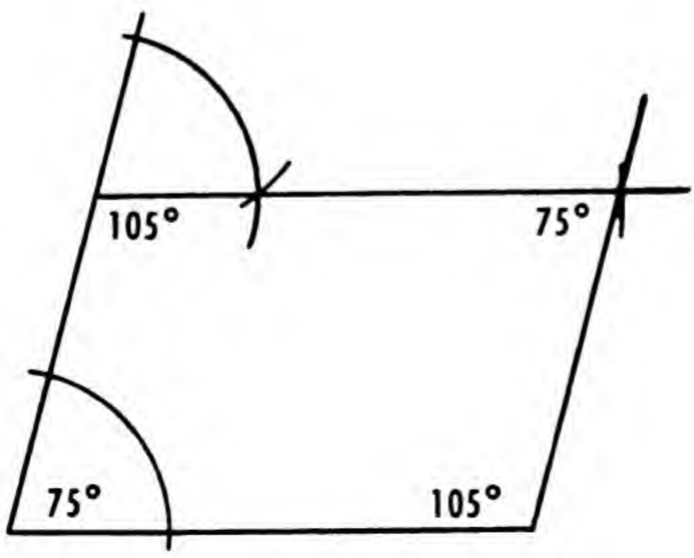


Figure 275.

6.

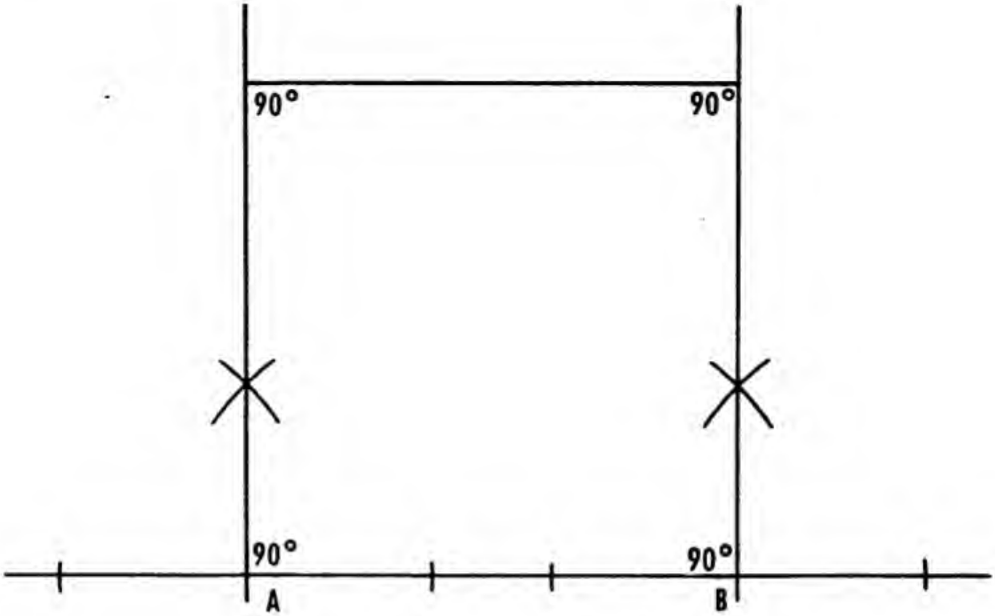


Figure 276.

7.

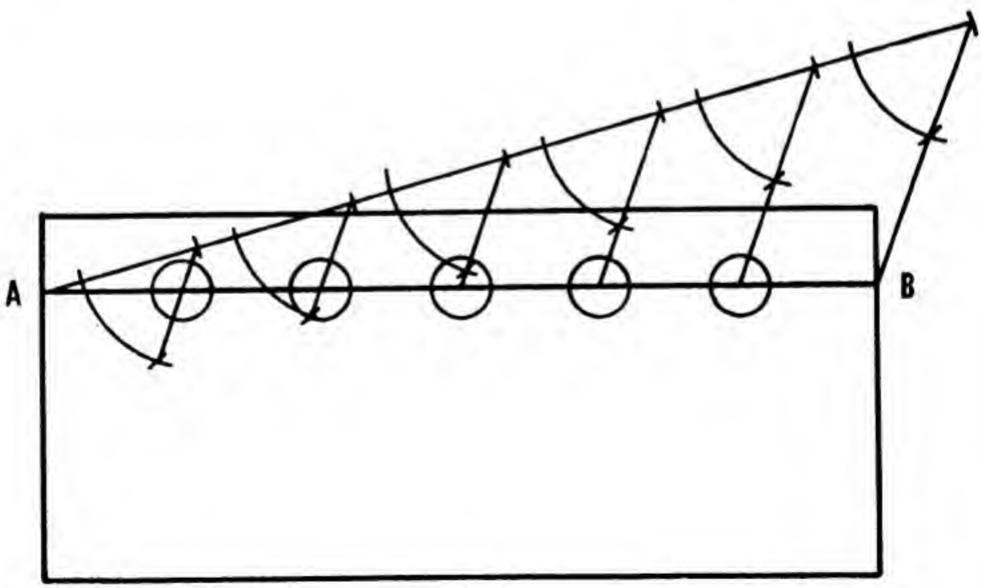


Figure 277.

8.

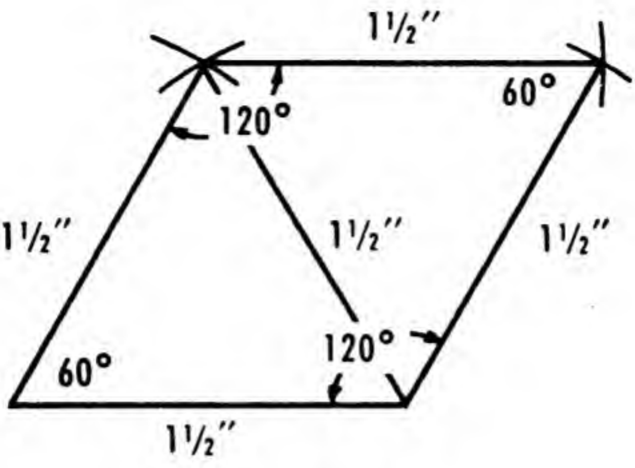


Figure 278.

9. Each base angle is 70°.

10. (a) 3″; (b) 4½″; (c) 3¾″; (d) 5″.

SECTION IV

1. See conditions (a) and (c) of paragraph 67.

2.

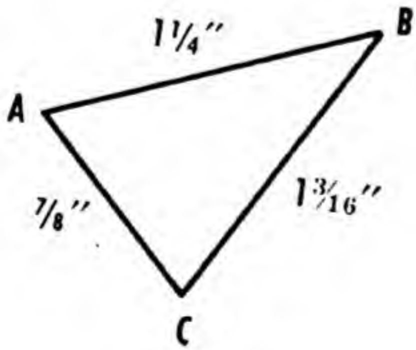


Figure 279.

3. (a) $\frac{AB}{5} = \frac{8}{12}$; $AB = 3\frac{1}{3}$; (b) $\frac{B'C}{6} = \frac{12}{8}$; $B'C' = 9$.

4.

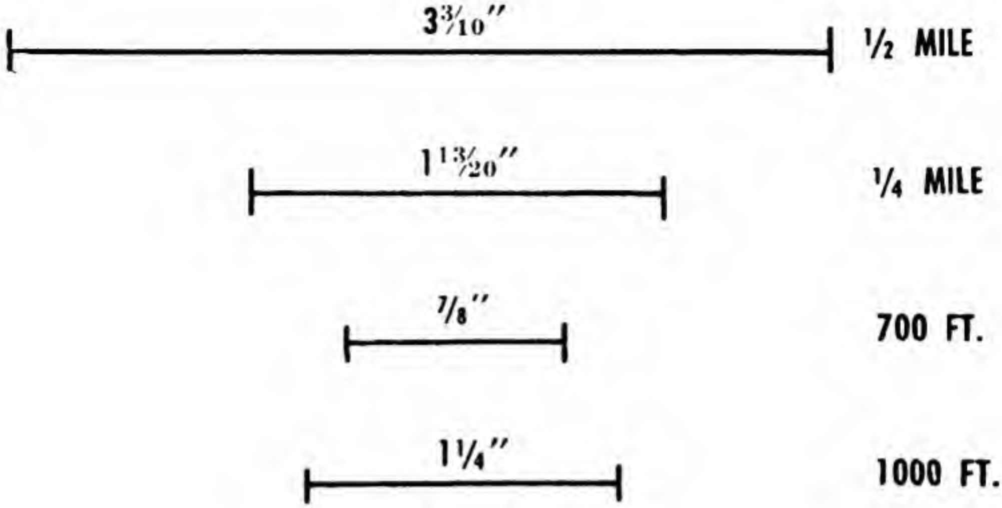


Figure 280.

5. SCALE: 1 in. = 20 miles —
   2½″ = 50 miles; 6¾″ = 135 miles; 1⅞″ = 37½ miles; 5″ = 100 miles; 7½″ = 150 miles.
6. 18 in. by 12⅜ in.
7. 1/80.
8. 25 ft.
9. See figure 73.
10. See figure 75 and 76.
11. Turned off and the wall plug removed.
12. Bad and shorted.
13. Ordinary choke coils have only two leads coming out while a transformer has three or more leads.
14. Smaller.
15. 25 volts.
16. Center to center.

SECTION V

1. Ordinate.
2. Abscissa.
3. Rectangular coordinates.
4. 17 cents; 49 percent.
5. −2 volts.
6. −5 volts.
7. None.
8. Knob *A* on position 2; knob *B* on 60.
9. 16.1 μh.

SECTION VI

1. 76.65 sq. ft.
2. (a) 15 sq. ft.; (b) 42 sq. in.; (c) 14 sq. ft.
3. (a) 120 sq. ft.; (b) 9,900 sq. yds.; (c) 570 sq. cm.
4. (a) 66 sq. in.; (b) 100 sq. in.; (c) 57.6 sq. in.
5. 78.5 sq. in.
6. 81 sq. ft.
7. 4.5 cu. ft.
8. 18,850 cu. in. or 10.91 cu. ft.
9. 342.08 lbs.
10. 4,312 lbs.
11. 7.30 cu. in.
12. 5.84 sq. in.
13. 29.50 sq. in.

SECTION VII

1. (a) $Z = 15.8$; (b) $R = 13.7$; (c) $X = 36.7$; (d) $R = 28.1$; (e) $Z = 5.83$; (f) $R = 13.4$.
2. 16.3 nautical miles.
3. Lags; 90°.
4. 76.3 ohms.
5. 180°.
6.

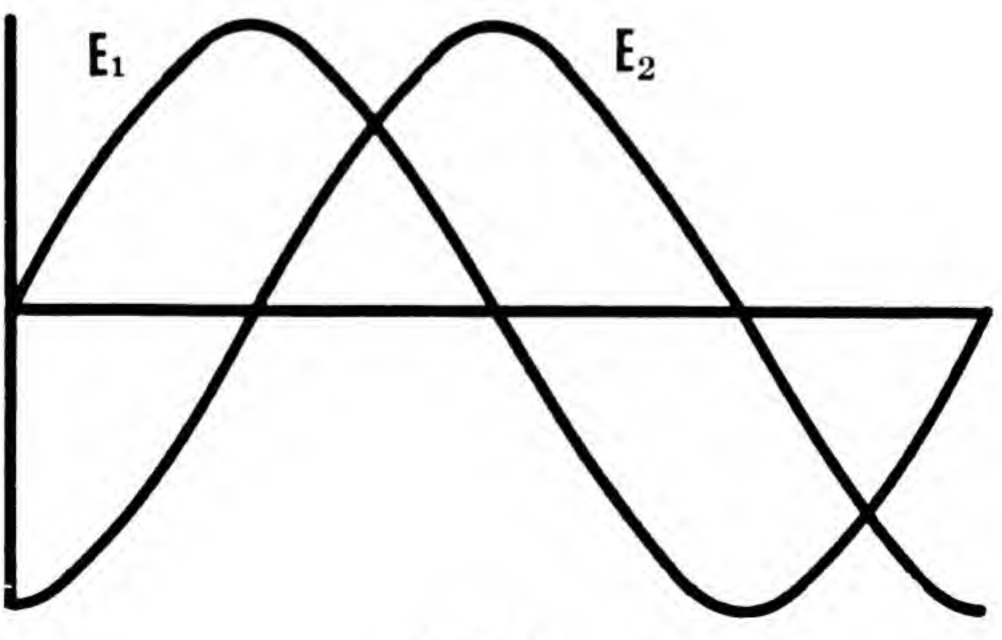


Figure 281.

7. 207.2 volts.
8. $Z = 61$ ohms; $I = 1.81$ amperes.
9. 32.5 inches.
10. $R = 51.2$ ohms.

SECTION VIII

1. 52.5°.
2. 87.7 yards.
3. 76.6°.
4. 1087.3 feet.
5. 162.6 feet.
6. (a) $R = 109.7$; $X = 87$; (b) $X = 23.8$, $Z = 66.4$; (c) $\Theta = 73.2°$, $X = 300$.
7. –160.7 volts.
8. 223.3 volts.
9. –66.1 volts.
10. 0 volts.
11. (a) 0.524 radian; (b) 5.99 radians; (c) 1.07 radians.
12. (a) 177.6°; (b) 22.9°; (c) 418.3°.

SECTION IX

1. (a) 21.4 volts at 32.8°.
   (b) 154.8 volts at 69.6°.
   (c) 24.5 volts at 0°.
   (d) 63.6 volts at 239.9°.
   (e) 80.6 volts at –45°.
2. (a) $x = 24$, $y = 18$; $24 + j18$.
   (b) $x = 5.1$, $y = 13.5$; $5.1 + j13.5$.
   (c) $x = -113.6$, $y = 27.8$; $-113.6 + j27.8$.
   (d) $x = 0$, $y = -40$; $-j40$.
   (e) $x = 47.3$, $y = -175.7$; $47.3 - j175.7$.
   (f) $x = 46.8$, $y = 22.6$; $46.8 + j22.6$.
   (g) $x = -9.1$, $y = 18.2$; $-9.1 + j18.2$.
   (h) $x = 18.3$, $y = -6.45$; $18.3 - j6.45$.
   (i) $x = -101$, $y = 0$; -101.
3. Refer to appendix B, answers to pretests, for answers.

APPENDIX D

TABLES

Table I.—Natural Sines, Cosines, and Tangents
0°–14.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Table I.—Natural Sines, Cosines, and Tangents.—(*Continued*)
15°–29.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Table I.—Natural Sines, Cosines, and Tangents.—(*Continued*)

30°–44.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8509	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5105	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7076	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7313	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.0293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7060	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Table I.—Natural Sines, Cosines, and Tangents.—(*Continued*)

45°–59.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Table I.—Natural Sines, Cosines, and Tangents.—(*Continued*)

60°–74.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
70	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
Degs.	Function	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'

Table I.—Natural Sines, Cosines, and Tangents.—(*Continued*)

75°–89.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, LOGARITHMS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS OF Nos. FROM 1 TO 1000

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia.	
							Circum.	Area
1	1	1	1.0000	1.0000	0.00000	1000.000	3.142	0.7854
2	4	8	1.4142	1.2599	0.30103	500.000	6.283	3.1416
3	9	27	1.7321	1.4422	0.47712	333.333	9.425	7.0686
4	16	64	2.0000	1.5874	0.60206	250.000	12.566	12.5664
5	25	125	2.2361	1.7100	0.69897	200.000	15.708	19.6350
6	36	216	2.4495	1.8171	0.77815	166.667	18.850	28.2743
7	49	343	2.6458	1.9129	0.84510	142.857	21.991	38.4845
8	64	512	2.8284	2.0000	0.90308	125.000	25.133	50.2655
9	81	729	3.0000	2.0801	0.95424	111.111	28.274	63.6173
10	100	1000	3.1623	2.1544	1.00000	100.000	31.416	78.5398
11	121	1331	3.3166	2.2240	1.04139	90.9091	34.558	95.0332
12	144	1728	3.4641	2.2894	1.07918	83.3333	37.699	113.097
13	169	2197	3.6056	2.3513	1.11394	76.9231	40.841	132.732
14	196	2744	3.7417	2.4101	1.14613	71.4286	43.982	153.938
15	225	3375	3.8730	2.4662	1.17609	66.6667	47.124	176.715
16	256	4096	4.0000	2.5198	1.20412	62.5000	50.265	201.062
17	289	4913	4.1231	2.5713	1.23045	58.8235	53.407	226.980
18	324	5832	4.2426	2.6207	1.25527	55.5556	56.549	254.469
19	361	6859	4.3589	2.6684	1.27875	52.6316	59.690	283.529
20	400	8000	4.4721	2.7144	1.30103	50.0000	62.832	314.159
21	441	9261	4.5826	2.7589	1.32222	47.6190	65.973	346.361
22	484	10648	4.6904	2.8020	1.34242	45.4545	69.115	380.133
23	529	12167	4.7958	2.8439	1.36173	43.4783	72.257	415.476
24	576	13824	4.8990	2.8845	1.38021	41.6667	75.398	452.389
25	625	15625	5.0000	2.9240	1.39794	40.0000	78.540	490.874
26	676	17576	5.0990	2.9625	1.41497	38.4615	81.681	530.929
27	729	19683	5.1962	3.0000	1.43136	37.0370	84.823	572.555
28	784	21952	5.2915	3.0366	1.44716	35.7143	87.965	615.752
29	841	24389	5.3852	3.0723	1.46240	34.4828	91.106	660.520
30	900	27000	5.4772	3.1072	1.47712	33.3333	94.248	706.858
31	961	29791	5.5678	3.1414	1.49136	32.2581	97.389	754.768
32	1024	32768	5.6569	3.1748	1.50515	31.2500	100.531	804.248
33	1089	35937	5.7446	3.2075	1.51851	30.3030	103.673	855.299
34	1156	39304	5.8310	3.2396	1.53148	29.4118	106.814	907.920
35	1225	42875	5.9161	3.2711	1.54407	28.5714	109.956	962.113
36	1296	46656	6.0000	3.3019	1.55630	27.7778	113.097	1017.88
37	1369	50653	6.0828	3.3322	1.56820	27.0270	116.239	1075.21
38	1444	54872	6.1644	3.3620	1.57978	26.3158	119.381	1134.11
39	1521	59319	6.2450	3.3912	1.59106	25.6410	122.522	1194.59
40	1600	64000	6.3246	3.4200	1.60206	25.0000	125.66	1256.64
41	1681	68921	6.4031	3.4482	1.61278	24.3902	128.81	1320.25
42	1764	74088	6.4807	3.4760	1.62325	23.8095	131.95	1385.44
43	1849	79507	6.5574	3.5034	1.63347	23.2558	135.09	1452.20
44	1936	85184	6.6332	3.5303	1.64345	22.7273	138.23	1520.53

921364—51——20

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
45	2025	91125	6.7082	3.5569	1.65321	22.2222	141.37	1590.43
46	2116	97336	6.7823	3.5830	1.66276	21.7391	144.51	1661.90
47	2209	103823	6.8557	3.6088	1.67210	21.2766	147.65	1734.94
48	2304	110592	6.9282	3.6342	1.68124	20.8333	150.80	1809.56
49	2401	117649	7.0000	3.6593	1.69020	20.4082	153.94	1885.74
50	2500	125000	7.0711	3.6840	1.69897	20.0000	157.08	1963.50
51	2601	132651	7.1414	3.7084	1.70757	19.6078	160.22	2042.82
52	2704	140608	7.2111	3.7325	1.71600	19.2308	163.36	2123.72
53	2809	148877	7.2801	3.7563	1.72428	18.8679	166.50	2206.18
54	2916	157464	7.3485	3.7798	1.73239	18.5185	169.65	2290.22
55	3025	166375	7.4162	3.8030	1.74036	18.1818	172.79	2375.83
56	3136	175616	7.4833	3.8259	1.74819	17.8571	175.93	2463.01
57	3249	185193	7.5498	3.8485	1.75587	17.5439	179.07	2551.76
58	3364	195112	7.6158	3.8709	1.76343	17.2414	182.21	2642.08
59	3481	205379	7.6811	3.8930	1.77085	16.9492	185.35	2733.97
60	3600	216000	7.7460	3.9149	1.77815	16.6667	188.50	2827.43
61	3721	226981	7.8102	3.9365	1.78533	16.3934	191.64	2922.47
62	3844	238328	7.8740	3.9579	1.79239	16.1290	194.78	3019.07
63	3969	250047	7.9373	3.9791	1.79934	15.8730	197.92	3117.25
64	4096	262144	8.0000	4.0000	1.80618	15.6250	201.06	3216.99
65	4225	274625	8.0623	4.0207	1.81291	15.3846	204.20	3318.31
66	4356	287496	8.1240	4.0412	1.81954	15.1515	207.35	3421.19
67	4489	300763	8.1854	4.0615	1.82607	14.9254	210.49	3525.65
68	4624	314432	8.2462	4.0817	1.83251	14.7059	213.63	3631.68
69	4761	328509	8.3066	4.1016	1.83885	14.4928	216.77	3739.28
70	4900	343000	8.3666	4.1213	1.84510	14.2857	219.91	3848.45
71	5041	357911	8.4261	4.1408	1.85126	14.0845	223.05	3959.19
72	5184	373248	8.4853	4.1602	1.85733	13.8889	226.19	4071.50
73	5329	389017	8.5440	4.1793	1.86332	13.6986	229.34	4185.39
74	5476	405224	8.6023	4.1983	1.86923	13.5135	232.48	4300.84
75	5625	421875	8.6603	4.2172	1.87506	13.3333	235.62	4417.86
76	5776	438976	8.7178	4.2358	1.88081	13.1579	238.76	4536.46
77	5929	456533	8.7750	4.2543	1.88649	12.9870	241.90	4656.63
78	6084	474552	8.8318	4.2727	1.89209	12.8205	245.04	4778.36
79	6241	493039	8.8882	4.2908	1.89763	12.6582	248.19	4901.67
80	6400	512000	8.9443	4.3089	1.90309	12.5000	251.33	5026.55
81	6561	531441	9.0000	4.3267	1.90849	12.3457	254.47	5153.00
82	6724	551368	9.0554	4.3445	1.91381	12.1951	257.61	5281.02
83	6889	571787	9.1104	4.3621	1.91908	12.0482	260.75	5410.61
84	7056	592704	9.1652	4.3795	1.92428	11.9048	263.89	5541.77
85	7225	614125	9.2195	4.3968	1.92942	11.7647	267.04	5674.50
86	7396	636056	9.2736	4.4140	1.93450	11.6279	270.18	5808.80
87	7569	658503	9.3274	4.4310	1.93952	11.4943	273.32	5944.68
88	7744	681472	9.3808	4.4480	1.94448	11.3636	276.46	6082.12
89	7921	704969	9.4340	4.4647	1.94939	11.2360	279.60	6221.14

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
90	8100	729000	9.4868	4.4814	1.95424	11.1111	282.74	6361.73
91	8281	753571	9.5394	4.4979	1.95904	10.9890	285.88	6503.88
92	8464	778688	9.5917	4.5144	1.96379	10.8696	289.03	6647.61
93	8649	804357	9.6437	4.5307	1.96848	10.7527	292.17	6792.91
94	8836	830584	9.6954	4.5468	1.97313	10.6383	295.31	6939.78
95	9025	857375	9.7468	4.5629	1.97772	10.5263	298.45	7088.22
96	9216	884736	9.7980	4.5789	1.98227	10.4167	301.59	7238.23
97	9409	912673	9.8489	4.5947	1.98677	10.3093	304.73	7389.81
98	9604	941192	9.8995	4.6104	1.99123	10.2041	307.88	7542.96
99	9801	970299	9.9499	4.6261	1.99564	10.1010	311.02	7697.69
100	10000	1000000	10.0000	4.6416	2.00000	10.00000	314.16	7853.98
101	10201	1030301	10.0499	4.6570	2.00432	9.90099	317.30	8011.85
102	10404	1061208	10.0995	4.6723	2.00860	9.80392	320.44	8171.28
103	10609	1092727	10.1489	4.6875	2.01284	9.70874	323.58	8332.29
104	10816	1124864	10.1980	4.7027	2.01703	9.61538	326.73	8494.87
105	11025	1157625	10.2470	4.7177	2.02119	9.52381	329.87	8659.01
106	11236	1191016	10.2956	4.7326	2.02531	9.43396	333.01	8824.73
107	11449	1225043	10.3441	4.7475	2.02938	9.34579	336.15	8992.02
108	11664	1259712	10.3923	4.7622	2.03342	9.25926	339.29	9160.88
109	11881	1295029	10.4403	4.7769	2.03743	9.17431	342.43	9331.32
110	12100	1331000	10.4881	4.7914	2.04139	9.09091	345.58	9503.32
111	12321	1367631	10.5357	4.8059	2.04532	9.00901	348.72	9676.89
112	12544	1404928	10.5830	4.8203	2.04922	8.92857	351.86	9852.03
113	12769	1442897	10.6301	4.8346	2.05308	8.84956	355.00	10028.7
114	12996	1481544	10.6771	4.8488	2.05690	8.77193	358.14	10207.0
115	13225	1520875	10.7238	4.8629	2.06070	8.69565	361.28	10386.9
116	13456	1560896	10.7703	4.8770	2.06446	8.62069	364.42	10568.3
117	13689	1601613	10.8167	4.8910	2.06819	8.54701	367.57	10751.3
118	13924	1643032	10.8628	4.9049	2.07188	8.47458	370.71	10935.9
119	14161	1685159	10.9087	4.9187	2.07555	8.40336	373.85	11122.0
120	14400	1728000	10.9545	4.9324	2.07918	8.33333	376.99	11309.7
121	14641	1771561	11.0000	4.9461	2.08279	8.26446	380.13	11499.0
122	14884	1815848	11.0454	4.9597	2.08636	8.19672	383.27	11689.9
123	15129	1860867	11.0905	4.9732	2.08991	8.13008	386.42	11882.3
124	15376	1906624	11.1355	4.9866	2.09342	8.06452	389.56	12076.3
125	15625	1953125	11.1803	5.0000	2.09691	8.00000	392.70	12271.8
126	15876	2000376	11.2250	5.0133	2.10037	7.93651	395.84	12469.0
127	16129	2048383	11.2694	5.0265	2.10380	7.87402	398.98	12667.7
128	16384	2097152	11.3137	5.0397	2.10721	7.81250	402.12	12868.0
129	16641	2146689	11.3578	5.0528	2.11059	7.75194	405.27	13069.8
130	16900	2197000	11.4018	5.0653	2.11394	7.69231	408.41	13273.2
131	17161	2248091	11.4455	5.0788	2.11727	7.63359	411.55	13478.2
132	17424	2299968	11.4891	5.0916	2.12057	7.57576	414.69	13684.8
133	17689	2352637	11.5326	5.1045	2.12385	7.51880	417.83	13892.9
134	17956	2406104	11.5758	5.1172	2.12710	7.46269	420.97	14102.6

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	Area
135	18225	2460375	11.6190	5.1299	2.13033	7 40741	424.12	14313.9
136	18496	2515456	11.6619	5.1426	2.13354	7.35294	427.26	14526.7
137	18769	2571353	11.7047	5.1551	2.13672	7.29927	430.40	14741.1
138	19044	2628072	11.7473	5.1676	2.13988	7.24638	433 54	14957.1
139	19321	2685619	11.7898	5.1801	2.14301	7.19424	436.68	15174.7
140	19600	2744000	11.8322	5.1925	2.14613	7.14286	439.82	15393.8
141	19881	2803221	11.8743	5.2048	2.14922	7.09220	442.96	15614.5
142	20164	2863288	11.9164	5.2171	2.15229	7.04225	446 11	15836.8
143	20449	2924207	11.9583	5.2293	2.15534	6.99301	449.25	16060.6
144	20736	2985984	12.0000	5.2415	2.15836	6.94444	452.39	16286.0
145	21025	3048625	12.0416	5.2536	2.16137	6.89655	455.53	16513.0
146	21316	3112136	12.0830	5.2656	2.16435	6.84932	458.67	16741.5
147	21609	3176523	12.1244	5.2776	2.16732	6.80272	461.81	16971.7
148	21904	3241792	12.1655	5.2896	2.17026	6.75676	464 96	17203.4
149	22201	3307949	12.2066	5.3015	2.17319	6.71141	468.10	17436.6
150	22500	3375000	12.2474	5.3133	2.17609	6.66667	471.24	17671.5
151	22801	3442951	12.2882	5.3251	2.17898	6.62252	474.38	17907.9
152	23104	3511808	12.3288	5.3368	2.18184	6.57895	477.52	18145.8
153	23409	3581577	12.3693	5.3485	2.18469	6.53595	480.66	18385.4
154	23716	3652264	12.4097	5.3601	2.18752	6.49351	483.81	18626.5
155	24025	3723875	12.4499	5.3717	2.19033	6.45161	486.95	18869.2
156	24336	3796416	12.4900	5.3832	2.19312	6.41026	490.09	19113.4
157	24649	3869893	12.5300	5.3947	2.19590	6.36943	493.23	19359.3
158	24964	3944312	12.5698	5.4061	2.19866	6.32911	496.37	19606.7
159	25281	4019679	12.6095	5.4175	2.20140	6.28931	499.51	19855.7
160	25600	4096000	12.6491	5.4288	2.20412	6.25000	502.65	20106.2
161	25921	4173281	12.6886	5.4401	2.20683	6.21118	505.80	20358.3
162	26244	4251528	12.7279	5.4514	2.20952	6.17284	508.94	20612.0
163	26569	4330747	12.7671	5.4626	2.21219	6.13497	512.08	20867.2
164	26896	4410944	12.8062	5.4737	2.21484	6.09756	515.22	21124.1
165	27225	4492125	12.8452	5.4848	2.21748	6.06061	518.36	21382.5
166	27556	4574296	12.8841	5.4959	2.22011	6.02410	521.50	21642.4
167	27889	4657463	12.9228	5.5069	2.22272	5.98802	524.65	21904.0
168	28224	4741632	12.9615	5.5178	2.22531	5.95238	527.79	22167.1
169	28561	4826809	13.0000	5.5288	2.22789	5.91716	530.93	22431.8
170	28900	4913000	13.0384	5.5397	2.23045	5.88235	534.07	22698.0
171	29241	5000211	13.0767	5.5505	2.23300	5.84795	537.21	22965.8
172	29584	5088448	13.1149	5.5613	2.23553	5.81395	540.35	23235.2
173	29929	5177717	13.1529	5.5721	2.23805	5.78035	543.50	23506.2
174	30276	5268024	13.1909	5.5828	2.24055	5.74713	546.64	23778.7
175	30625	5359375	13.2288	5.5934	2.24304	5.71429	549.78	24052.8
176	30976	5451776	13.2665	5.6041	2.24551	5.68182	552.92	24328.5
177	31329	5545233	13.3041	5.6147	2.24797	5.64972	556.06	24605.7
178	31684	5639752	13.3417	5.6252	2.25042	5.61798	559.20	24884.6
179	32041	5735339	13.3791	5.6357	2.25285	5.58659	562.35	25164.9

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
180	32400	5832000	13.4164	5.6462	2.25527	5.55556	565.49	25446.9
181	32761	5929741	13.4536	5.6567	2.25768	5.52486	568.63	25730.4
182	33124	6028568	13.4907	5.6671	2.26007	5.49451	571.77	26015.5
183	33489	6128487	13.5277	5.6774	2.26245	5.46448	574.91	26302.2
184	33856	6229504	13.5647	5.6877	2.26482	5.43478	578.05	26590.4
185	34225	6331625	13.6015	5.6980	2.26717	5.40541	581.19	26880.3
186	34596	6434856	13.6382	5.7083	2.26951	5.37634	584.34	27171.6
187	34969	6539203	13.6748	5.7185	2.27184	5.34759	587.48	27464.6
188	35344	6644672	13.7113	5.7287	2.27416	5.31915	590.62	27759.1
189	35721	6751269	13.7477	5.7388	2.27646	5.29101	593.76	28055.2
190	36100	6859000	13.7840	5.7489	2.27875	5.26316	596.90	28352.9
191	36481	6967871	13.8203	5.7590	2.28103	5.23560	600.04	28652.1
192	36864	7077888	13.8564	5.7690	2.28330	5.20833	603.19	28952.9
193	37249	7189057	13.8924	5.7790	2.28556	5.18135	606.33	29255.3
194	37636	7301384	13.9284	5.7890	2.28780	5.15464	609.47	29559.2
195	38025	7414875	13.9642	5.7989	2.29003	5.12821	612.61	29864.8
196	38416	7529536	14.0000	5.8088	2.29226	5.10204	615.75	30171.9
197	38809	7645373	14.0357	5.8186	2.29447	5.07614	618.89	30480.5
198	39204	7762392	14.0712	5.8285	2.29667	5.05051	622.04	30790.7
199	39601	7880599	14.1067	5.8383	2.29885	5.02513	625.18	31102.6
200	40000	8000000	14.1421	5.8480	2.30103	5.00000	628.32	31415.9
201	40401	8120601	14.1774	5.8578	2.30320	4.97512	631.46	31730.9
202	40804	8242408	14.2127	5.8675	2.30535	4.95050	634.60	32047.4
203	41209	8365427	14.2478	5.8771	2.30750	4.92611	637.74	32365.5
204	41616	8489664	14.2829	5.8868	2.30963	4.90196	640.89	32685.1
205	42025	8615125	14.3178	5.8964	2.31175	4.87805	644.03	33006.4
206	42436	8741816	14.3527	5.9059	2.31387	4.85437	647.17	33329.2
207	42849	8869743	14.3875	5.9155	2.31597	4.83092	650.31	33653.5
208	43264	8998912	14.4222	5.9250	2.31806	4.80769	653.45	33979.5
209	43681	9129329	14.4568	5.9345	2.32015	4.78469	656.59	34307.0
210	44100	9261000	14.4914	5.9439	2.32222	4.76190	659.73	34636.1
211	44521	9393931	14.5258	5.9533	2.32428	4.73934	662.88	34966.7
212	44944	9528128	14.5602	5.9627	2.32634	4.71698	666.02	35298.9
213	45369	9663597	14.5945	5.9721	2.32838	4.69484	669.16	35632.7
214	45796	9800344	14.6287	5.9814	2.33041	4.67290	672.30	35968.1
215	46225	9938375	14.6629	5.9907	2.33244	4.65116	675.44	36305.0
216	46656	10077696	14.6969	6.0000	2.33445	4.62963	678.58	36643.5
217	47089	10218313	14.7309	6.0092	2.33646	4.60829	681.73	36983.6
218	47524	10360232	14.7648	6.0185	2.33846	4.58716	684.87	37325.3
219	47961	10503459	14.7986	6.0277	2.34044	4.56621	688.01	37668.5
220	48400	10648000	14.8324	6.0368	2.34242	4.54545	691.15	38013.3
221	48841	10793861	14.8661	6.0459	2.34439	4.52489	694.29	38359.6
222	49284	10941048	14.8997	6.0550	2.34635	4.50450	697.43	38707.6
223	49729	11089567	14.9332	6.0641	2.34830	4.48431	700.58	39057.1
224	50176	11239424	14.9666	6.0732	2.35025	4.46429	703.72	39408.1

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
225	50625	11390625	15.0000	6.0822	2.35218	4.44444	706.86	39760.8
226	51076	11543176	15.0333	6.0912	2.35411	4.42478	710.00	40115.0
227	51529	11697083	15.0665	6.1002	2.35603	4.40529	713.14	40470.8
228	51984	11852352	15.0997	6.1091	2.35793	4.38596	716.28	40828.1
229	52441	12008989	15.1327	6.1180	2.35984	4.36681	719.42	41187.1
230	52900	12167000	15.1658	6.1269	2.36173	4.34783	722.57	41547.6
231	53361	12326391	15.1987	6.1358	2.36361	4.32900	725.71	41909.6
232	53824	12487168	15.2315	6.1446	2.36549	4.31034	728.85	42273.3
233	54289	12649337	15.2643	6.1534	2.36736	4.29185	731.99	42638.5
234	54756	12812904	15.2971	6.1622	2.36922	4.27350	735.13	43005.3
235	55225	12977875	15.3297	6.1710	2.37107	4.25532	738.27	43373.6
236	55696	13144256	15.3623	6.1797	2.37291	4.23729	741.42	43743.5
237	56169	13312053	15.3948	6.1885	2.37475	4.21941	744.56	44115.0
238	56644	13481272	15.4272	6.1972	2.37658	4.20168	747.70	44488.1
239	57121	13651919	15.4596	6.2058	2.37840	4.18410	750.84	44862.7
240	57600	13824000	15.4919	6.2145	2.38021	4.16667	753.98	45238.9
241	58081	13997521	15.5242	6.2231	2.38202	4.14938	757.12	45616.7
242	58564	14172488	15.5563	6.2317	2.38382	4.13223	760.27	45996.1
243	59049	14348907	15.5885	6.2403	2.38561	4.11523	763.41	46377.0
244	59536	14526784	15.6205	6.2488	2.38739	4.09836	766.55	46759.5
245	60025	14706125	15.6525	6.2573	2.38917	4.08163	769.69	47143.5
246	60516	14886936	15.6844	6.2658	2.39094	4.06504	772.83	47529.2
247	61009	15069223	15.7162	6.2743	2.39270	4.04858	775.97	47916.4
248	61504	15252992	15.7480	6.2828	2.39445	4.03226	779.12	48305.1
249	62001	15438249	15.7797	6.2912	2.39620	4.01606	782.26	48695.5
250	62500	15625000	15.8114	6.2996	2.39794	4.00000	785.40	49087.4
251	63001	15813251	15.8430	6.3080	2.39967	3.98406	788.54	49480.9
252	63504	16003008	15.8745	6.3164	2.40140	3.96825	791.68	49875.9
253	64009	16194277	15.9060	6.3247	2.40312	3.95257	794.82	50272.6
254	64516	16387064	15.9374	6.3330	2.40483	3.93701	797.96	50670.7
255	65025	16581375	15.9687	6.3413	2.40654	3.92157	801.81	51070.5
256	65536	16777216	16.0000	6.3496	2.40824	3.90625	804.25	51471.9
257	66049	16974593	16.0312	6.3579	2.40993	3.89105	807.39	51874.8
258	66564	17173512	16.0624	6.3661	2.41162	3.87597	810.53	52279.2
259	67081	17373979	16.0935	6.3743	2.41330	3.86100	813.67	52685.3
260	67600	17576000	16.1245	6.3825	2.41497	3.84615	816.81	53092.9
261	68121	17779581	16.1555	6.3907	2.41664	3.83142	819.96	53502.1
262	68644	17984728	16.1864	6.3988	2.41830	3.81679	823.10	53912.9
263	69169	18191447	16.2173	6.4070	2.41996	3.80228	826.24	54325.2
264	69696	18399744	16.2481	6.4151	2.42160	3.78788	829.38	54739.1
265	70225	18609625	16.2788	6.4232	2.42325	3.77358	832.52	55154.6
266	70756	18821096	16.3095	6.4312	2.42488	3.75940	835.66	55571.6
267	71289	19034163	16.3401	6.4393	2.42651	3.74532	838.81	55990.3
268	71824	19248832	16.3707	6.4473	2.42813	3.73134	841.95	56410.4
269	72361	19465109	16.4012	6.4553	2.42975	3.71747	845.09	56832.2

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	Area
270	72900	19683000	16.4317	6.4633	2.43136	3.70370	848.23	57255.5
271	73441	19902511	16.4621	6.4713	2.43297	3.69004	851.37	57680.4
272	73984	20123648	16.4924	6.4792	2.43457	3.67647	854.51	58106.9
273	74529	20346417	16.5227	6.4872	2.43616	3.66300	857.66	58534.9
274	75076	20570824	16.5529	6.4951	2.43775	3.64964	860.80	58964.6
275	75625	20796875	16.5831	6.5030	2.43933	3.63636	863.94	59395.7
276	76176	21024576	16.6132	6.5108	2.44091	3.62319	867.08	59828.5
277	76729	21253933	16.6433	6.5187	2.44248	3.61011	870.22	60262.8
278	77284	21484952	16.6733	6.5265	2.44404	3.59712	873.36	60698.7
279	77841	21717639	16.7033	6.5343	2.44560	3.58423	876.50	61136.2
280	78400	21952000	16.7332	6.5421	2.44716	3.57143	879.65	61575.2
281	78961	22188041	16.7631	6.5499	2.44871	3.55872	882.79	62015.8
282	79524	22425768	16.7929	6.5577	2.45025	3.54610	885.93	62458.0
283	80089	22665187	16.8226	6.5654	2.45179	3.53357	889.07	62901.8
284	80656	22906304	16.8523	6.5731	2.45332	3.52113	892.21	63347.1
285	81225	23149125	16.8819	6.5808	2.45484	3.50877	895.35	63794.0
286	81796	23393656	16.9115	6.5885	2.45637	3.49650	898.50	64242.4
287	82369	23639903	16.9411	6.5962	2.45788	3.48432	901.64	64692.5
288	82944	23887872	16.9706	6.6039	2.45939	3.47222	904.78	65144.1
289	83521	24137569	17.0000	6.6115	2.46090	3.46021	907.92	65597.2
290	84100	24389000	17.0294	6.6191	2.46240	3.44828	911.06	66052.0
291	84681	24642171	17.0587	6.6267	2.46389	3.43643	914.20	66508.3
292	85264	24897088	17.0880	6.6343	2.46538	3.42466	917.35	66966.2
293	85849	25153757	17.1172	6.6419	2.46687	3.41297	920.49	67425.6
294	86436	25412184	17.1464	6.6494	2.46835	3.40136	923.63	67886.7
295	87025	25672375	17.1756	6.6569	2.46982	3.38983	926.77	68349.3
296	87616	25934336	17.2047	6.6644	2.47129	3.37838	929.91	68813.5
297	88209	26198073	17.2337	6.6719	2.47276	3.36700	933.05	69279.2
298	88804	26463592	17.2627	6.6794	2.47422	3.35570	936.19	69746.5
299	89401	26730899	17.2916	6.6869	2.47567	3.34448	939.34	70215.4
300	90000	27000000	17.3205	6.6943	2.47712	3.33333	942.48	70685.8
301	90601	27270901	17.3494	6.7018	2.47857	3.32226	945.62	71157.9
302	91204	27543608	17.3781	6.7092	2.48001	3.31126	948.76	71631.5
303	91809	27818127	17.4069	6.7166	2.48144	3.30033	951.90	72106.6
304	92416	28094464	17.4356	6.7240	2.48287	3.28947	955.04	72583.4
305	93025	28372625	17.4642	6.7313	2.48430	3.27869	958.19	73061.7
306	93636	28652616	17.4929	6.7387	2.48572	3.26797	961.33	73541.5
307	94249	28934443	17.5214	6.7460	2.48714	3.25733	964.47	74023.0
308	94864	29218112	17.5499	6.7533	2.48855	3.24675	967.61	74506.0
309	95481	29503629	17.5784	6.7606	2.48996	3.23625	970.75	74990.6
310	96100	29791000	17.6068	6.7679	2.49136	3.22581	973.89	75476.8
311	96721	30080231	17.6352	6.7752	2.49276	3.21543	977.04	75964.5
312	97344	30371328	17.6635	6.7824	2.49415	3.20513	980.18	76453.8
313	97969	30664297	17.6918	6.7897	2.49554	3.19489	983.32	76944.7
314	98596	30959144	17.7200	6.7969	2.49693	3.18471	986.46	77437.1

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
315	99225	31255875	17.7482	6.8041	2.49831	3.17460	989.60	77931.1
316	99856	31554496	17.7764	6.8113	2.49969	3.16456	992.74	78426.7
317	100489	31855013	17.8045	6.8185	2.50106	3.15457	995.88	78923.9
318	101124	32157432	17.8326	6.8256	2.50243	3.14465	999.03	79422.6
319	101761	32461759	17.8606	6.8328	2.50379	3.13480	1002.2	79922.9
320	102400	32768000	17.8885	6.8399	2.50515	3.12500	1005.3	80424.8
321	103041	33076161	17.9165	6.8470	2.50651	3.11527	1008.5	80928.2
322	103684	33386248	17.9444	6.8541	2.50786	3.10559	1011.6	81433.2
323	104329	33698267	17.9722	6.8612	2.50920	3.09598	1014.7	81939.8
324	104976	34012244	18.0000	6.8683	2.51055	3.08642	1017.9	82448.0
325	105625	34328125	18.0278	6.8753	2.51188	3.07692	1021.0	82957.7
326	106276	34645976	18.0555	6.8824	2.51322	3.06749	1024.2	83469.0
327	106929	34965783	18.0831	6.8894	2.51455	3.05810	1027.3	83981.8
328	107584	35287552	18.1108	6.8964	2.51587	3.04878	1030.4	84496.3
329	108241	35611289	18.1384	6.9034	2.51720	3.03951	1033.6	85012.3
330	108900	35937000	18.1659	6.9104	2.51851	3.03030	1036.7	85529.9
331	109561	36264691	18.1934	6.9174	2.51983	3.02115	1039.9	86049.0
332	110224	36594368	18.2209	6.9244	2.52114	3.01205	1043.0	86569.7
333	110889	36926037	18.2483	6.9313	2.52244	3.00300	1046.2	87092.0
334	111556	37259704	18.2757	6.9382	2.52375	2.99401	1049.3	87615.9
335	112225	37595375	18.3030	6.9451	2.52504	2.98507	1052.4	88141.3
336	112896	37933056	18.3303	6.9521	2.52634	2.97619	1055.6	88668.3
337	113569	38272753	18.3576	6.9589	2.52763	2.96736	1058.7	89196.9
338	114244	38614472	18.3848	6.9658	2.52892	2.95858	1061.9	89727.0
339	114921	38958219	18.4120	6.9727	2.53020	2.94985	1065.0	90258.7
340	115600	39304000	18.4391	6.9795	2.53148	2.94118	1068.1	90792.0
341	116281	39651821	18.4662	6.9864	2.53275	2.93255	1071.3	91326.9
342	116964	40001688	18.4932	6.9932	2.53403	2.92398	1074.4	91863.3
343	117649	40353607	18.5203	7.0000	2.53529	2.91545	1077.6	92401.3
344	118336	40707584	18.5472	7.0068	2.53656	2.90698	1080.7	92940.9
345	119025	41063625	18.5742	7.0136	2.53782	2.89855	1083.8	93482.0
346	119716	41421736	18.6011	7.0203	2.53908	2.89017	1087.0	94024.7
347	120409	41781923	18.6279	7.0271	2.54033	2.88184	1090.1	94569.0
348	121104	42144192	18.6548	7.0338	2.54158	2.87356	1093.3	95114.9
349	121801	42508549	18.6815	7.0406	2.54283	2.86533	1096.4	95662.3
350	122500	42875000	18.7083	7.0473	2.54407	2.85714	1099.6	96211.3
351	123201	43243551	18.7350	7.0540	2.54531	2.84900	1102.7	96761.8
352	123904	43614208	18.7617	7.0607	2.54654	2.84091	1105.8	97314.0
353	124609	43986977	18.7883	7.0674	2.54777	2.83286	1109.0	97867.7
354	125316	44361864	18.8149	7.0740	2.54900	2.82486	1112.1	98423.0
355	126025	44738875	18.8414	7.0807	2.55023	2.81690	1115.3	98979.8
356	126736	45118016	18.8680	7.0873	2.55145	2.80899	1118.4	99538.2
357	127449	45499293	18.8944	7.0940	2.55267	2.80112	1121.5	100098.0
358	128164	45882712	18.9209	7.1006	2.55388	2.79330	1124.7	100660.0
359	128881	46268279	18.9473	7.1072	2.55509	2.78552	1127.8	101223.0

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
360	129600	46656000	18.9737	7.1138	2.55630	2.77778	1131.0	101788
361	130321	47045881	19.0000	7.1204	2.55751	2.77008	1134.1	102354
362	131044	47437928	19.0263	7.1269	2.55871	2.76243	1137.3	102922
363	131769	47832147	19.0526	7.1335	2.55991	2.75482	1140.4	103491
364	132496	48228544	19.0788	7.1400	2.56110	2.74725	1143.5	104062
365	133225	48627125	19.1050	7.1466	2.56229	2.73973	1146.7	104635
366	133956	49027896	19.1311	7.1531	2.56348	2.73224	1149.8	105209
367	134689	49430863	19.1572	7.1596	2.56467	2.72480	1153.0	105785
368	135424	49836032	19.1833	7.1661	2.56585	2.71739	1156.1	106362
369	136161	50243409	19.2094	7.1726	2.56703	2.71003	1159.2	106941
370	136900	50653000	19.2354	7.1791	2.56820	2.70270	1162.4	107521
371	137641	51064811	19.2614	7.1855	2.56937	2.69542	1165.5	108103
372	138384	51478848	19.2873	7.1920	2.57054	2.68817	1168.7	108687
373	139129	51895117	19.3132	7.1984	2.57171	2.68097	1171.8	109272
374	139876	52313624	19.3391	7.2048	2.57287	2.67380	1175.0	109858
375	140625	52734375	19.3649	7.2112	2.57403	2.66667	1178.1	110447
376	141376	53157376	19.3907	7.2177	2.57519	2.65957	1181.2	111036
377	142129	53582633	19.4165	7.2240	2.57634	2.65252	1184.4	111628
378	142884	54010152	19.4422	7.2304	2.57749	2.64550	1187.5	112221
379	143641	54439939	19.4679	7.2368	2.57864	2.63852	1190.7	112815
380	144400	54872000	19.4936	7.2432	2.57978	2.63158	1193.8	113411
381	145161	55306341	19.5192	7.2495	2.58093	2.62467	1196.9	114009
382	145924	55742968	19.5448	7.2558	2.58206	2.61780	1200.1	114608
383	146689	56181887	19.5704	7.2622	2.58320	2.61097	1203.2	115209
384	147456	56623104	19.5959	7.2685	2.58433	2.60417	1206.4	115812
385	148225	57066625	19.6214	7.2748	2.58546	2.59740	1209.5	116416
386	148996	57512456	19.6469	7.2811	2.58659	2.59067	1212.7	117021
387	149769	57960603	19.6723	7.2874	2.58771	2.58398	1215.8	117628
388	150544	58411072	19.6977	7.2936	2.58883	2.57732	1218.9	118237
389	151321	58863869	19.7231	7.2999	2.58995	2.57069	1222.1	118847
390	152100	59319000	19.7484	7.3061	2.59106	2.56410	1225.2	119459
391	152881	59776471	19.7737	7.3124	2.59218	2.55755	1228.4	120072
392	153664	60236288	19.7990	7.3186	2.59329	2.55102	1231.5	120687
393	154449	60698457	19.8242	7.3248	2.59439	2.54453	1234.6	121304
394	155236	61162984	19.8494	7.3310	2.59550	2.53807	1237.8	121922
395	156025	61629875	19.8746	7.3372	2.59660	2.53165	1240.9	122542
396	156816	62099136	19.8997	7.3434	2.59770	2.52525	1244.1	123163
397	157609	62570773	19.9249	7.3496	2.59879	2.51889	1247.2	123786
398	158404	63044792	19.9499	7.3558	2.59988	2.51256	1250.4	124410
399	159201	63521199	19.9750	7.3619	2.60097	2.50627	1253.5	125036
400	160000	64000000	20.0000	7.3681	2.60206	2.50000	1256.6	125664
401	160801	64481201	20.0250	7.3742	2.60314	2.49377	1259.8	126293
402	161604	64964808	20.0499	7.3803	2.60423	2.48756	1262.9	126923
403	162409	65450827	20.0749	7.3864	2.60531	2.48139	1266.1	127556
404	163216	65939264	20.0998	7.3925	2.60638	2.47525	1269.2	128190

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
405	164025	66430125	20.1246	7.3986	2.60746	2.46914	1272.3	128825
406	164836	66923416	20.1494	7.4047	2.60853	2.46305	1275.5	129462
407	165649	67419143	20.1742	7.4108	2.60959	2.45700	1278.6	130100
408	166464	67917312	20.1990	7.4169	2.61066	2.45098	1281.8	130741
409	167281	68417929	20.2237	7.4229	2.61172	2.44499	1284.9	131382
410	168100	68921000	20.2485	7.4290	2.61278	2.43902	1288.1	132025
411	168921	69426531	20.2731	7.4350	2.61384	2.43309	1291.2	132670
412	169744	69934528	20.2978	7.4410	2.61490	2.42718	1294.3	133317
413	170569	70444997	20.3224	7.4470	2.61595	2.42131	1297.5	133965
414	171396	70957944	20.3470	7.4530	2.61700	2.41546	1300.6	134614
415	172225	71473375	20.3715	7.4590	2.61805	2.40964	1303.8	135265
416	173056	71991296	20.3961	7.4650	2.61909	2.40385	1306.9	135918
417	173889	72511713	20.4206	7.4710	2.62014	2.39808	1310.0	136572
418	174724	73034632	20.4450	7.4770	2.62118	2.39234	1313.2	137228
419	175561	73560059	20.4695	7.4829	2.62221	2.38664	1316.3	137885
420	176400	74088000	20.4939	7.4889	2.62325	2.38095	1319.5	138544
421	177241	74618461	20.5183	7.4948	2.62428	2.37530	1322.6	139205
422	178084	75151448	20.5426	7.5007	2.62531	2.36967	1325.8	139867
423	178929	75686967	20.5670	7.5067	2.62634	2.36407	1328.9	140531
424	179776	76225024	20.5913	7.5126	2.62737	2.35849	1332.0	141196
425	180625	76765625	20.6155	7.5185	2.62839	2.35294	1335.2	141863
426	181476	77308776	20.6398	7.5244	2.62941	2.34742	1338.3	142531
427	182329	77854483	20.6640	7.5302	2.63043	2.34192	1341.5	143201
428	183184	78402752	20.6882	7.5361	2.63144	2.33645	1344.6	143872
429	184041	78953589	20.7123	7.5420	2.63246	2.33100	1347.7	144545
430	184900	79507000	20.7364	7.5478	2.63347	2.32558	1350.9	145220
431	185761	80062991	20.7605	7.5537	2.63448	2.32019	1354.0	145896
432	186624	80621568	20.7846	7.5595	2.63548	2.31482	1357.2	146574
433	187489	81182737	20.8087	7.5654	2.63649	2.30947	1360.3	147254
434	188356	81746504	20.8327	7.5712	2.63749	2.30415	1363.5	147934
435	189225	82312875	20.8567	7.5770	2.63849	2.29885	1366.6	148617
436	190096	82881856	20.8806	7.5828	2.63949	2.29358	1369.7	149301
437	190969	83453453	20.9045	7.5886	2.64048	2.28833	1372.9	149987
438	191844	84027672	20.9284	7.5944	2.64147	2.28311	1376.0	150674
439	192721	84604519	20.9523	7.6001	2.64246	2.27790	1379.2	151363
440	193600	85184000	20.9762	7.6059	2.64345	2.27273	1382.3	152053
441	194481	85766121	21.0000	7.6117	2.64444	2.26757	1385.4	152745
442	195364	86350888	21.0238	7.6174	2.64542	2.26244	1388.6	153439
443	196249	86938307	21.0476	7.6232	2.64640	2.25734	1391.7	154134
444	197136	87528384	21.0713	7.6289	2.64738	2.25225	1394.9	154830
445	198025	88121125	21.0950	7.6346	2.64836	2.24719	1398.0	155528
446	198916	88716536	21.1187	7.6403	2.64933	2.24215	1401.2	156228
447	199809	89314623	21.1424	7.6460	2.65031	2.23714	1404.3	156930
448	200704	89915392	21.1660	7.6517	2.65128	2.23214	1407.4	157633
449	201601	90518849	21.1896	7.6574	2.65225	2.22717	1410.6	158337

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	Area
450	202500	91125000	21.2132	7.6631	2.65321	2.22222	1413.7	159043
451	203401	91733851	21.2368	7.6688	2.65418	2.21730	1416.9	159751
452	204304	92345408	21.2603	7.6744	2.65514	2.21239	1420.0	160460
453	205209	92959677	21.2838	7.6801	2.65610	2.20751	1423.1	161171
454	206116	93576664	21.3073	7.6857	2.65706	2.20264	1426.3	161883
455	207025	94196375	21.3307	7.6914	2.65801	2.19780	1429.4	162597
456	207936	94818816	21.3542	7.6970	2.65896	2.19298	1432.6	163313
457	208849	95443993	21.3776	7.7026	2.65992	2.18818	1435.7	164030
458	209764	96071912	21.4009	7.7082	2.66087	2.18341	1438.9	164748
459	210681	96702579	21.4243	7.7138	2.66181	2.17865	1442.0	165468
460	211600	97336000	21.4476	7.7194	2.66276	2.17391	1445.1	166190
461	212521	97972181	21.4709	7.7250	2.66370	2.16920	1448.3	166914
462	213444	98611128	21.4942	7.7306	2.66464	2.16450	1451.4	167639
463	214369	99252847	21.5174	7.7362	2.66558	2.15983	1454.6	168365
464	215296	99897344	21.5407	7.7418	2.66652	2.15517	1457.7	169093
465	216225	100544625	21.5639	7.7473	2.66745	2.15054	1460.8	169823
466	217156	101194696	21.5870	7.7529	2.66839	2.14592	1464.0	170554
467	218089	101847563	21.6102	7.7584	2.66932	2.14133	1467.1	171287
468	219024	102503232	21.6333	7.7639	2.67025	2.13675	1470.3	172021
469	219961	103161709	21.6564	7.7695	2.67117	2.13220	1473.4	172757
470	220900	103823000	21.6795	7.7750	2.67210	2.12766	1476.5	173494
471	221841	104487111	21.7025	7.7805	2.67302	2.12314	1479.7	174234
472	222784	105154048	21.7256	7.7860	2.67394	2.11864	1482.8	174974
473	223729	105823817	21.7486	7.7915	2.67486	2.11417	1486.0	175716
474	224676	106496424	21.7715	7.7970	2.67578	2.10971	1489.1	176460
475	225625	107171875	21.7945	7.8025	2.67669	2.10526	1492.3	177205
476	226576	107850176	21.8174	7.8079	2.67761	2.10084	1495.4	177952
477	227529	108531333	21.8403	7.8134	2.67852	2.09644	1498.5	178701
478	228484	109215352	21.8632	7.8188	2.67943	2.09205	1501.7	179451
479	229441	109902239	21.8861	7.8243	2.68034	2.08768	1504.8	180203
480	230400	110592000	21.9089	7.8297	2.68124	2.08333	1508.0	180956
481	231361	111284641	21.9317	7.8352	2.68215	2.07900	1511.1	181711
482	232324	111980168	21.9545	7.8406	2.68305	2.07469	1514.3	182467
483	233289	112678587	21.9773	7.8460	2.68395	2.07039	1517.4	183225
484	234256	113379904	22.0000	7.8514	2.68485	2.06612	1520.5	183984
485	235225	114084125	22.0227	7.8568	2.68574	2.06186	1523.7	184745
486	236196	114791256	22.0454	7.8622	2.68664	2.05761	1526.8	185508
487	237169	115501303	22.0681	7.8676	2.68753	2.05339	1530.0	186272
488	238144	116214272	22.0907	7.8730	2.68842	2.04918	1533.1	187038
489	239121	116930169	22.1133	7.8784	2.68931	2.04499	1536.2	187805
490	240100	117649000	22.1359	7.8837	2.69020	2.04082	1539.4	188574
491	241081	118370771	22.1585	7.8891	2.69108	2.03666	1542.5	189345
492	242064	119095488	22.1811	7.8944	2.69197	2.03252	1545.7	190117
493	243049	119823157	22.2036	7.8998	2.69285	2.02840	1548.8	190890
494	244036	120553784	22.2261	7.9051	2.69373	2.02429	1551.9	191665

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
495	245025	121287375	22.2486	7.9105	2.69461	2.02020	1555.1	192442
496	246016	122023936	22.2711	7.9158	2.69548	2.01613	1558.2	193221
497	247009	122763473	22.2935	7.9211	2.69636	2.01207	1561.4	194000
498	248004	123505992	22.3159	7.9264	2.69723	2.00803	1564.5	194782
499	249001	124251499	22.3383	7.9317	2.69810	2.00401	1567.7	195565
500	250000	125000000	22.3607	7.9370	2.69897	2.00000	1570.8	196350
501	251001	125751501	22.3830	7.9423	2.69984	1.99601	1573.9	197136
502	252004	126506008	22.4054	7.9476	2.70070	1.99203	1577.1	197923
503	253009	127263527	22.4277	7.9528	2.70157	1.98807	1580.2	198713
504	254016	128024064	22.4499	7.9581	2.70243	1.98413	1583.4	199504
505	255025	128787625	22.4722	7.9634	2.70329	1.98020	1586.5	200296
506	256036	129554216	22.4944	7.9686	2.70415	1.97629	1589.7	201090
507	257049	130323843	22.5167	7.9739	2.70501	1.97239	1592.8	201886
508	258064	131096512	22.5389	7.9791	2.70586	1.96850	1595.9	202683
509	259081	131872229	22.5610	7.9843	2.70672	1.96464	1599.1	203482
510	260100	132651000	22.5832	7.9896	2.70757	1.96078	1602.2	204282
511	261121	133432831	22.6053	7.9948	2.70842	1.95695	1605.4	205084
512	262144	134217728	22.6274	8.0000	2.70927	1.95312	1608.5	205887
513	263169	135005697	22.6495	8.0052	2.71012	1.94932	1611.6	206692
514	264196	135796744	22.6716	8.0104	2.71096	1.94553	1614.8	207499
515	265225	136590875	22.6936	8.0156	2.71181	1.94175	1617.9	208307
516	266256	137388096	22.7156	8.0208	2.71265	1.93798	1621.1	209117
517	267289	138188413	22.7376	8.0260	2.71349	1.93424	1624.2	209928
518	268324	138991832	22.7596	8.0311	2.71433	1.93050	1627.3	210741
519	269361	139798359	22.7816	8.0363	2.71517	1.92678	1630.5	211556
520	270400	140608000	22.8035	8.0415	2.71600	1.92308	1633.6	212372
521	271441	141420761	22.8254	8.0466	2.71684	1.91939	1636.8	213189
522	272484	142236648	22.8473	8.0517	2.71767	1.91571	1639.9	214008
523	273529	143055667	22.8692	8.0569	2.71850	1.91205	1643.1	214829
524	274576	143877824	22.8910	8.0620	2.71933	1.90840	1646.2	215651
525	275625	144703125	22.9129	8.0671	2.72016	1.90476	1649.3	216475
526	276676	145531576	22.9347	8.0723	2.72099	1.90114	1652.5	217301
527	277729	146363183	22.9565	8.0774	2.72181	1.89753	1655.6	218128
528	278784	147197952	22.9783	8.0825	2.72263	1.89394	1658.8	218956
529	279841	148035889	23.0000	8.0876	2.72346	1.89036	1661.9	219787
530	280900	148877000	23.0217	8.0927	2.72428	1.88679	1665.0	220618
531	281961	149721291	23.0434	8.0978	2.72509	1.88324	1668.2	221452
532	283024	150568768	23.0651	8.1028	2.72591	1.87970	1671.3	222287
533	284089	151419437	23.0868	8.1079	2.72673	1.87617	1674.5	223123
534	285156	152273304	23.1084	8.1130	2.72754	1.87266	1677.6	223961
535	286225	153130375	23.1301	8.1180	2.72835	1.86916	1680.8	224801
536	287296	153990656	23.1517	8.1231	2.72916	1.86567	1683.9	225642
537	288369	154854153	23.1733	8.1281	2.72997	1.86220	1687.0	226484
538	289444	155720872	23.1948	8.1332	2.73078	1.85874	1690.2	227329
539	290521	156590819	23.2164	8.1382	2.73159	1.85529	1693.3	228175

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
540	291600	157464000	23.2379	8.1433	2.73239	1.85185	1696.5	229022
541	292681	158340421	23.2594	8.1483	2.73320	1.84843	1699.6	229871
542	293764	159220088	23.2809	8.1533	2.73400	1.84502	1702.7	230722
543	294849	160103007	23.3024	8.1583	2.73480	1.84162	1705.9	231574
544	295936	160989184	23.3238	8.1633	2.73560	1.83824	1709.0	232428
545	297025	161878625	23.3452	8.1683	2.73640	1.83486	1712.2	233283
546	298116	162771336	23.3666	8.1733	2.73719	1.83150	1715.3	234140
547	299209	163667323	23.3880	8.1783	2.73799	1.82815	1718.5	234998
548	300304	164566592	23.4094	8.1833	2.73878	1.82482	1721.6	235858
549	301401	165469149	23.4307	8.1882	2.73957	1.82149	1724.7	236720
550	302500	166375000	23.4521	8.1932	2.74036	1.81818	1727.9	237583
551	303601	167284151	23.4734	8.1982	2.74115	1.81488	1731.0	238448
552	304704	168196608	23.4947	8.2031	2.74194	1.81159	1734.2	239314
553	305809	169112377	23.5160	8.2081	2.74273	1.80832	1737.3	240182
554	306916	170031464	23.5372	8.2130	2.74351	1.80505	1740.4	241051
555	308025	170953875	23.5584	8.2180	2.74429	1.80180	1743.6	241922
556	309136	171879616	23.5797	8.2229	2.74507	1.79856	1746.7	242795
557	310249	172808693	23.6008	8.2278	2.74586	1.79533	1749.9	243669
558	311364	173741112	23.6220	8.2327	2.74663	1.79211	1753.0	244545
559	312481	174676879	23.6432	8.2377	2.74741	1.78891	1756.2	245422
560	313600	175616000	23.6643	8.2426	2.74819	1.78571	1759.3	246301
561	314721	176558481	23.6854	8.2475	2.74896	1.78253	1762.4	247181
562	315844	177504328	23.7065	8.2524	2.74974	1.77936	1765.6	248063
563	316969	178453547	23.7276	8.2573	2.75051	1.77620	1768.7	248947
564	318096	179406144	23.7487	8.2621	2.75128	1.77305	1771.9	249832
565	319225	180362125	23.7697	8.2670	2.75205	1.76991	1775.0	250719
566	320356	181321496	23.7908	8.2719	2.75282	1.76678	1778.1	251607
567	321489	182284263	23.8118	8.2768	2.75358	1.76367	1781.3	252497
568	322624	183250432	23.8328	8.2816	2.75435	1.76056	1784.4	253388
569	323761	184220009	23.8537	8.2865	2.75511	1.75747	1787.6	254281
570	324900	185193000	23.8747	8.2913	2.75587	1.75439	1790.7	255176
571	326041	186169411	23.8956	8.2962	2.75664	1.75131	1793.9	256072
572	327184	187149248	23.9165	8.3010	2.75740	1.74825	1797.0	256970
573	328329	188132517	23.9374	8.3059	2.75815	1.74520	1800.1	257869
574	329476	189119224	23.9583	8.3107	2.75891	1.74216	1803.3	258770
575	330625	190109375	23.9792	8.3155	2.75967	1.73913	1806.4	259672
576	331776	191102976	24.0000	8.3203	2.76042	1.73611	1809.6	260576
577	332929	192100033	24.0208	8.3251	2.76118	1.73310	1812.7	261482
578	334084	193100552	24.0416	8.3300	2.76193	1.73010	1815.8	262389
579	335241	194104539	24.0624	8.3348	2.76268	1.72712	1819.0	263298
580	336400	195112000	24.0832	8.3396	2.76343	1.72414	1822.1	264208
581	337561	196122941	24.1039	8.3443	2.76418	1.72117	1825.3	265120
582	338724	197137368	24.1247	8.3491	2.76492	1.71821	1828.4	266033
583	339889	198155287	24.1454	8.3539	2.76567	1.71527	1831.6	266948
584	341056	199176704	24.1661	8.3587	2.76641	1.71233	1834.7	267865

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
585	342225	200201625	24.1868	8.3634	2.76716	1.70940	1837.8	268783
586	343396	201230056	24.2074	8.3682	2.76790	1.70649	1841.0	269701
587	344569	202262003	24.2281	8.3730	2.76864	1.70358	1844.1	270624
588	345744	203297472	24.2487	8.3777	2.76938	1.70068	1847.3	271547
589	346921	204336469	24.2693	8.3825	2.77012	1.69779	1850.4	272471
590	348100	205379000	24.2899	8.3872	2.77085	1.69492	1853.5	273397
591	349281	206425071	24.3105	8.3919	2.77159	1.69205	1856.7	274325
592	350464	207474688	24.3311	8.3967	2.77232	1.68919	1859.8	275254
593	351649	208527857	24.3516	8.4014	2.77305	1.68634	1863.0	276184
594	352836	209584584	24.3721	8.4061	2.77379	1.68350	1866.1	277117
595	354025	210644875	24.3926	8.4108	2.77452	1.68067	1869.3	278051
596	355216	211708736	24.4131	8.4155	2.77525	1.67785	1872.4	278986
597	356409	212776173	24.4336	8.4202	2.77597	1.67504	1875.5	279923
598	357604	213847192	24.4540	8.4249	2.77670	1.67224	1878.7	280862
599	358801	214921799	24.4745	8.4296	2.77743	1.66945	1881.8	281802
600	360000	216000000	24.4949	8.4343	2.77815	1.66667	1885.0	282743
601	361201	217081801	24.5153	8.4390	2.77887	1.66389	1888.1	283687
602	362404	218167208	24.5357	8.4437	2.77960	1.66113	1891.2	284631
603	363609	219256227	24.5561	8.4484	2.78032	1.65837	1894.4	285578
604	364816	220348864	24.5764	8.4530	2.78104	1.65563	1897.5	286526
605	366025	221445125	24.5967	8.4577	2.78176	1.65289	1900.7	287475
606	367236	222545016	24.6171	8.4623	2.78247	1.65017	1903.8	288426
607	368449	223648543	24.6374	8.4670	2.78319	1.64745	1907.0	289379
608	369664	224755712	24.6577	8.4716	2.78390	1.64474	1910.1	290333
609	370881	225866529	24.6779	8.4763	2.78462	1.64204	1913.2	291289
610	372100	226981000	24.6982	8.4809	2.78533	1.63934	1916.4	292247
611	373321	228099131	24.7184	8.4856	2.78604	1.63666	1919.5	293206
612	374544	229220928	24.7386	8.4902	2.78675	1.63399	1922.7	294166
613	375769	230346397	24.7588	8.4948	2.78746	1.63132	1925.8	295128
614	376996	231475544	24.7790	8.4994	2.78817	1.62866	1928.9	296092
615	378225	232608375	24.7992	8.5040	2.78888	1.62602	1932.1	297057
616	379456	233744896	24.8193	8.5086	2.78958	1.62338	1935.2	298024
617	380689	234885113	24.8395	8.5132	2.79029	1.62075	1938.4	298992
618	381924	236029032	24.8596	8.5178	2.79099	1.61812	1941.5	299962
619	383161	237176659	24.8797	8.5224	2.79169	1.61551	1944.7	300934
620	384400	238328000	24.8998	8.5270	2.79239	1.61290	1947.8	301907
621	385641	239483061	24.9199	8.5316	2.79309	1.61031	1950.9	302882
622	386884	240641848	24.9399	8.5362	2.79379	1.60772	1954.1	303858
623	388129	241804367	24.9600	8.5408	2.79449	1.60514	1957.2	304836
624	389376	242970624	24.9800	8.5453	2.79518	1.60256	1960.4	305815
625	390625	244140625	25.0000	8.5499	2.79588	1.60000	1963.5	306796
626	391876	245314376	25.0200	8.5544	2.79657	1.59744	1966.6	307779
627	393129	246491883	25 0400	8.5590	2.79727	1.59490	1969.8	308763
628	394384	247673152	25.0599	8.5635	2.79796	1.59236	1972.9	309748
629	395641	248858189	25.0799	8.5681	2.79865	1.58983	1976.1	310736

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia.	
							Circum.	Area
630	396900	250047000	25.0998	8.5726	2.79934	1.58730	1979.2	311725
631	398161	251239591	25.1197	8.5772	2.80003	1.58479	1982.4	312715
632	399424	252435968	25.1396	8.5817	2.80072	1.58228	1985.5	313707
633	400689	253636137	25.1595	8.5862	2.80140	1.57978	1988.6	314700
634	401956	254840104	25.1794	8.5907	2.80209	1.57729	1991.8	315696
635	403225	256047875	25.1992	8.5952	2.80277	1.57480	1994.9	316692
636	404496	257259456	25.2190	8.5997	2.80346	1.57233	1998.1	317690
637	405769	258474853	25.2389	8.6043	2.80414	1.56986	2001.2	318690
638	407044	259694072	25.2587	8.6088	2.80482	1.56740	2004.3	319692
639	408321	260917119	25.2784	8.6132	2.80550	1.56495	2007.5	320695
640	409600	262144000	25.2982	8.6177	2.80618	1.56250	2010.6	321699
641	410881	263374721	25.3180	8.6222	2.80686	1.56006	2013.8	322705
642	412164	264609288	25.3377	8.6267	2.80754	1.55763	2016.9	323713
643	413449	265847707	25.3574	8.6312	2.80821	1.55521	2020.0	324722
644	414736	267089984	25.3772	8.6357	2.80889	1.55280	2023.2	325733
645	416025	268336125	25.3969	8.6401	2.80956	1.55039	2026.3	326745
646	417316	269586136	25.4165	8.6446	2.81023	1.54799	2029.5	327759
647	418609	270840023	25.4362	8.6490	2.81090	1.54560	2032.6	328775
648	419904	272097792	25.4558	8.6535	2.81158	1.54321	2035.8	329792
649	421201	273359449	25.4755	8.6579	2.81224	1.54083	2038.9	330810
650	422500	274625000	25.4951	8.6624	2.81291	1.53846	2042.0	331831
651	423801	275894451	25.5147	8.6668	2.81358	1.53610	2045.2	332853
652	425104	277167808	25.5343	8.6713	2.81425	1.53374	2048.3	333876
653	426409	278445077	25.5539	8.6757	2.81491	1.53139	2051.5	334901
654	427716	279726264	25.5734	8.6801	2.81558	1.52905	2054.6	335927
655	429025	281011375	25.5930	8.6845	2.81624	1.52672	2057.7	336955
656	430336	282300416	25.6125	8.6890	2.81690	1.52439	2060.9	337985
657	431649	283593393	25.6320	8.6934	2.81757	1.52207	2064.0	339016
658	432964	284890312	25.6515	8.6978	2.81823	1.51976	2067.2	340049
659	434281	286191179	25.6710	8.7022	2.81889	1.51745	2070.3	341084
660	435600	287496000	25.6905	8.7066	2.81954	1.51515	2073.5	342119
661	436921	288804781	25.7099	8.7110	2.82020	1.51286	2076.6	343157
662	438244	290117528	25.7294	8.7154	2.82086	1.51057	2079.7	344196
663	439569	291434247	25.7488	8.7198	2.82151	1.50830	2082.9	345237
664	440896	292754944	25.7682	8.7241	2.82217	1.50602	2086.0	346279
665	442225	294079625	25.7876	8.7285	2.82282	1.50376	2089.2	347323
666	443556	295408296	25.8070	8.7329	2.82347	1.50150	2092.3	348368
667	444889	296740963	25.8263	8.7373	2.82413	1.49925	2095.4	349415
668	446224	298077632	25.8457	8.7416	2.82478	1.49701	2098.6	350464
669	447561	299418309	25.8650	8.7460	2.82543	1.49477	2101.7	351514
670	448900	300763000	25.8844	8.7503	2.82607	1.49254	2104.9	352565
671	450241	302111711	25.9037	8.7547	2.82672	1.49031	2108.0	353618
672	451584	303464448	25.9230	8.7590	2.82737	1.48810	2111.2	354673
673	452929	304821217	25.9422	8.7634	2.82802	1.48588	2114.3	355730
674	454276	306182024	25.9615	8.7677	2.82866	1.48368	2117.4	356788

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia.	
							Circum.	Area
675	455625	307546875	25.9808	8.7721	2.82930	1.48148	2120.6	357847
676	456976	308915776	26.0000	8.7764	2.82995	1.47929	2123.7	358908
677	458329	310288733	26.0192	8.7807	2.83059	1.47711	2126.9	359971
678	459684	311665752	26.0384	8.7850	2.83123	1.47493	2130.0	361035
679	461041	313046839	26.0576	8.7893	2.83187	1.47275	2133.1	362101
680	462400	314432000	26.0768	8.7937	2.83251	1.47059	2136.3	363168
681	463761	315821241	26.0960	8.7980	2.83315	1.46843	2139.4	364237
682	465124	317214568	26.1151	8.8023	2.83378	1.46628	2142.6	365308
683	466489	318611987	26.1343	8.8066	2.83442	1.46413	2145.7	366380
684	467856	320013504	26.1534	8.8109	2.83506	1.46199	2148.9	367453
685	469225	321419125	26.1725	8.8152	2.83569	1.45985	2152.0	368528
686	470596	322828856	26.1916	8.8194	2.83632	1.45773	2155.1	369605
687	471969	324242703	26.2107	8.8237	2.83696	1.45560	2158.3	370684
688	473344	325660672	26.2298	8.8280	2.83759	1.45349	2161.4	371764
689	474721	327082769	26.2488	8.8323	2.83822	1.45138	2164.[illegible]	372845
690	476100	328509000	26.2679	8.8366	2.83885	1.44928	2167.7	373928
691	477481	329939371	26.2869	8.8408	2.83948	1.44718	2170.8	375013
692	478864	331373888	26.3059	8.8451	2.84011	1.44509	2174.0	376099
693	480249	332812557	26.3249	8.8493	2.84073	1.44300	2177.1	377187
694	481636	334255384	26.3439	8.8536	2.84136	1.44092	2180.3	378276
695	483025	335702375	26.3629	8.8578	2.84198	1.43885	2183.4	379367
696	484416	337153536	26.3818	8.8621	2.84261	1.43678	2186.6	380459
697	485809	338608873	26.4008	8.8663	2.84323	1.43472	2189.7	381554
698	487204	340068392	26.4197	8.8706	2.84386	1.43267	2192.8	382649
699	488601	341532099	26.4386	8.8748	2.84448	1.43062	2196.0	383746
700	490000	343000000	26.4575	8.8790	2.84510	1.42857	2199.1	384845
701	491401	344472101	26.4764	8.8833	2.84572	1.42653	2202.3	385945
702	492804	345948408	26.4953	8.8875	2.84634	1.42450	2205.4	387047
703	494209	347428927	26.5141	8.8917	2.84696	1.42248	2208.5	388151
704	495616	348913664	26.5330	8.8959	2.84757	1.42046	2211.7	389256
705	497025	350402625	26.5518	8.9001	2.84819	1.41844	2214.8	390363
706	498436	351895816	26.5707	8.9043	2.84880	1.41643	2218.0	391471
707	499849	353393243	26.5895	8.9085	2.84942	1.41443	2221.1	392580
708	501264	354894912	26.6083	8.9127	2.85003	1.41243	2224.3	393692
709	502681	356400829	26.6271	8.9169	2.85065	1.41044	2227.4	394805
710	504100	357911000	26.6458	8.9211	2.85126	1.40845	2230.5	395919
711	505521	359425431	26.6646	8.9253	2.85187	1.40647	2233.7	397035
712	506944	360944128	26.6833	8.9295	2.85248	1.40449	2236.8	398153
713	508369	362467097	26.7021	8.9337	2.85309	1.40253	2240.0	399272
714	509796	363994344	26.7208	8.9378	2.85370	1.40056	2243.1	400393
715	511225	365525875	26.7395	8.9420	2.85431	1.39860	2246.2	401515
716	512656	367061696	26.7582	8.9462	2.85491	1.39665	2249.4	402639
717	514089	368601813	26.7769	8.9503	2.85552	1.39470	2252.5	403765
718	515524	370146232	26.7955	8.9545	2.85612	1.39276	2255.7	404892
719	516961	371694959	26.8142	8.9587	2.85673	1.39082	2258.8	406020

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
720	518400	373248000	26.8328	8.9628	2.85733	1.38889	2261.9	407150
721	519841	374805361	26.8514	8.9670	2.85794	1.38696	2265.1	408282
722	521284	376367048	26.8701	8.9711	2.85854	1.38504	2268.2	409416
723	522729	377933067	26.8887	8.9752	2.85914	1.38313	2271.4	410550
724	524176	379503424	26.9072	8.9794	2.85974	1.38122	2274.5	411687
725	525625	381078125	26.9258	8.9835	2.86034	1.37931	2277.7	412825
726	527076	382657176	26.9444	8.9876	2.86094	1.37741	2280.8	413965
727	528529	384240583	26.9629	8.9918	2.86153	1.37552	2283.9	415106
728	529984	385828352	26.9815	8.9959	2.86213	1.37363	2287.1	416248
729	531441	387420489	27.0000	9.0000	2.86273	1.37174	2290.2	417393
730	532900	389017000	27.0185	9.0041	2.86332	1.36986	2293.4	418539
731	534361	390617891	27.0370	9.0082	2.86392	1.36799	2296.5	419686
732	535824	392223168	27.0555	9.0123	2.86451	1.36612	2299.7	420835
733	537289	393832837	27.0740	9.0164	2.86510	1.36426	2302.8	421986
734	538756	395446904	27.0924	9.0205	2.86570	1.36240	2305.9	423138
735	540225	397065375	27.1109	9.0246	2.86629	1.36054	2309.1	424293
736	541696	398688256	27.1293	9.0287	2.86688	1.35870	2312.2	425448
737	543169	400315553	27.1477	9.0328	2.86747	1.35685	2315.4	426604
738	544644	401947272	27.1662	9.0369	2.86806	1.35501	2318.5	427762
739	546121	403583419	27.1846	9.0410	2.86864	1.35318	2321.6	428922
740	547600	405224000	27.2029	9.0450	2.86923	1.35135	2324.8	430084
741	549081	406869021	27.2213	9.0491	2.86982	1.34953	2327.9	431247
742	550564	408518488	27.2397	9.0532	2.87040	1.34771	2331.1	432412
743	552049	410172407	27.2580	9.0572	2.87099	1.34590	2334.2	433578
744	553536	411830784	27.2764	9.0613	2.87157	1.34409	2337.3	434746
745	555025	413493625	27.2947	9.0654	2.87216	1.34228	2340.5	435916
746	556516	415160936	27.3130	9.0694	2.87274	1.34048	2343.6	437087
747	558009	416832723	27.3313	9.0735	2.87332	1.33869	2346.8	438259
748	559504	418508992	27.3496	9.0775	2.87390	1.33690	2349.9	439433
749	561001	420189749	27.3679	9.0816	2.87448	1.33511	2353.1	440609
750	562500	421875000	27.3861	9.0856	2.87506	1.33333	2356.2	441786
751	564001	423564751	27.4044	9.0896	2.87564	1.33156	2359.3	442965
752	565504	425259008	27.4226	9.0937	2.87622	1.32979	2362.5	444146
753	567009	426957777	27.4408	9.0977	2.87680	1.32802	2365.6	445328
754	568516	428661064	27.4591	9.1017	2.87737	1.32626	2368.8	446511
755	570025	430368875	27.4773	9.1057	2.87795	1.32450	2371.9	447697
756	571536	432081216	27.4955	9.1098	2.87852	1.32275	2375.0	448883
757	573049	433798093	27.5136	9.1138	2.87910	1.32100	2378.2	450072
758	574564	435519512	27.5318	9.1178	2.87967	1.31926	2381.3	451262
759	576081	437245479	27.5500	9.1218	2.88024	1.31752	2384.5	452453
760	577600	438976000	27.5681	9.1258	2.88081	1.31579	2387.6	453646
761	579121	440711081	27.5862	9.1298	2.88138	1.31406	2390.8	454841
762	580644	442450728	27.6043	9.1338	2.88196	1.31234	2393.9	456037
763	582169	444194947	27.6225	9.1378	2.88252	1.31062	2397.0	457234
764	583696	445943744	27.6405	9.1418	2.88309	1.30890	2400.2	458434

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SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia.	
							Circum.	Area
765	585225	447697125	27.6586	9.1458	2.88366	1.30719	2403.3	459635
766	586756	449455096	27.6767	9.1498	2.88423	1.30548	2406.5	460837
767	588289	451217663	27.6948	9.1537	2.88480	1.30378	2409.6	462042
768	589824	452984832	27.7128	9.1577	2.88536	1.30208	2412.7	463247
769	591361	454756609	27.7308	9.1617	2.88593	1.30039	2415.9	464454
770	592900	456533000	27.7489	9.1657	2.88649	1.29870	2419.0	465663
771	594441	458314011	27.7669	9.1696	2.88705	1.29702	2422.2	466873
772	595984	460099648	27.7849	9.1736	2.88762	1.29534	2425.3	468085
773	597529	461889917	27.8029	9.1775	2.88818	1.29366	2428.5	469298
774	599076	463684824	27.8209	9.1815	2.88874	1.29199	2431.6	470513
775	600625	465484375	27.8388	9.1855	2.88930	1.29032	2434.7	471730
776	602176	467288576	27.8568	9.1894	2.88986	1.28866	2437.9	472948
777	603729	469097433	27.8747	9.1933	2.89042	1.28700	2441.0	474168
778	605284	470910952	27.8927	9.1973	2.89098	1.28535	2444.2	475389
779	606841	472729139	27.9106	9.2012	2.89154	1.28370	2447.3	476612
780	608400	474552000	27.9285	9.2052	2.89209	1.28205	2450.4	477836
781	609961	476379541	27.9464	9.2091	2.89265	1.28041	2453.6	479062
782	611524	478211768	27.9643	9.2130	2.89321	1.27877	2456.7	480290
783	613089	480048687	27.9821	9.2170	2.89376	1.27714	2459.9	481519
784	614656	481890304	28.0000	9.2209	2.89432	1.27551	2463.0	482750
785	616225	483736625	28.0179	9.2248	2.89487	1.27389	2466.2	483982
786	617796	485587656	28.0357	9.2287	2.89542	1.27226	2469.3	485216
787	619369	487443403	28.0535	9.2326	2.89597	1.27065	2472.4	486451
788	620944	489303872	28.0713	9.2365	2.89653	1.26904	2475.6	487688
789	622521	491169069	28.0891	9.2404	2.89708	1.26743	2478.7	488927
790	624100	493039000	28.1069	9.2443	2.89763	1.26582	2481.9	490167
791	625681	494913671	28.1247	9.2482	2.89818	1.26422	2485.0	491409
792	627264	496793088	28.1425	9.2521	2.89873	1.26263	2488.1	492652
793	628849	498677257	28.1603	9.2560	2.89927	1.26103	2491.3	493897
794	630436	500566184	28.1780	9.2599	2.89982	1.25945	2494.4	495143
795	632025	502459875	28.1957	9.2638	2.90037	1.25786	2497.6	496391
796	633616	504358336	28.2135	9.2677	2.90091	1.25628	2500.7	497641
797	635209	506261573	28.2312	9.2716	2.90146	1.25471	2503.8	498892
798	636804	508169592	28.2489	9.2754	2.90200	1.25313	2507.0	500145
799	638401	510082399	28.2666	9.2793	2.90255	1.25156	2510.1	501399
800	640000	512000000	28.2843	9.2832	2.90309	1.25000	2513.3	502655
801	641601	513922401	28.3019	9.2870	2.90363	1.24844	2516.4	503912
802	643204	515849608	28.3196	9.2909	2.90417	1.24688	2519.6	505171
803	644809	517781627	28.3373	9.2948	2.90472	1.24533	2522.7	506432
804	646416	519718464	28.3549	9.2986	2.90526	1.24378	2525.8	507694
805	648025	521660125	28.3725	9.3025	2.90580	1.24224	2529.0	508958
806	649636	523606616	28.3901	9.3063	2.90634	1.24069	2532.1	510223
807	651249	525557943	28.4077	9.3102	2.90687	1.23916	2535.3	511490
808	652864	527514112	28.4253	9.3140	2.90741	1.23762	2538.4	512758
809	654481	529475129	28.4429	9.3179	2.90795	1.23609	2541.5	514028

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
810	656100	531441000	28.4605	9.3217	2.90849	1.23457	2544.7	515300
811	657721	533411731	28.4781	9.3255	2.90902	1.23305	2547.8	516573
812	659344	535387328	28.4956	9.3294	2.90956	1.23153	2551.0	517848
813	660969	537367797	28.5132	9.3332	2.91009	1.23001	2554.1	519124
814	662596	539353144	28.5307	9.3370	2.91062	1.22850	2557.3	520402
815	664225	541343375	28.5482	9.3408	2.91116	1.22699	2560.4	521681
816	665856	543338496	28.5657	9.3447	2.91169	1.22549	2563.5	522962
817	667489	545338513	28.5832	9.3485	2.91222	1.22399	2566.7	524245
818	669124	547343432	28.6007	9.3523	2.91275	1.22249	2569.8	525529
819	670761	549353259	28.6182	9.3561	2.91328	1.22100	2573.0	526814
820	672400	551368000	28.6356	9.3599	2.91381	1.21951	2576.1	528102
821	674041	553387661	28.6531	9.3637	2.91434	1.21803	2579.2	529391
822	675684	555412248	28.6705	9.3675	2.91487	1.21655	2582.4	530681
823	677329	557441767	28.6880	9.3713	2.91540	1.21507	2585.5	531973
824	678976	559476224	28.7054	9.3751	2.91593	1.21359	2588.7	533267
825	680625	561515625	28.7228	9.3789	2.91645	1.21212	2591.8	534562
826	682276	563559976	28.7402	9.3827	2.91698	1.21065	2595.0	535858
827	683929	565609283	28.7576	9.3865	2.91751	1.20919	2598.1	537157
828	685584	567663552	28.7750	9.3902	2.91803	1.20773	2601.2	538456
829	687241	569722789	28.7924	9.3940	2.91855	1.20627	2604.4	539758
830	688900	571787000	28.8097	9.3978	2.91908	1.20482	2607.5	541061
831	690561	573856191	28.8271	9.4016	2.91960	1.20337	2610.7	542365
832	692224	575930368	28.8444	9.4053	2.92012	1.20192	2613.8	543671
833	693889	578009537	28.8617	9.4091	2.92065	1.20048	2616.9	544979
834	695556	580093704	28.8791	9.4129	2.92117	1.19904	2620.1	546288
835	697225	582182875	28.8964	9.4166	2.92169	1.19760	2623.2	547599
836	698896	584277056	28.9137	9.4204	2.92221	1.19617	2626.4	548912
837	700569	586376253	28.9310	9.4241	2.92273	1.19474	2629.5	550226
838	702244	588480472	28.9482	9.4279	2.92324	1.19332	2632.7	551541
839	703921	590589719	28.9655	9.4316	2.92376	1.19189	2635.8	552858
840	705600	592704000	28.9828	9.4354	2.92428	1.19048	2638.9	554177
841	707281	594823321	29.0000	9.4391	2.92480	1.18906	2642.1	555497
842	708964	596947688	29.0172	9.4429	2.92531	1.18765	2645.2	556819
843	710649	599077107	29.0345	9.4466	2.92583	1.18624	2648.4	558142
844	712336	601211584	29.0517	9.4503	2.92634	1.18483	2651.5	559467
845	714025	603351125	29.0689	9.4541	2.92686	1.18343	2654.6	560794
846	715716	605495736	29.0861	9.4578	2.92737	1.18203	2657.8	562122
847	717409	607645423	29.1033	9.4615	2.92788	1.18064	2660.9	563452
848	719104	609800192	29.1204	9.4652	2.92840	1.17925	2664.1	564783
849	720801	611960049	29.1376	9.4690	2.92891	1.17786	2667.2	566116
850	722500	614125000	29.1548	9.4727	2.92942	1.17647	2670.4	567450
851	724201	616295051	29.1719	9.4764	2.92993	1.17509	2673.5	568786
852	725904	618470208	29.1890	9.4801	2.93044	1.17371	2676.6	570124
853	727609	620650477	29.2062	9.4838	2.93095	1.17233	2679.8	571463
854	729316	622835864	29.2233	9.4875	2.93146	1.17096	2682.9	572803

SQUARES, CUBES, SQUARE ROOTS, ETC.—Continued

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
855	731025	625026375	29.2404	9.4912	2.93197	1.16959	2686.1	574146
856	732736	627222016	29.2575	9.4949	2.93247	1.16822	2689.2	575490
857	734449	629422793	29.2746	9.4986	2.93298	1.16686	2692.3	576835
858	736164	631628712	29.2916	9.5023	2.93349	1.16550	2695.5	578182
859	737881	633839779	29.3087	9.5060	2.93399	1.16414	2698.6	579530
860	739600	636056000	29.3258	9.5097	2.93450	1.16279	2701.8	580880
861	741321	638277381	29.3428	9.5134	2.93500	1.16144	2704.9	582232
862	743044	640503928	29.3598	9.5171	2.93551	1.16009	2708.1	583585
863	744769	642735647	29.3769	9.5207	2.93601	1.15875	2711.2	584940
864	746496	644972544	29.3939	9.5244	2.93651	1.15741	2714.3	586297
865	748225	647214625	29.4109	9.5281	2.93702	1.15607	2717.5	587655
866	749956	649461896	29.4279	9.5317	2.93752	1.15473	2720.6	589014
867	751689	651714363	29.4449	9.5354	2.93802	1.15340	2723.8	590375
868	753424	653972032	29.4618	9.5391	2.93852	1.15207	2726.9	591738
869	755161	656234909	29.4788	9.5427	2.93902	1.15075	2730.0	593102
870	756900	658503000	29.4958	9.5464	2.93952	1.14943	2733.2	594468
871	758641	660776311	29.5127	9.5501	2.94002	1.14811	2736.3	595835
872	760384	663054848	29.5296	9.5537	2.94052	1.14679	2739.5	597204
873	762129	665338617	29.5466	9.5574	2.94101	1.14548	2742.6	598575
874	763876	667627624	29.5635	9.5610	2.94151	1.14416	2745.8	599947
875	765625	669921875	29.5804	9.5647	2.94201	1.14286	2748.9	601320
876	767376	672221376	29.5973	9.5683	2.94250	1.14155	2752.0	602696
877	769129	674526133	29.6142	9.5719	2.94300	1.14025	2755.2	604073
878	770884	676836152	29.6311	9.5756	2.94349	1.13895	2758.3	605451
879	772641	679151439	29.6479	9.5792	2.94399	1.13766	2761.5	606831
880	774400	681472000	29.6648	9.5828	2.94448	1.13636	2764.6	608212
881	776161	683797841	29.6816	9.5865	2.94498	1.13507	2767.7	609595
882	777924	686128968	29.6985	9.5901	2.94547	1.13379	2770.9	610980
883	779689	688465387	29.7153	9.5937	2.94596	1.13250	2774.0	612366
884	781456	690807104	29.7321	9.5973	2.94645	1.13122	2777.2	613754
885	783225	693154125	29.7489	9.6010	2.94694	1.12994	2780.3	615143
886	784996	695506456	29.7658	9.6046	2.94743	1.12867	2783.5	616534
887	786769	697864103	29.7825	9.6082	2.94792	1.12740	2786.6	617927
888	788544	700227072	29.7993	9.6118	2.94841	1.12613	2789.7	619321
889	790321	702595369	29.8161	9.6154	2.94890	1.12486	2792.9	620717
890	792100	704969000	29.8329	9.6190	2.94939	1.12360	2796.0	622114
891	793881	707347971	29.8496	9.6226	2.94988	1.12233	2799.2	623513
892	795664	709732288	29.8664	9.6262	2.95036	1.12108	2802.3	624913
893	797449	712121957	29.8831	9.6298	2.95085	1.11982	2805.4	626315
894	799236	714516984	29.8998	9.6334	2.95134	1.11857	2808.6	627718
895	801025	716917375	29.9166	9.6370	2.95182	1.11732	2811.7	629124
896	802816	719323136	29.9333	9.6406	2.95231	1.11607	2814.9	630530
897	804609	721734273	29.9500	9.6442	2.95279	1.11483	2818.0	631938
898	806404	724150792	29.9666	9.6477	2.95328	1.11359	2821.2	633348
899	808201	726572699	29.9833	9.6513	2.95376	1.11235	2824.3	634760

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
900	810000	729000000	30.0000	9.6549	2.95424	1.11111	2827.4	636173
901	811801	731432701	30.0167	9.6585	2.95472	1.10988	2830.6	637587
902	813604	733870808	30.0333	9.6620	2.95521	1.10865	2833.7	639003
903	815409	736314327	30.0500	9.6656	2.95569	1.10742	2836.9	640421
904	817216	738763264	30.0666	9.6692	2.95617	1.10619	2840.0	641840
905	819025	741217625	30.0832	9.6727	2.95665	1.10497	2843.1	643261
906	820836	743677416	30.0998	9.6763	2.95713	1.10375	2846.3	644683
907	822649	746142643	30.1164	9.6799	2.95761	1.10254	2849.4	646107
908	824464	748613312	30.1330	9.6834	2.95809	1.10132	2852.6	647533
909	826281	751089429	30.1496	9.6870	2.95856	1.10011	2855.7	648960
910	828100	753571000	30.1662	9.6905	2.95904	1.09890	2858.8	650388
911	829921	756058031	30.1828	9.6941	2.95952	1.09769	2862.0	651818
912	831744	758550528	30.1993	9.6976	2.95999	1.09649	2865.1	653250
913	833569	761048497	30.2159	9.7012	2.96047	1.09529	2868.3	654684
914	835396	763551944	30.2324	9.7047	2.96095	1.09409	2871.4	656118
915	837225	766060875	30.2490	9.7082	2.96142	1.09290	2874.6	657555
916	839056	768575296	30.2655	9.7118	2.96190	1.09170	2877.7	658993
917	840889	771095213	30.2820	9.7153	2.96237	1.09051	2880.8	660433
918	842724	773620632	30.2985	9.7188	2.96284	1.08932	2884.0	661874
919	844561	776151559	30.3150	9.7224	2.96332	1.08814	2887.1	663317
920	846400	778688000	30.3315	9.7259	2.96379	1.08696	2890.3	664761
921	848241	781229961	30.3480	9.7294	2.96426	1.08578	2893.4	666207
922	850084	783777448	30.3645	9.7329	2.96473	1.08460	2896.5	667654
923	851929	786330467	30.3809	9.7364	2.96520	1.08342	2899.7	669103
924	853776	788889024	30.3974	9.7400	2.96567	1.08225	2902.8	670554
925	855625	791453125	30.4138	9.7435	2.96614	1.08108	2906.0	672006
926	857476	794022776	30.4302	9.7470	2.96661	1.07991	2909.1	673460
927	859329	796597983	30.4467	9.7505	2.96708	1.07875	2912.3	674915
928	861184	799178752	30.4631	9.7540	2.96755	1.07759	2915.4	676372
929	863041	801765089	30.4795	9.7575	2.96802	1.07643	2918.5	677831
930	864900	804357000	30.4959	9.7610	2.96848	1.07527	2921.7	679291
931	866761	806954491	30.5123	9.7645	2.96895	1.07411	2924.8	680752
932	868624	809557568	30.5287	9.7680	2.96942	1.07296	2928.0	682216
933	870489	812166237	30.5450	9.7715	2.96988	1.07181	2931.1	683680
934	872356	814780504	30.5614	9.7750	2.97035	1.07066	2934.2	685147
935	874225	817400375	30.5778	9.7785	2.97081	1.06952	2937.4	686615
936	876096	820025856	30.5941	9.7819	2.97128	1.06838	2940.5	688084
937	877969	822656953	30.6105	9.7854	2.97174	1.06724	2943.7	689555
938	879844	825293672	30.6268	9.7889	2.97220	1.06610	2946.8	691028
939	881721	827936019	30.6431	9.7924	2.97267	1.06496	2950.0	692502
940	883600	830584000	30.6594	9.7959	2.97313	1.06383	2953.1	693978
941	885481	833237621	30.6757	9.7993	2.97359	1.06270	2956.2	695455
942	887364	835896888	30.6920	9.8028	2.97405	1.06157	2959.4	696934
943	889249	838561807	30.7083	9.8063	2.97451	1.06045	2962.5	698415
944	891136	841232384	30.7246	9.8097	2.97497	1.05932	2965.7	699897
945	893025	843908625	30.7409	9.8132	2.97543	1.05820	2968.8	701380
946	894916	846590536	30.7571	9.8167	2.97589	1.05708	2971.9	702865
947	896809	849278123	30.7734	9.8201	2.97635	1.05597	2975.1	704352
948	898704	851971392	30.7896	9.8236	2.97681	1.05485	2978.2	705840
949	900601	854670349	30.8058	9.8270	2.97727	1.05374	2981.4	707330

SQUARES, CUBES, SQUARE ROOTS, ETC.—Concluded

No.	Square	Cube	Square Root	Cube Root	Log.	1000 x Recip.	No. = Dia. Circum.	No. = Dia. Area
950	902500	857375000	30.8221	9.8305	2.97772	1.05263	2984.5	708822
951	904401	860085351	30.8383	9.8339	2.97818	1.05152	2987.7	710315
952	906304	862801408	30.8545	9.8374	2.97864	1.05042	2990.8	711809
953	908209	865523177	30.8707	9.8408	2.97909	1.04932	2993.9	713306
954	910116	868250664	30.8869	9.8443	2.97955	1.04822	2997.1	714803
955	912025	870983875	30.9031	9.8477	2.98000	1.04712	3000.2	716303
956	913936	873722816	30.9192	9.8511	2.98046	1.04603	3003.4	717804
957	915849	876467493	30.9354	9.8546	2.98091	1.04493	3006.5	719306
958	917764	879217912	30.9516	9.8580	2.98137	1.04384	3009.6	720810
959	919681	881974079	30.9677	9.8614	2.98182	1.04275	3012.8	722316
960	921600	884736000	30.9839	9.8648	2.98227	1.04167	3015.9	723823
961	923521	887503681	31.0000	9.8683	2.98272	1.04058	3019.1	725332
962	925444	890277128	31.0161	9.8717	2.98318	1.03950	3022.2	726842
963	927369	893056347	31.0322	9.8751	2.98363	1.03842	3025.4	728354
964	929296	895841344	31.0483	9.8785	2.98408	1.03734	3028.5	729867
965	931225	898632125	31.0644	9.8819	2.98453	1.03627	3031.6	731382
966	933156	901428696	31.0805	9.8854	2.98498	1.03520	3034.8	732899
967	935089	904231063	31.0966	9.8888	2.98543	1.03413	3037.9	734417
968	937024	907039232	31.1127	9.8922	2.98588	1.03306	3041.1	735937
969	938961	909853209	31.1288	9.8956	2.98632	1.03199	3044.2	737458
970	940900	912673000	31.1448	9.8990	2.98677	1.03093	3047.3	738981
971	942841	915498611	31.1609	9.9024	2.98722	1.02987	3050.5	740506
972	944784	918330048	31.1769	9.9058	2.98767	1.02881	3053.6	742032
973	946729	921167317	31.1929	9.9092	2.98811	1.02775	3056.8	743559
974	948676	924010424	31.2090	9.9126	2.98856	1.02669	3059.9	745088
975	950625	926859375	31.2250	9.9160	2.98900	1.02564	3063.1	746619
976	952576	929714176	31.2410	9.9194	2.98945	1.02459	3066.2	748151
977	954529	932574833	31.2570	9.9227	2.98989	1.02354	3069.3	749685
978	956484	935441352	31.2730	9.9261	2.99034	1.02249	3072.5	751221
979	958441	938313739	31.2890	9.9295	2.99078	1.02145	3075.6	752758
980	960400	941192000	31.3050	9.9329	2.99123	1.02041	3078.8	754296
981	962361	944076141	31.3209	9.9363	2.99167	1.01937	3081.9	755837
982	964324	946966168	31.3369	9.9396	2.99211	1.01833	3085.0	757378
983	966289	949862087	31.3528	9.9430	2.99255	1.01729	3088.2	758922
984	968256	952763904	31.3688	9.9464	2.99300	1.01626	3091.3	760466
985	970225	955671625	31.3847	9.9497	2.99344	1.01523	3094.5	762013
986	972196	958585256	31.4006	9.9531	2.99388	1.01420	3097.6	763561
987	974169	961504803	31.4166	9.9565	2.99432	1.01317	3100.8	765111
988	976144	964430272	31.4325	9.9598	2.99476	1.01215	3103.9	766662
989	978121	967361669	31.4484	9.9632	2.99520	1.01112	3107.0	768214
990	980100	970299000	31.4643	9.9666	2.99564	1.01010	3110.2	769769
991	982081	973242271	31.4802	9.9699	2.99607	1.00908	3113.3	771325
992	984064	976191488	31.4960	9.9733	2.99651	1.00806	3116.5	772882
993	986049	979146657	31.5119	9.9766	2.99695	1.00705	3119.6	774441
994	988036	982107784	31.5278	9.9800	2.99739	1.00604	3122.7	776002
995	990025	985074875	31.5436	9.9833	2.99782	1.00503	3125.9	777564
996	992016	988047936	31.5595	9.9866	2.99826	1.00402	3129.0	779128
997	994009	991026973	31.5753	9.9900	2.99870	1.00301	3132.2	780693
998	996004	994011992	31.5911	9.9933	2.99913	1.00200	3135.3	782260
999	998001	997002999	31.6070	9.9967	2.99957	1.00100	3138.5	783828

APPENDIX E

GLOSSARY—INDEX

ABSCISSA—Chap. 5, par. 127.

ACUTE ANGLE—Chap. 1, par. 6.

ADJACENT—Lying close to, but not necessarily in contact with. Chap. 1, par. 11.

ALPHA (α)—Greek letter used to designate angles. Chap. 8, par. 257.

ALTITUDE—The perpendicular distance from the base of a figure to the summit.

ANGLE(S)—Chap. 1, par. 2.
- ACUTE—Chap. 1, par. 6.
- COMPLEMENTARY—Chap. 3, par. 56–57.
- CORRESPONDING—Chap. 3, par. 43.
- OF DEPRESSION—Chap. 8, par. 239.
- OF ELEVATION—Chap. 8, par 239.
- OBTUSE—Chap. 1, par. 6.
- RIGHT—Chap. 1, par. 6.
- STRAIGHT—Chap. 1, par. 6.
- TANGENT OF—Chap. 8, par. 233.
- SUPPLEMENTARY—Chap. 3, par. 56–57.
- VERTICAL—Chap. 3, par. 58.

ANGULAR VELOCITY—Chap. 8, par. 272–273.

ARC—A segment, or piece, of a curve. Chap. 2, par. 18.

AREA—The surface contents of any figure.
- OF CIRCLE—Chap. 6, par. 171.
- OF IRREGULAR OBJECT—Chap. 6, par. 180.
- OF PARALLELOGRAM—Chap. 6, par. 163.
- OF RECTANGLE—Chap. 6, par. 162.
- OF SPHERE—Chap. 6, par. 179.
- OF SQUARE—Chap. 6, par. 161.
- OF TRAPEZOID—Chap. 6, par. 166.
- OF TRIANGLE—Chap. 6, par. 164.
- UNDER SINE CURVE—Chap. 10, par. 311.

BAR GRAPH—Chap. 5, par. 127.

BASE-LINE METHOD (dimensions)—Chap. 4, par. 116.

BEARING—The direction of one point or object with respect to another, or to the points of the compass.
- RELATIVE BEARING—Chap. 1, par. 11.
- TRUE BEARING—Chap. 1, par. 10.

BISECT—To divide into two equal parts.

BISECTOR—Chap. 2, par. 27.

CALIBRATION—The process of determining the correct values for different scale readings of a meter or other device. Also the determination of the markings of a dial which correspond to specific measurements, such as those of voltage, current, frequency, and so forth.

CALIBRATION CURVE—Chap. 5, par. 152–153.

CAPACITANCE—Chap. 5, par. 157.

CAPACITIVE REACTANCE—Chap. 7, par. 228.

CAPACITOR—Chap. 7, par. 206.

CARDINAL POINTS (of compass)—Chap. 1, par. 10.

CENTER-TAPPED—Chap. 4, par. 101.

CHASSIS PUNCH—Chap. 4, par. 119–120.

CHORD—The part of a straight line between two points at which the straight line intersects a curve. Chap. 2, par. 24.

CIRCLE—A closed plane curve, all points of which are equally distant from a fixed point in the plane called the center. Chap. 6, par. 171.

AREA OF—Chap. 6, par. 171.

TANGENT TO—Chap. 2, par. 38.

CIRCLE GRAPH—Chap. 5, par. 157.

CIRCULAR MIL (cir mil or c.m.)—Chap. 6, par. 172–173.

COMBINATION SQUARE—Chap. 4, par. 117.

COMPASS—(1) A device for determining direction by means of a magnetic needle swinging on a free pivot and pointing to the magnetic north. (2) An instrument for describing circles, transferring measurements, and so forth. Chap. 2, par. 18.

CARD—A circular card of a mariner's compass on which are marked the 32 points and the 360° of a circle. Chap. 1, par. 11.

ROSE—Chap. 3, par. 43.

COMPLEMENTARY ANGLES—Chap. 3, par. 56–57.

COMPLEX NUMBER—Chap. 9, par. 297.

COMPUTE—To determine by calculation. Chap. 10.

CONGRUENT—Coinciding throughout (said of figures).

TRIANGLES—Chap. 2, par. 39.

CONTINUOUS LINE METHOD (dimensions)—Chap. 4, par. 115.

COORDINATE SYSTEM—Chap. 5, par. 127.

CORRESPONDING ANGLES—Chap. 3, par. 43.

COSECANT (csc)—Chap. 8, par 236.

COSINE (cos)—Chap. 8, par. 236.

COTANGENT (cot)—Chap. 8, par. 236.

COUNTERCLOCKWISE (CCW)—Chap. 7, par. 221.

COUNTERELECTROMOTIVE FORCE—Chap. 7, par. 213.

CURVES, FAMILY OF—Chap. 5, par. 142.

CYCLE—Chap. 7, par. 207.

DECIBEL (db)—One-tenth of a bel. A unit used in electrical work for measuring relative signal strength. Chap. 5, par. 149.

DEGREE (°)—Chap. 1, par. 7.

DEPENDENT VARIABLE—Chap. 10, par. 300.

DEPRESSION, ANGLE OF—Chap. 8, par. 239.

DERIVATIVE—Anything obtained or deduced from another. Chap. 10, par. 319.

DERIVATIVES, TABLE OF—Chap. 10, par. 325.

DIAGONAL—Chap. 2, par. 31.

DIFFERENTIAL—An infinitesimally small change assigned to a variable. Chap. 10, par. 304.

DIFFERENTIAL CALCULUS—Chap. 10, par. 315.

DIFFERENTIATION—Chap. 10, par. 322.

DIFFERENTIATOR CIRCUIT—Chap. 10, par. 330.

DIMENSION—Measure in a single line, as length, width, height, or circumference. Something having length only is said to be of one dimension; having length and width, two dimensions; having volume, three dimensions.

DIVIDERS—An instrument, consisting of two adjustable legs meeting at an angle. It is used to divide lines and lay off distances on drawings. Chap. 2, par. 18.

DIVISION OF POLAR VECTORS—Chap. 7, par. 229.

DRAFTSMAN—One who draws plans or sketches, as of machinery or structures.

DUPLICATE—To make a copy or transcript of.

ELECTRONIC INTEGRATION—Chap. 10, par. 313.

ELEVATION, ANGLE OF—Chap. 8, par. 239.

ENGINEER'S SCALE—Chap. 4, par. 122.

EQUIDISTANT—Equally distant.

EQUILATERAL—Having all sides equal.

TRIANGLE—Chap. 2, par. 35.

FAMILY OF CURVES—Chap. 5, par. 142.

FIELD-CHANGE KITS—Chap. 4, par. 122.

FREQUENCY—Chap. 7, par. 207.

FUNCTION—A magnitude the values of which correspond to the values of another magnitude. Chap. 7, par. 10.

GAUGE—An instrument for the measurement of dimensions or other physical characteristics. Chap. 3.

GENERATE—To trace out (a line, figure, or solid), as by the motion of a point.

GENERATOR—A machine that changes mechanical energy into electrical energy. Chap. 7.

GEOMETRIC—According to principles of geometry.

GEOMETRY—That branch of mathematics which investigates the relations, properties, and measurement of solids, surfaces, lines, and angles.

GRAPH—A pictorial presentation of the relation between two or more variable quantities.

BAR—Chap. 5, par. 127.

CIRCLE—Chap. 5, par. 131.

LINE—Chap. 5, par. 133.

POLAR—Chap. 5, par. 147.

GRAPHIC SCALE—Chap. 4, par. 101.

GRID VOLTAGE-PLATE CURRENT CURVE (E_g—I_p)—Chap. 5, par. 144.

GYROCOMPASS—A compass consisting of a continuously driven gyroscope whose spinning axis is confined to a horizontal plane, so that the earth's rotation causes it to assume a position parallel to the earth's axis and thus point to true north.

GYROSCOPE—A wheel or disk mounted to spin rapidly about an axis, and free to rotate about one or both of two axes perpendicular to each other and to the axis of spin.

HALF-SIZE SCALE—Chap. 4, par. 83.

HEADING—Direction in which a ship advances or points, as the ship's heading is true north. Chap. 1.

HEXAGON—A closed plane figure bounded by six straight lines. Chap. 2, par. 34.

HORIZONTAL COMPONENT—Chap. 9, par. 295.

HYPOTENUSE—Side of a right triangle opposite the right angle.

IMPEDANCE—Chap. 7, par. 230.

INDEPENDENT VARIABLE—Chap. 10, par. 300.

INDUCTANCE—Chap. 5, par. 144; Chap. 7.

INDUCTIVE REACTANCE—Chap. 7, par. 228.

INDUCTOR—Chap. 7, par. 206.

INFINITESIMAL—Extremely small, approaching zero as a limit. Chap. 10, par. 321.

IN PHASE—Chap. 7, par. 207.

INSTANTANEOUS AMPLITUDE—Chap. 8, par. 255.

INTEGRAL CALCULUS—Chap. 10, par. 305.

INTEGRALS, TABLE OF—Chap. 10, par. 310.

INTEGRATION—Chap. 10, par. 307.

INTERCARDINAL POINTS—Chap. 1, par. 11.

IRREGULAR OBJECT, AREA OF—Chap. 6, par. 180.

ISOSCELES TRIANGLE—Chap. 3, par. 53.

KEY—A small section of a scale drawn in the lower right-hand corner of a drawing. Chap. 4, par. 78.

LAYOUT—An outline, usually with a diagram providing directions for work. Chap. 4, par. 110.

LIMITS—Chap. 10, par. 302.

LINE GRAPH—Chap. 5, par. 133.

LIST—To lean to one side or the other, as the ship lists to port.

LOCATION DIMENSIONS—Chap. 4, par. 114.

LOWER LIMIT—Chap. 10, par. 306.

LUBBER LINE—Chap. 1, par. 15.

MANEUVERING BOARD—Chap. 5, par. 151.

MIL—Chap. 6, par. 172.

MINUTE (′)—Chap. 1, par. 7.

MULTIPLICATION OF POLAR VECTORS—Chap. 7, par. 229.

NEGATIVE—Chap. 7, par. 207.

NEGATIVE-POWER CURVE—Chap. 7, par. 215.

NOMOGRAPH—Chap. 5, par. 154.

OBLIQUE—Neither perpendicular nor horizontal; slanting. Chap. 3, par. 46; Chap. 6, par. 163.

OBTUSE ANGLE—Chap. 1, par. 6.

OCTAGON—A closed plane figure bounded by eight straight lines. Chap. 2, par. 33.

OCTAL BASE—Chap. 4, par. 91.

OHMMETER—Chap. 4, par. 93.

ORDINATE—Chap. 5, par. 127.

OSCILLOSCOPE—An apparatus for showing visually on the screen of a cathode ray tube, wave forms of a rapidly varying quantity such as an alternating voltage. Chap. 10, par. 314.

OUT OF PHASE—Chap. 7, par. 210.

PARABOLA—A conic section; the intersection of a cone with a plane parallel to its side. Chap. 10, par. 301.

PARALLAX—Chap. 1, par. 15; Chap. 4, par. 118.

PARALLEL LINES—Chap. 3, par. 43.

PARALLELOGRAM—Chap. 3, par. 46.

  AREA OF—Chap. 6, par. 163.

  METHOD OF VECTOR ADDITION—Chap. 9, par. 284.

PARALLEL RULES—Chap. 3, par. 43.

PELORUS—Chap. 1, par. 10.

PENTAGON—A closed plane figure bounded by five straight lines. Chap. 2, par. 36.

PERPENDICULAR ($\perp$)—Chap. 2, par. 23.

PERPENDICULAR BISECTOR—Chap. 2, par. 23.

PHASE—Chap. 7.

  IN PHASE—Chap. 7, par. 207.

  OUT OF PHASE—Chap. 7, par. 210.

PHI (Φ)—Greek letter used to designate angles. Chap. 8, par. 234.

PIVOT—A point, fixed pin, or short axis about which something turns.

PLANE—A surface in which, if any two points are taken, the straight line that joins them lies wholly in that surface. Chap. 3.

PLANIMETER—Chap. 10, par. 312.

PLATE CHARACTERISTIC—Chap. 5, par. 142.

POLAR GRAPH—Chap. 5, par. 147.

POLAR VECTOR—Chap. 7, par. 229.

DIVISION OF—Chap. 7, par. 229.

MULTIPLICATION OF—Chap. 7, par. 229.

POLYGON—A figure, generally plane and closed, having many angles, and hence many sides. Chap. 3, par. 55.

METHOD OF VECTOR ADDITION—Chap. 9, par. 285.

POSITIVE—Chap. 7, par. 207.

POSITIVE-POWER CURVE—Chap. 7, par. 215.

PRICK PUNCH—Chap. 4, par. 119.

PRIMARY—Chap. 4, par. 104.

PRIME—A symbol (′) placed to the right of and above a letter or number. Used to denote (1) different variables with the same letter, as X, X′, and so forth; (2) feet, as 2′, read 2 feet; (3) minutes in measuring angles, as 3°10′, read 3 degrees 10 minutes.

PROPORTIONAL—Chap. 4, par. 61.

PROTRACTOR—Chap. 1, par. 9.

QUADRANT—The quarter of a circle, an arc of 90°, Chap. 8, par. 258.

QUADRILATERAL—A plane figure of four sides and four angles. Chap. 2, par. 31.

QUARTER-SIZE SCALE—Chap. 4, par. 83.

RADIAN—Chap. 8, par. 272.

RADIATION PATTERN—Chap. 5, par. 149.

RADIUS—Chap. 2, par. 19.

RATE OF CHANGE—Chap. 10, par. 323.

RATIO—Chap. 4, par. 61.

REACTANCE—The opposition in ohms offered to the flow of alternating current by inductance or capacitance in a circuit. Chap. 7, par. 206.

RECTANGLE—Chap. 2, par. 30.

AREA OF—Chap. 6, par. 162.

RECTANGULAR COMPONENTS—Chap. 9, par. 293.

RECTANGULAR COORDINATES—Chap. 5, par. 127.

REGULAR FIGURE—Chap. 2, par. 32.

RELATIVE BEARING—Chap. 1, par. 11.

RELATIVE-POWER OUTPUT—Chap. 5, par. 134.

REPRESENTATIVE FRACTION (RF)—Chap. 4, par. 74.

RHOMBUS—Chap. 3, par. 46.

RIGHT ANGLE—Chap. 1, par. 6.

RIGHT TRIANGLE—Chap. 3, par. 53.

ROTATE—To turn, as a wheel, round an axis.

SCALAR—A quantity fully described by a number. Chap. 9.

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